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Head of the Department
Department of Electrical & Electronics Engg
Vidya Jyothi Institute of Technology
HYDERABAD-500 075.

PRINCIPAL
Vidya Jyothi Institute of Technology
Himayatnagar (Vill), C.B. Post.,
Hyderabad-75.

Vidya Jyothi Institute of Technology

(An Autonomous Institution)

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Aziz Nagar Gate, C.B. Post, Hyderabad-500 075

DEPARTMENT OF ELECTRICAL AND ELECTRONICS ENGINEERING

Vision of the Institution

- To develop into a reputed Institution at National and International level in Engineering, Technology and Management by generation and dissemination of knowledge through intellectual, cultural and ethical efforts with human values.
- To fosters scientific temper in promoting the world class professional and technical expertise.

Mission of the Institution

- To create state-of-the-art infrastructure facilities for optimization of knowledge acquisition.
- To nurture the students holistically and make them competent to excel in the global scenario.
- To promote R&D and consultancy through strong industry-institute interaction to address the societal problems.

Name of the Faculty: V. Vijaya Lakshmi **Designation:** Assoc. Prof

Program & Regulation: B.Tech & R19 **Academic Year:** 2020-2021

Course Code: A33204 **Course Name:** Network Analysis **Credits:** 03

Department: EEE **Year** II **Semester** I **Section** – A

Vision of the Department

- To become a reputed department in the impartation of professional and technical expertise in the field of Electrical and Electronics Engineering

Mission of the Department

- Imparting Quality Technical Education by provision of state-of-the-art laboratories.
- Preparing the students to think innovatively and find effective solutions to address engineering and societal problems with a multi-disciplinary approach maintaining continuous industry interaction
- Encouraging team work and preparing the students for lifelong learning with ethical responsibility for a successful professional career.

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DEPARTMENT OF ELECTRICAL AND ELECTRONICS ENGINEERING

Program Educational Objectives:

. **PEO1:** Equip graduates with a sound foundation in mathematics, science and engineering fundamentals, necessary to build a protective career

PEO2: Graduates will excel in giving solutions to real time problems through technical expertise and operational skill set in the field of Electrical Engineering

PEO3: Graduates will act with integrity in catering the need based requirements blended with ethics and professionalism.

Program Outcomes (POs)

Engineering Graduates will be able to:

1. Engineering knowledge: Apply the knowledge of mathematics, science, engineering fundamentals, and an engineering specialization to the solution of complex engineering problems.

2. Problem analysis: Identify, formulate, review research literature, and analyze complex engineering problems reaching substantiated conclusions using first principles of mathematics, natural sciences, and engineering sciences.

3. Design/development of solutions: Design solutions for complex engineering problems and design system components or processes that meet the specified needs with appropriate consideration for the public health and safety, and the cultural, societal, and environmental considerations.

4. Conduct investigations of complex problems: Use research-based knowledge and research methods including design of experiments, analysis and interpretation of data, and synthesis of the information to provide valid conclusions.

5. Modern tool usage: Create, select, and apply appropriate techniques, resources, and modern engineering and IT tools including prediction and modeling to complex engineering activities with an understanding of the limitations.

6. The engineer and society: Apply reasoning informed by the contextual knowledge to assess societal, health, safety, legal and cultural issues and the consequent responsibilities relevant to the professional engineering practice.

7. Environment and sustainability: Understand the impact of the professional engineering solutions in societal and environmental contexts, and demonstrate the knowledge of, and need for sustainable development.

8. Ethics: Apply ethical principles and commit to professional ethics and responsibilities and norms of the engineering practice.

9. Individual and team work: Function effectively as an individual, and as a member or leader in diverse teams, and in multidisciplinary settings.

10. Communication: Communicate effectively on complex engineering activities with the engineering community and with society at large, such as, being able to comprehend and write effective reports and design documentation, make effective presentations, and give and receive clear instructions.

11. Project management and finance: Demonstrate knowledge and understanding of the engineering and management principles and apply these to one's own work, as a member and leader in a team, to manage projects and in multidisciplinary environments.

12. Life-long learning: Recognize the need for, and have the preparation and ability to engage in independent and life-long learning in the broadest context of technological change.

Program Specific Outcomes (PSOs)

PSO 1: Apply the fundamentals of Electrical and Electronics Engineering to analyze and synthesize problems of Electric Circuits, Electronic Circuits, Control Systems, Electrical Machines and Power Systems.

PSO 2: Apply the appropriate techniques and modern engineering hardware and software tools in Electrical Engineering to engage in life-long learning and to successfully adapt in multi-disciplinary environments.

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B.Tech II & III Year Revised Academic Calendar for the Academic Year 2020-21

FIRST SEMESTER		Commencement of Class Work 17.07.2020	
	FROM	**TO**	**DURATION**
I Spell of Instructions (Online)	17.07.2020	09.10.2020	12 WEEKS
Mid -II & End Semester Examinations of Previous Semester	14.10.2020	12.11.2020	5 WEEKS
Practical Examinations of Previous Semester	16.11.2020	21.11.2020	1 WEEK
Revision of Syllabi of Current Semester	23.11.202	05.12.2020	2 WEEKS
Betterment Examinations of Previous Semester	02.12.2020	05.12.2020	4 DAYS
I Mid Examinations of Current Semester	07.12.2020	15.12.2020	1 WEEK
Practical Classes of Current Semester	16.12.2020	19.12.2020	4 DAYS
II Spell of Instructions (Online)	21.12.2020	20.02.2021	9 WEEKS
Practical Examinations	24.02.2021	03.03.2021	1 WEEK
II Mid & End Semester Examinations	05.03.2021	22.03.2021	2 WEEKS
Betterment Examinations	24.03.2021	27.03.2021	4 DAYS
SECOND SEMESTER		**Commencement of Class Work 30.03.2021**	
I Spell of Instructions	30.03.2021	22.05.2021	8 WEEKS
I Mid Examinations	24.05.2021	29.05.2021	1 WEEK
II Spell of Instructions	31.05.2021	24.07.2021	8 WEEKS
II Mid Examinations	26.07.2021	31.07.2021	1 WEEK
Practical Examinations	02.08.2021	07.08.2021	1 WEEK
Betterment Examinations	09.08.2021	12.08.2021	4 DAYS
End Semester Examinations	13.08.2021	28.08.2021	2 WEEKS

COE **DEAN EXAMS.** **DIRECTOR**

Course Objectives

- To introduce the basic concepts of circuit analysis which is the foundation for all of the Electrical Engineering discipline
- To emphasize on the basic analysis of circuits which includes three phase circuits, two port networks, transient analysis and network topology

Course Outcomes (COs)

CO1	Apply network theorems for the analysis of electrical networks.
CO2	Obtain the transient and steady-state response of electrical circuits.
CO3	Apply graph theory to formulate network equations.
CO4	Analyze two port networks.
CO5	Evaluate circuits in the sinusoidal steady-state (Three-phase).

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DEPARTMENT OF ELECTRICAL AND ELECTRONICS ENGINEERING

COs Mapping with POs & PSOs

Course Outcomes	Program Outcomes												Program Specific Outcomes	
	1	2	3	4	5	6	7	8	9	10	11	12	PSO1	PSO2
CO1	3	3	2	3	3	-	-	-	-	-	-	3	1	1
CO2	3	3	2	3	3	-	-	-	-	-	-	3	1	1
CO3	3	3	2	2	-	-	-	-	-	-	-	3	1	1
CO4	3	3	2	3	3	-	-	-	-	-	-	3	1	1
CO5	3	3	2	2	3	-	-	-	-	-	-	3	1	1
Average	3	3	2	2.6	3	-	-	-	-	-	-	3	1	1

Assessment Plan

S.No.	Test/Examination	Units/ Topics Covered	COs covered	Proposed Date	Maximum Marks
1	Assignment I	Unit-1,Unit-2 and Unit-3(Half)	CO1,CO2 and CO3	7.10.2020	5
2	Mid I	Unit-1,Unit-2 and Unit-3(Half)	CO1,CO2 and CO3	14.12.2020	20
3	Assignment II	Unit-3(Half),Unit-4 and Unit-5	CO3,CO4 and CO5	15.2.2021	5
4	Mid II	Unit-3(Half),Unit-4 and Unit-5	CO3,CO4 and CO5	20.3.2021	20

Direct Assessment (Internal Examination & External Examination)	Indirect Assessment (Course End Survey)
2.28	2.82

Course Faculty | Course Co-Ordinator | HOD

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DEPARTMENT OF ELECTRICAL&ELECTRONICS ENGINEERING

Academic year: 2020-21 **Year& Semester: II B.Tech- I Sem**
Section: EEE-A **W. E. F: 18-07-2020**

ONLINE CLASSES TIME TABLE

Day/ Hours	9.30 AM to 10.30 AM	10.45 AM to 11.45 AM	12.00 PM to 01.00 PM	02.00 PM to 03.00 PM
MON	EMF	NA	CAFT	MC-I
TUE	EDC	PS-I	EM-I	-------
WED	NA	CAFT	EMF	-------
THU	PS-I	EM-I	EDC	-------
FRI	CAFT	NA	EMF	MC-I
SAT	EM-I	EDC	PS-I	-------

Subject **Name of the Faculty**

CAFT	Complex Analysis & Fourier Transforms	Mrs.Ch.Sridevi
EDC	Electronic Devices & Circuits	Mrs.A.Pavithra
PS-I	Power Systems-I	Mr.S.Suresh
NA	Network Analysis	Mrs.V.Vijaya Lakshmi
EMF	Electro Magnetic Fields	Mr.B.Sudhakar Reddy
EM- I	Electrical Machines–I	Mr.P.Naga Muneendra
MC-I	Gender Sensitization	Mrs.M.Hepsiba

Class In charge **Mrs.V.Vijaya Lakshmi**

Time Table I/C **H.O.D**

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DEPARTMENT OF ELECTRICAL AND ELECTRONICS ENGINEERING

Lesson Plan

Name of the Faculty: V.Vijaya Lakshmi **Year/ Sem:**II-I

Course Name : Network Analysis **Course Code: A33204**

S .No	Lecture Hour	Teaching Aids required	Topics to be covered	Books no./Page No.
		Unit-I: Network Theorems(Dc & Ac), Mesh And Nodal Analysis		
1	L1	Online Class(PPT)	Introduction	TB1(62-93)
2	L2&L3	Online Class(GJB)	Analysis of Circuits using mesh methods	TB1(62-93)
3	L4&L5	Online Class(GJB)	Analysis of Circuits using Nodal methods,	TB1(62-93)
4	L6 & L7	Online Class(PPT)	Norton's theorem	TB1(111-120)
5	L8	Online Class(PPT)	Maximum Power Transfer theorem	TB1(127-130)
6	L9	Online Class(PPT)	Reciprocity theorem	TB1(133-134)
7	L10	Online Class(PPT)	Millmann's theorem	TB1(131-132)
8	L11	Online Class(PPT)	Compensation theorem	TB1(135-136)
9	L12	Online Class(GJB)	Problems	TB1(62-136)
		Unit-II: D.C and A.C Transient Analysis		
10	L13	Online Class(PPT)	Introduction	TB1(333-338)
11	L14	Online Class(GJB)	Transient response of series and parallel R-L circuits for D.C excitation- Initial conditions- Solution method using differential equation	TB1(339-345)
12	L15	Online Class(GJB)	Transient response of series and parallel R-C circuits for DC Excitation using differential equation-Initial conditions	TB1(349-350)
13	L16 & L17	Online Class(GJB)	Transient response of series R-L-C circuits for DC Excitation using differential equation-Initial conditions	TB1(333-338)

14	L18	Online Class(GJB)	Transient response of series and parallel R-L circuits for D.C excitation- Initial conditions- Solution method using laplace transforms	TB1(339-345)
15	L19	Online Class(GJB)	Transient response of series R-C circuits for DC Excitation using laplace transforms	TB1(349-350)
16	L20 & L21	Online Class(GJB)	Transient response of series R-L-C circuits for DC Excitation using laplace transforms	TB1(349-350)
17	L23	Online Class(GJB)	Transient response of series and parallel R-L circuits for sinusoidal excitation- Initial conditions- Solution method using differential	TB1(346-348)
18	L24	Online Class(GJB)	Transient response of series and parallel R-C circuits for sinusoidal Excitation using differential equation-Initial conditions	TB1(350-351)
19	L25	Online Class(GJB)	Transient response of series R-L-C circuits for sinusoidal Excitation using differential equation- Initial conditions	TB1(352-353)
20	L26	Online Class(GJB)	Transient response of series and parallel R-L circuits for sinusoidal excitation- Initial conditions- Solution method using laplace transforms	TB1(346-348)
21	L27	Online Class(GJB)	Transient response of series R-C circuits for sinusoidal Excitation using laplace transforms	TB1(350-351)
22	L28	Online Class(GJB)	Transient response of series R-L-C circuits for sinusoidal1 Excitation using laplace transforms	TB1(352-353)
23	L29 & L30	Online Class(GJB)	Problems	
			Unit-III: Network Topology	
24	L31	Online Class(PPT)	Introduction	TB1(691-697)
25	L32	Online Class(PPT)	Network Topology - Definitions, Graph, Tree	TB1(691-697)
26	L33 & L34	Online Class(PPT)	Incidence Matrix, Basic Cut Set Matrices for planar networks	TB1(698-702)
27	L35 & L36	Online Class(PPT)	Basic Tie set Matrices for planar networks	TB1(698-702)
28	L37 & L38	Online Class(PPT)	Loop method for analysis of Networks with Voltage and Current Sources	TB1(706-710)
29	L39 & L40	Online Class(PPT)	Nodal method for analysis of Networks with Voltage and Current Sources	TB1(706-710)
30	L41	Online Class(PPT)	Duality & Dual Networks	TB1(711-725)
31	L42 & L43	Online Class(GJB)	Problems	TB1(706-710)

Unit- IV: Two Port Networks				
32	L44	Online Class(PPT)	Introduction	TB1(566-597)
33	L45	Online	Two port network parameters – Z parameters	TB1(566-
34	L46	Online Class(PPT)	Two port network parameters – Y parameters	TB1(566-597)
35	L47	Online Class(PPT)	Two port network parameters –ABCD parameters	TB1(566-597)
36	L48	Online Class(PPT)	Two port network parameters –Hybrid parameters	TB1(566-597)
37	L49	Online Class(PPT)	Inter Relations	TB1(599-601)
38	L50	Online Class(PPT)	Series connection of two port networks	TB1(599-601)
39	L51	Online Class(PPT)	Parallel connection of two port networks	TB1(599-601)
40	L52	Online Class(PPT)	Cascaded connection of two port networks	TB1(677-680)
41	L53	Online Class(PPT)	Concept of transformed network	TB1(677-680)
42	L54	Online Class(PPT)	Two port network parameters using transformed variables	TB1(566-597)
43	L55	Online Class(PPT)	Problems	
Unit- V: Three Phase Ciruits				
44	L56	Online Class(PPT)	Introduction	TB1(301-302)
45	L57	Online Class(PPT)	Three phase Circuits – Generation of Three Phase Voltage	TB1(302-303)
46	L58	Online Class(PPT)	Review of Voltage and Current relations in Star system	TB1(307-330)
47	L59	Online Class(PPT)	Review of Voltage and Current relations in Delta system	TB1(305-306)
48	L60	Online Class(PPT)	Analysis of balanced three phase circuits- Measurement of active and reactive power	TB1(301-330)
49	L61 & L62	Online Class(PPT)	Analysis of unbalanced three phase circuits- Measurement of active and reactive power	TB1(301-302)

Faculty I/C

HOD

L - Lecture
A - Assignment
T - Text Books
R - References

GJB - Google Jam Board
PPT - Power Point Presentation
MD - Model Demo
FV - Field Visit

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DEPARTMENT OF ELECTRICAL AND ELECTRONICS ENGINEERING

Course Delivery Plan & Record of class work

Unit-I

S No	Proposed		Topics To Be Covered	Teaching Aids used	Execution	
	DATE	HOURS			DATE	HOURS
1	20.7.2020	1	Introduction	Online Class(PPT)	20.7.2020	1
2	22.7.2020&24.7.2020	2	Analysis of Circuits using mesh methods	Online Class(GJB)	22.7.2020 24.7.2020	2
3	27.7.2020&29.7.2020	2	Analysis of Circuits using Nodal methods	Online Class(GJB)	27.7.2020 29.7.2020	2
4	31.7.2020	1	Norton's theorem	Online Class(PPT)	31.7.2020	1
5	3.8.2020	1	Maximum Power Transfer theorem	Online Class(PPT)	3.8.2020	1
6	5.8.2020	1	Reciprocity theorem	Online Class(PPT)	5.8.2020	1
7	10.8.2020	1	Millmann's theorem	Online Class(PPT)	10.8.2020	1
8	12.8.2020	1	Compensation theorem	Online Class(PPT)	12.8.2020	1
9	14.8.2020	1	Problems	Online Class(GJB)	14.8.2020	1

Justification for deviation (if Any)

No deviation

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Unit-II

S No	Proposed		Topics To Be Covered	Teaching Aids used	Execution	
	DATE	HOURS			DATE	HOURS
1	17.8.2020	1	Introduction	Online Class(GJB)	17.8.2020	1
2	19.8.2020	1	Transient response of series and parallel R-L circuits for D.C excitation- Initial conditions- Solution method using differential equation	Online Class(GJB)	19.8.2020	1
3	21.8.2020	1	Transient response of series and parallel R-C circuits for DC Excitation using differential equation-Initial conditions	Online Class(GJB)	21.8.2020	1
4	24.8.2020	1	Transient response of series R-L-C circuits for DC Excitation using differential equation-Initial conditions	Online Class(GJB)	24.8.2020	1
5	26.8.2020	1	Transient response of series and parallel R-L circuits for D.C excitation- Initial conditions- Solution method using laplace transforms	Online Class(GJB)	26.8.2020	1
6	28.8.2020	1	Transient response of series R-C circuits for DC Excitation using laplace transforms	Online Class(GJB)	28.8.2020	1
7	31.8.2020	2	Transient response of series R-L-C circuits for DC Excitation using laplace transforms	Online Class(GJB)	31.8.2020	2
8	2.9.2020	1	Transient response of series and parallel R-L circuits for sinusoidal excitation- Initial conditions- Solution method using differential equation	Online Class(GJB)	2.9.2020	1
9	4.9.2020	1	Transient response of series and parallel R-C circuits for sinusoidal Excitation using differential equation-Initial conditions	Online Class(GJB)	4.9.2020	1
10	7.9.2020	1	Transient response of series R-L-C circuits for sinusoidal Excitation using differential equation-Initial conditions	Online Class(GJB)	7.9.2020	1
11	9.9.2020	1	Transient response of series and parallel R-L circuits for sinusoidal excitation- Initial conditions- Solution method using laplace transforms	Online Class(GJB)	9.9.2020	1
12	11.9.2020	1	Transient response of series R-C circuits for sinusoidal Excitation using laplace	Online Class(GJB)	11.9.2020	1

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13	14.9.2020	1	Transient response of series R-L-C circuits for sinusoidal1 Excitation using laplace transforms	Online Class(GJB)	14.9.2020		1
14	16.9.2020	.	Problems	Online Class(GJB)	16.9.2020		1

Justification for deviation (if Any)

No deviation

Course faculty

HOD

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Unit-III

S No	Proposed		Topics To Be Covered	Teaching Aids used	Execution	
	DATE	HOURS			DATE	HOURS
1	18.9.2020	1	Introduction	Online Class(PPT)	18.9.2020	1
2	28.9.2020	1	Network Topology - Definitions, Graph, Tree	Online Class(PPT)	28.9.2020	1
3	30.9.2020&5 .10.2020	2	Incidence Matrix, Basic Cut Set Matrices for planar networks	Online Class(PPT)	30.9.2020 5.10.2020	2
4	7.10.2020&9 .10.2020	2	Basic Tie set Matrices for planar networks	Online Class(PPT)	7.10.2020 9.10.2020	2
5	12.10.2020& 23.11.2020	2	Loop method for analysis of Networks with	Online Class(PPT)	12.10.2020 23.11.2020	2
6	25.11.2020	1	Nodal method for analysis of Networks with Voltage and Current Sources	Online Class(PPT)	25.11.2020	1
7	27.11.2020	1	Duality & Dual Networks	Online Class(PPT)	27.11.2020	1
8	30.11.2020	1	Problems	Online Class(GJB)	30.11.2020	1

Justification for deviation (if Any)

No deviation

Course faculty **HOD**

Unit-IV

S No	Proposed		Topics To Be Covered	Teaching Aids used	Execution	
	DATE	HOURS			DATE	HOURS
1	2.12.2020	1	Introduction	Online(PPT)	2.12.2020	1
2	3.12.2020	1	Two port network parameters – Z parameters	Online(PPT)	3.12.2020	1
3	21.12.2020	1	Two port network parameters – Y parameters	Online(PPT)	21.12.2020	1
4	23.12.2020	1	Two port network parameters – ABCD parameters	Online(PPT)	23.12.2020	1
5	25.12.2020	1	Two port network parameters – Hybrid parameters	Online(PPT)	25.12.2020	1
6	28.12.2020	1	Inter Relations	Online(PPT)	28.12.2020	1
7	30.12.2020	1	Series connection of two port networks	Online(PPT)	30.12.2020	1
8	4.1.2021	1	Parallel connection of two port networks	Online(PPT)	4.1.2021	1
9	6.1.2021	1	Cascaded connection of two port networks	Online(PPT)	6.1.2021	1
10	8.1.2021	1	Concept of transformed network	Online(PPT)	8.1.2021	1
11	11.1.2020	1	Two port network parameters using transformed variables	Online(PPT)	11.1.2021	1
12	13.1.2020	1	Problems	Online(PPT)	13.1.2021	1

Justification for deviation (if Any)

No deviation

Course faculty

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Unit-V

S No	Proposed		Topics To Be Covered	Teaching Aids used	Execution	
	DATE	HOURS			DATE	HOURS
1	15.1.2021	1	Introduction	Online Class(PPT)	15.1.2021	1
2	18.1.2021	1	Three phase Circuits – Generation of Three Phase Voltage	Online Class(PPT)	18.1.2021	1
3	20.1.2021 & 22.1.2021	2	Review of Voltage and Current relations in Star system	Online Class(PPT)	20.1.2021 22.1.2021	2
4	25.1.2021 & 27.1.2021	2	Review of Voltage and Current relations in Delta system	Online Class(PPT)	25.1.2021 27.1.2021	2
5	29.1.2021 &1.2.2021	2	Analysis of balanced three phase circuits- Measurement of active and reactive power	Online Class(PPT)	29.1.2021 1.2.2021	2
6	3.2.2021 &5.2.2021	2	Analysis of unbalanced three phase circuits- Measurement of active and reactive power	Online Class(PPT)	3.2.2021 5.2.2021	2

Justification for deviation (if Any)

No deviation

Course faculty

HOD

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DEPARTMENT OF ELECTRICAL AND ELECTRONICS ENGINEERING

Student List

II-A Section

S.No	Roll.No	Name of the Student
1	19911A0201	A Pavani
2	19911A0202	Ahankari Aditya
3	19911A0203	Ajmeera Rakesh
4	19911A0205	Algubelli Sindhu
5	19911A0206	Annaldas Aravind
6	19911A0207	Anthamgari Swathi Priya
7	19911A0208	Badagani Ganesh
8	19911A0209	Bandela Sumith
9	19911A0210	Banoth Sravan
10	19911A0211	Bhooma Rajashekar Goud
11	19911A0212	Bhukya Bhargav
12	19911A0213	Bhukya Naveen
13	19911A0214	Buchalwar Swennik
14	19911A0215	Chakali Praveen
15	19911A0216	Chennam Alekhya
16	19911A0217	Chitikela Chetana
17	19911A0218	Chittiprolu Anusha
18	19911A0219	Dara Uday Bhargav
19	19911A0220	Dasari Srinivas
20	19911A0221	Devaradeshi Udhay Kiran
21	19911A0222	Edula Bharath Kumar Goud
22	19911A0223	Gade Suprathika
23	19911A0224	Giddalapati Vamsi Ram Reddy
24	19911A0225	Goli Sai Prashanth
25	19911A0226	Gone Meghana
26	19911A0227	Gorla Anjali Raj
27	19911A0228	Govindreddy Asritha
28	19911A0229	Guguloth Kalyan
29	19911A0230	Gugulothu Rahul
30	19911A0231	Jarpala Nageswari
31	19911A0232	Jatavath Pavan
32	19911A0233	Joshi Sai Vivekanand
33	19911A0234	Kanaveni Akhila
34	19911A0235	Karike Srinu
35	19911A0236	Karukuri Sai Ram
36	19911A0237	Katna Srisailam
37	19911A0238	Kolluru Maheshwar
38	19911A0239	Koppera Vijendhar
39	19911A0240	Kottapeta Sanjeev Kumar Goud

40	19911A0241	Kurva Krishna
41	20915A0201	Boini Ramnesh
42	20915A0202	Bonike Naveen
43	20915A0203	Bowreshetty Sai Kiran
44	20915A0204	D Hemanth Sagar
45	20915A0205	G Vishnu Vardhan
46	20915A0206	Gandham Sree Charan Deep
47	20915A0207	Gumdala Manikanta Goud
48	20915A0208	Karumuru Hepsibha
49	20915A0209	Kondoju Aparna
50	20915A0210	Koppula Akanksha
51	20915A0211	Kothuri Abhishek
52	20915A0212	Kummari Yamuna
53	20915A0213	M Raghavulu

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DEPARTMENT OF ELECTRICAL AND ELECTRONICS ENGINEERING

Syllabus Covered As Per Course Delivery Plan

Details/Duration	First 4 Weeks	Second 4 Weeks	Third 4 Weeks	End Of Semester
Percentage of Syllabus covered	25%	50%	75%	100%
Signature of staff with date	[signature]	[signature]	[signature]	[signature]
Signature of HOD with date	[signature] 31/8/20	[signature] 30/11/20	[signature] 3/1/21	[signature] 5/2/21
Signature of Auditor with date	C.N. [signature] 3/1/20	C.N. [signature] 30/11/20	C.N. [signature] 3/1/21	C.N. [signature] 5/2/21

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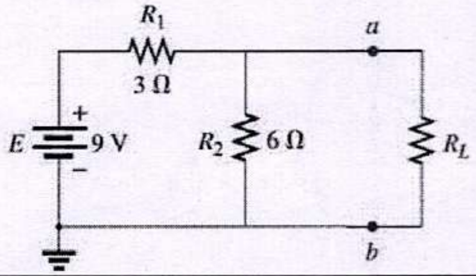
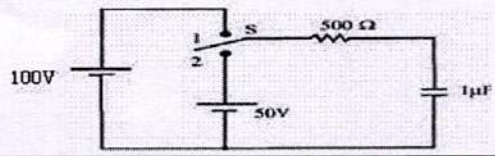
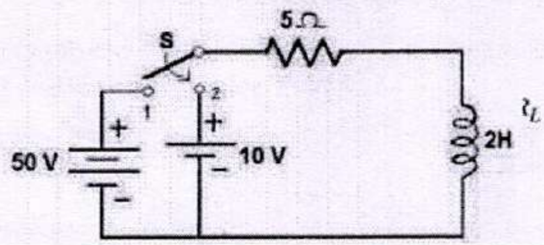
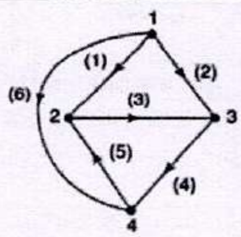
DEPARTMENT OF ELECTRICAL AND ELECTRONICS ENGINEERING

ASSIGNMENT – I (AY – 2020-21)

COURSE NAME: Network Analysis **Year & Semester: II-I**

S.No.	Questions	COs	POs	B.L
1	State Norton's Theorem and find Norton Equivalent for the circuit shown  	1	1,2,8	2
2	State and prove Maximum power transfer Theorem for both DC and AC excitation.	1	1,2,8,9	3
3	Switch is moved from position 1 to 2 at t = 0 find the voltage $V_R(t)$ and $V_C(t)$ for t=0  	2	1,2,8,9	4
4	In the circuit shown in figure, switch S is in position1 for a long time and brought to position 2 at time t=0.Determine the circuit current  	2	1,2,8,9	4
5	 For the following graph find the no.of possible trees.	3	1,2,8	2

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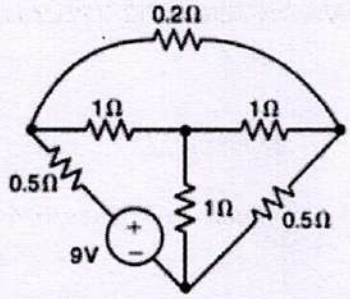
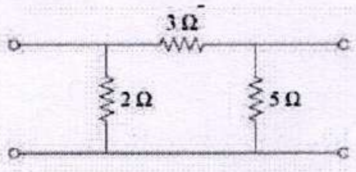
DEPARTMENT OF ELECTRICAL AND ELECTRONICS ENGINEERING

ASSIGNMENT – II (AY – 2020-21)

COURSE NAME: Network Analysis **Year & Semester: II-I**

S.No.	Questions	COs	POs	B.L
1	 Write down the incidence matrix and cutset matrices for the circuit shown in the figure	3	1,2,8	2
2	Find the Z parameters for the following networks from basics. 	4	1,2,8,9	3
3	Derive conditions for reciprocity and symmetry for Z, ABCD parameters	4	1,2,8,9	3
4	Obtain power equation using 2 wattmeter method for balanced and unbalanced systems.	5	1,2,8,9	5
5	A three phase balanced delta connected load of $(4+j8)\Omega$ is connected across a 400V, 3-Ø balanced supply. Determine the phase currents and line currents.Assume the phase of sequence to be RYB. Also calculate the power drawn by load.	5	1,2,8,9	2

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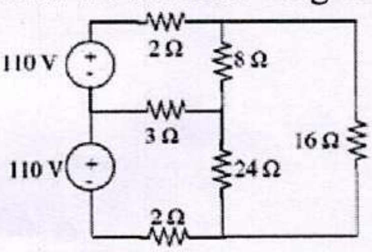
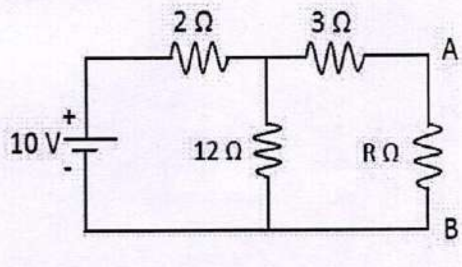
II B.Tech I Semester I Mid Examination, December-2020

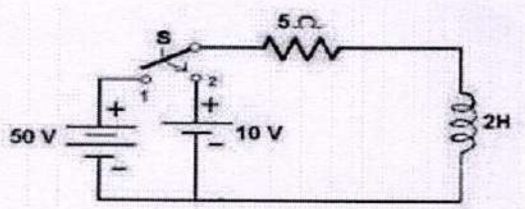
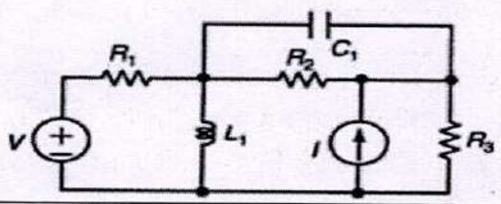
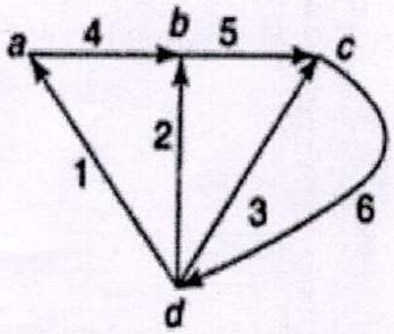
Subject Name: Network Analysis **BRANCH:EEE**
Time: 1 Hour **Max Marks:20**

Note: This question paper contains six questions. Answer any three questions.

Bloom's Level:

Remember	L1
Understand	L2
Apply	L3
Analyze	L4
Evaluate	L5
Create	L6

<table>
<tr><th colspan="2">ANSWER ALL THE QUESTIONS</th><th>Bloom's Level</th><th>CO</th><th>PO</th><th>Marks</th></tr>
<tr><td>1</td><td>Using nodal analysis find all branch currents for the following circuit

</td><td>L4</td><td>1</td><td>2</td><td>7M</td></tr>
<tr><td colspan="6">[OR]</td></tr>
<tr><td>2</td><td>Find Maximum power delivered to the load by using maximum power transfer theorem.

</td><td>L2</td><td>1</td><td>2</td><td>7M</td></tr>
<tr><td colspan="6"></td></tr>
</table>

3	In the circuit shown in figure, switch S is in position 1 for a long time and brought to position 2 at time t=0.Determine the circuit current. 	L5	2	3	7M
[OR]					
4	A series RC circuit with R = 100 Ω and C = 25 μF has a sinusoidal excitation V(t) = 250 Sin 500t. Find the total current assuming that the capacitor is initially Uncharged.	L4	2	3	7M
5	For the circuit shown draw the graph and find the no. of possible trees 	L4	3	2	6M
[OR]					
6	For the graph shown write the tieset matrix. 	L5	3	3	6M

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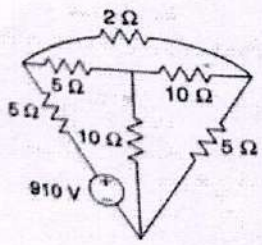
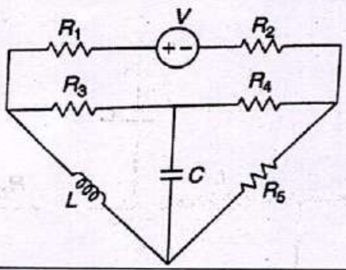
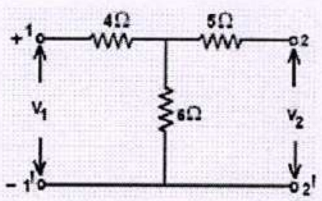
II B.Tech I Semester II Mid Examination, February-2021

Subject Name: Network Analysis **BRANCH: EEE**
Time: 90 Min **Max Marks:20**

Bloom's Level:

Remember	L1
Understand	L2
Apply	L3
Analyze	L4
Evaluate	L5
Create	L6

ANSWER ALL THE QUESTIONS		Bloom's Level	CO	PO	Marks
1.a)	Calculate twig voltages using KCL equation for the network shown 	L4	3	2	6M
	[OR]				
b)	Draw the dual of the network shown in the figure 	L3	3	1	6M
2(a)	Find the Z parameters for the circuit shown in Figure 	L4	4	2	7M
	[OR]				
b)	Derive expressions for combined network when two two-port networks connected in cascade	L3	4	2	7M

3.a)	Derive the relation between line and phase currents and line voltage and phase Voltages in delta connection of a 3- Phase balanced system	L3	5	2	7M
	[OR]				
b)	If loads are 4+j8 , 3+j4, 15+j20 supply is 3 phase 3wire 400v, 50Hz obtain line currents and power delivered	L3	5	2	7M

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R18

Subject Code: A23204

B.Tech II Year II Semester Regular Examination, OCTOBER/NOVEMBER-2020

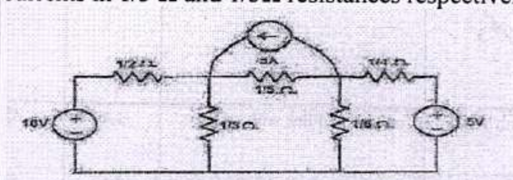
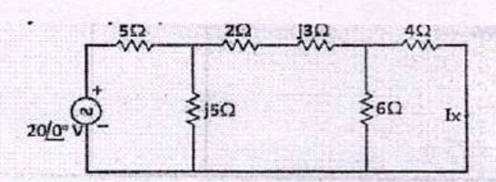
SUBJECT: Network Analysis **BRANCH:** EEE

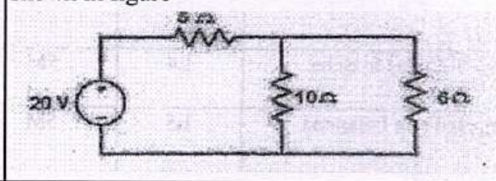
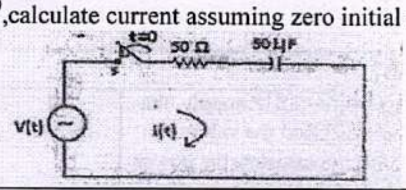
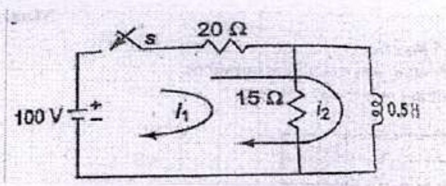
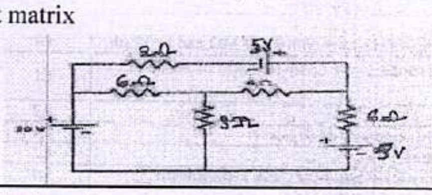
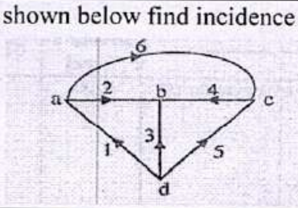
Time: 3 Hours **Max. Marks:** 75

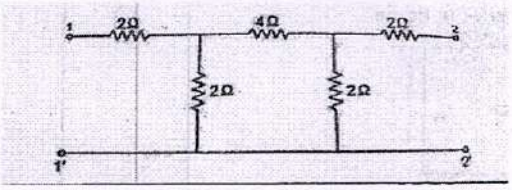
Note: This question paper contains two parts Part A and PartB
Part A is compulsory which carries 25 Marks.Answer all questions
Part B consists of 5 questions.Answer all the questions.

Bloom's Level:

Remember	L1	Analyze	L4
Understand	L2	Evaluate	L5
Apply	L3	Create	L6

	PART-A ANSWER ALL THE QUESTIONS	CO	PO	Bloom's Level	Marks
1	State Maximum Power Transfer theorem	1	1,12	L2	2M
2	Discuss and explain on supernode	1	1,12	L2	3M
3	Calculate the timeconstant of simple RL circuit with R=10MΩ and l=10μh	2	1,2,5	L3	2M
4	Review on the significance of initial conditions in a step response of series RL and RC circuits	2	1,4,5	L2	3M
5	Define a graph,tree with an example	3	1,12	L1	2M
7	Define the duality and dual elements with examples	3	1,12	L1	3M
8	Write a short notes on ABCD parameters	4	1,12	L1	2M
9	Derive the condition for reciprocity for Z parameters and Y parameters	4	1,4,5	L3	3M
10	Write the expression for active power and reactive power in a balanced three phase circuit	5	1,3,5	L1	2M
	PART-B ANSWER ALL THE QUESTIONS	**CO**	**PO**	**Bloom's Level**	**Marks**
11.i).a	Using the node voltage analysis,compute all the node voltages and currents in 1/3 Ω and 1/5Ω resistances respectively 	1	1,2,3,5	L3	5M
b)	Derive thevenin's theorem condition for DC excitation and verify with an example	1	1,2,3	L3	5M
	[OR]				
ii.a)	Verify reciprocity theorem for the circuit shown in the figure 	1	1,2,3	L2	5M

b)	Using Norton's theorem compute the current through 6Ω resistance shown in figure 	1	1,2,3,5	L3	5M
	[OR]				
12.i.a)	The series RC circuit shown in figure has sinusoidal voltage source $v(t)=100\sin(100t+\Phi)$ volts.If the switch is closed when $\Phi=90^0$,calculate current assuming zero initial charge 	2	1,2,3,5	L3	5M
b)	A series RC circuit with R=I00Ω andC=2SμF has a sinusoidal excitation V(t)=250Sin500t.Compute the total current assuming that the capacitor is initially uncharged	2	1,2,3,5	L3	5M
	[OR]				
ii.a)	For the circuit shown in the figure determine the currents i_1 and i_2 when the switch is closed at t=0 	2	1,2,3,5	L3	5M
b)	A series RC circuit with R=100Ω and L=5mH has a sinusoidal excitation of V(t)=250*sin500t.Find the total current assuming that the inductor is initially not charged	2	1,2,3,5	L4	5M
13.i.a)	For the following network obtain (A)basic tieset matrix and (B) basic cutset matrix 	3	1,2,3	L3	5M
b)	Determine the branch currents in terms of link currents using tie-set matrix with an example	3	1,2,3	L3	5M
	[OR]				
ii.a)	For the graph shown below find incidence and cutset matrices 	3	1,2,3	L4	5M
b)	Explain graph,oriented graph,subgraph,tree,cotree,and1ink with neat diagrams for any typical circuit.	3	1,2,3	L3	5M
14.i.a)	The Z parameters of a two port network are Z_{11}=10ohm, Z_{22}=15ohm,Z_{12}=Z_{21}=5ohms.Find the h parameters	4	1,2,3	L3	5M

b)	Find Y parameters for the circuit shown in the figure 	4	1,2,3	L3	5M
	[OR]				
ii.	Derive expressions for ABCD parameters of two-port networks connected in cascade.	4	1,2,3	L4	10M
15.i.a)	Derive the relation between line and phase currents and Voltages in delta connection of a 3-phase(balanced) system.	5	1,2,3	L3	5M
b)	b) A balanced star-connected load 4+j3per phase isconnected to a balanced 3- phase 400v supply. The phase current is12A,find (A) The total active power (B) Reactive power and' (C) Total apparent power (D) Power factor	5	1,2,3	L4	5M
	[OR]				
ii.	Three identical impedances are star connected across 440V,50Hz supply.The three line currents are IR=20L-40^0;ly=20L-160^0;IB=20L80^0;find the values of the elements,total power and the readings of wattmeter to measure power.	5	1,2,3	L3	5M

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CONTENT BEYOND SYLLABUS

S.No.	Date	Topics Covered	Details Of The Resource Person	Mapping With POs, PSOs
1	8.2.2021	Substitution theorem and Tellegen's theorem	V.Vijaya Lakshmi	3,1
2	10.2.2021	Network functions, Driving Points and Transfer functions –using Transformed (S)Variables	V.Vijaya Lakshmi	3,1

TUTORIAL CLASSES

S.No.	Date	Period	Topics Covered	Related COs
2	12.2.2021	1	Problems on transients	2
3	15.2.2021	1	Problems on tieset and cutest	3
4	17.2.2021	1	Problems on Network parameters	4
5	20.2.2021	1	Problems on star and delta connected loads	5

Faculty Name: V.Vijaya Lakshmi

Course: Network Analysis

Class-Section: II-A

Innovative Method:Google-It Report Writing

Objective of the Method:

Students are expected to Google the content of the topic available in open source. He/ She is also expected to go through, understand& explore beyond class room. Student will be reporting the concept understood in writing.

Aim of the method:

1. To maximize the learning experience.
2. To identify and prioritize content
3. To identify gaps in understanding

Implementation/Portrayal of method:

1. Use this opportunity to clear up any misconceptions
2. Student will be in a position to present the report prepared.
3. The report can be used for further reference.

Topic Covered through activity: Norton's Theorem

Description of the Method: Students were asked to google about Norton theorem statement & procedure to apply the theorem for a given circuit. They were further asked to prepare a report accordingly.

Benefits of the method: students understood how to apply Norton's theorem for a given circuit. This activity enhances communication and writing skills, it further engages the students in self learning.

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DEPARTMENT OF ELECTRICAL AND ELECTRONICS ENGINEERING

Name of the Student: G Kalyan

Roll.No: 19911A0229

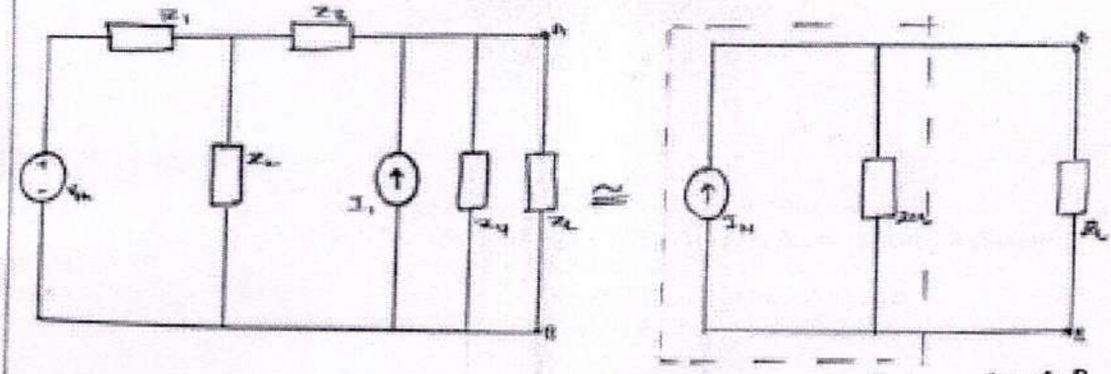
Year/Sem : II-I

Norton's theorem:

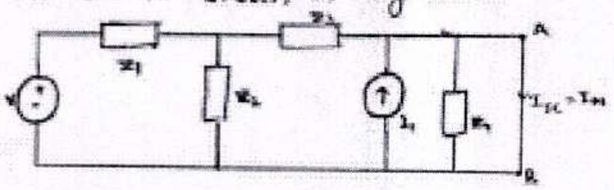
Statement: any combination of linear bilateral circuit elements and active sources. regardless of connection or complexity. connected to a given load Z_L. can be replaced by a simple two terminal network. consisting a single current source of I_N ampers and a single impedance Z_{eq} in parallel with it. across the two terminal of the load Z_L. the I_N is the short circuit current flowing through the short circuited path replaced instead of Z_L. It is also called Norton's current. the Z_{eq} is the equivalent impedance of the given network as viewed through the load terminals. with Z_L removed and all the active sources are replaced by short circuit while the independent current sources must be replaced by open circuit. while calculating Z_{eq}.

Explanation of Norton's theorem:

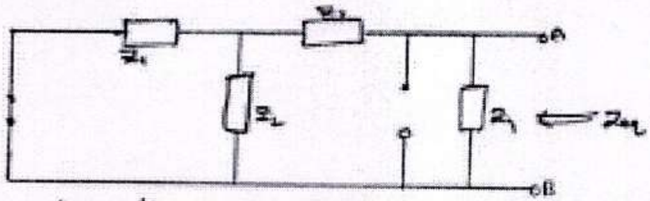
consider a network shown in fig(a) below. the terminals A-B are load terminals where load impedance Z_L is connected According to Norton's theorem the entire network can be replaced by a current resource I_N, and an equivalent impedance Z_{eq} in parallel with it. across the load terminals A-B as shown in fig(b)



-for obtaining current I_N. short the load terminals A.B. Calculate the current through the short circuited path by using any of the network amplification techniques. this is Norton's current I_N. It is shown in fig below



while the equivalent impedance Z_{eq} is to be obtained by the these procedure as in case of thevinion's theorem.

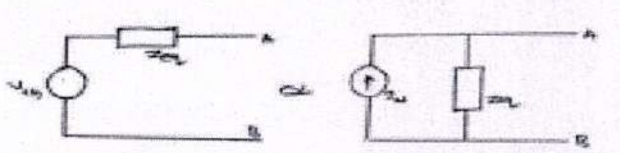


when the circuit is replaced by norton's equivalent across the load terminals, then the load current can be easily obtained by using current division in a parallel circuit as

$$I_L = I_N \frac{Z_{eq}}{Z_L + Z_{eq}}$$

-this theorem is also called dual of thevenin's theorem. This is because if the thevenin's equivalent voltage source is converted to an equivalent current source, the Norton's equivalent is obtained. This is shown in fig.

From source transformation we can write



$I_N = \frac{V_{th}}{Z_{eq}}$

or $Z_{eq} = \frac{V_{th}}{I_N}$

Steps to apply Norton's theorem:

step-1:- short the branch through which the current is to be calculated by removing the impedance between the terminals.

step-2:- obtain the current through this short circuited branch, using any of the network simplification techniques. this current is nothing but Norton's current I_N.

step-3:- Calculate the equivalent impedance Z_{eq} as viewed through the two terminals of interest by removing the branch impedance and making all the independent sources in active

step-4: Draw the Norton's equivalent across the terminals of interest, showing a current source I_N with the impedance Z_{eq} parallel with it. Reconnect the branch impedance now let it be Z_L the current through the branch of interest is.

$$I = I_N \times \frac{Z_{eq}}{Z_{eq}+Z_L}$$

Note: If dependent sources are present in the circuit thus

$$Z_{eq} = \frac{V_{th}}{-}$$

Outcome: Many students suggest that using the Google it-report writing strategy supported their ability to effectively explore and integrate thetopic and create flow and linkage amongst when writing their report.

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Aziz Nagar Gate, C.B. Post, Hyderabad-500 075

Department of Electrical and Electronics Engineering

Academic Year: 2020-2021 **BATCH: 2019-2023**

Course: NA

Faculty: V.Vijaya Lakshmi

S.No	Reg.No	MID I Threshold 60%							MID II Threshold 60%							Threshold 60%
		ASM - I (5)	PART-A			PART-B			ASM - II (5)	PART-A			PART-B			End Exam (75M)
			Q1(2M)	Q2(2M)	Q3 (2M)	Q4(5M)	Q5(5M)	Q6(4M)		Q1(2M)	Q2(2M)	Q3 (2M)	Q4(5M)	Q5(5M)	Q6(4M)	
1	18911A0217	5	2	2	1	5	5	4	5	2	2	2	5	5	4	5
2	18911A0229	5	2	2	1	5	5	4	5	2	2	1	5	5	4	45
3	18911A0233	0	AB	AB	AB	AB	AB	AB	0	AB	AB	AB	AB	AB	AB	A
4	18911A0239	5	1	1	2	3	3	2	5	1	1	2	3	3	2	27
5	18911A0244	5	1	1	2	3	3	2	5	1	1	2	3	3	2	45
6	18911A0251	5	1	1	1	3	3	2	5	1	1	2	3	3	2	20
7	18911A0279	5	1	1	1	2	2	2	5	1	1	1	2	2	2	26
8	19911A0201	5	1	1	2	4	4	3	5	1	1	1	4	4	3	45
9	19911A0202	5	2	2	1	4	4	3	5	2	2	1	4	4	3	26
10	19911A0203	5	AB	AB	AB	AB	AB	AB	5	AB	AB	AB	AB	AB	AB	A
11	19911A0205	5	1	1	2	3	3	2	5	1	1	1	3	3	2	50
12	19911A0206	5	2	2	1	5	5	4	5	2	2	2	5	5	4	47
13	19911A0207	5	2	2	2	5	5	4	5	2	2	1	5	5	4	49
14	19911A0208	5	2	2	1	4	4	3	5	2	2	2	4	4	3	29
15	19911A0209	5	2	2	1	5	5	4	5	2	2	1	5	5	4	45
16	19911A0210	5	2	2	1	4	4	3	5	2	2	2	4	4	3	52
17	19911A0211	5	1	1	1	4	4	3	5	1	1	2	4	4	3	45
18	19911A0212	5	1	1	2	4	4	3	5	1	1	1	4	4	3	12
19	19911A0213	5	2	2	2	5	5	4	5	2	2	1	5	5	4	45
20	19911A0214	5	2	2	1	5	5	4	5	2	2	1	5	5	4	47
21	19911A0215	5	2	2	1	5	5	4	5	2	2	2	5	5	4	48
22	19911A0216	5	1	1	2	3	3	2	5	1	1	1	3	3	2	45
23	19911A0217	5	2	2	2	5	5	4	5	2	2	1	5	5	4	55
24	19911A0218	5	2	2	1	5	5	4	5	2	2	1	5	5	4	45
25	19911A0219	5	2	2	1	5	5	4	5	2	2	1	5	5	4	2
26	19911A0220	5	2	2	1	5	5	4	5	2	2	2	5	5	4	26
46	19911A0221	5	2	2	2	4	4	3	5	2	2	1	4	4	3	49
47	19911A0222	5	1	1	1	2	2	1	5	1	1	0	2	2	1	26
48	19911A0223	5	2	2	2	5	5	4	5	2	2	1	5	5	4	32
49	19911A0224	5	2	2	2	4	4	3	5	2	2	1	4	4	3	48

50	19911A0225	5	AB	AB	AB	AB	AB	AB	5	AB	AB	AB	AB	AB	AB	A
51	19911A0226	5	1	1	1	4	4	3	5	1	1	1	4	4	3	28
52	19911A0227	5	2	2	1	4	4	3	5	2	2	2	4	4	3	48
34	19911A0228	5	2	2	1	5	5	4	5	2	2	1	5	5	4	64
35	19911A0229	5	2	2	2	5	5	4	5	2	2	2	5	5	4	45
36	19911A0230	5	1	1	1	3	3	2	5	1	1	1	3	3	2	49
37	19911A0231	5	2	2	1	5	5	4	5	2	2	2	5	5	4	45
38	19911A0232	5	2	2	2	4	4	3	5	2	2	1	4	4	3	47
39	19911A0233	5	2	2	2	4	4	3	5	2	2	1	4	4	3	45
40	19911A0234	5	1	1	2	3	3	2	5	1	1	1	3	3	2	57
41	19911A0235	5	1	1	2	2	2	2	5	1	1	0	2	2	2	18
42	19911A0236	5	1	1	1	4	4	3	5	1	1	1	4	4	3	9
43	19911A0237	5	1	1	2	3	3	2	5	1	1	2	3	3	2	26
44	19911A0238	5	2	2	1	5	5	4	5	2	2	2	5	5	4	31
45	19911A0239	0	AB	AB	AB	AB	AB	AB	0	AB	AB	AB	AB	AB	AB	0
46	19911A0240	5	2	2	2	4	4	3	5	2	2	1	4	4	3	48
47	19911A0241	5	1	1	2	4	4	3	5	1	1	1	4	4	3	28
48	19911A0242	5	1	1	1	4	4	3	5	1	1	1	4	4	3	9
49	19911A0243	5	2	2	2	5	5	4	5	2	2	2	5	5	4	5
50	19911A0244	5	1	1	1	3	3	2	5	1	1	2	3	3	2	28
51	19911A0245	5	2	2	1	5	5	4	5	2	2	2	5	5	4	27
52	19911A0246	5	1	1	2	2	2	1	5	1	1	0	2	2	1	45
53	19911A0247	5	2	2	1	5	5	4	5	2	2	2	5	5	4	46
54	19911A0248	5	2	2	2	4	4	3	5	2	2	1	4	4	3	48
55	19911A0249	5	2	2	2	5	5	4	5	2	2	1	5	5	4	27
56	19911A0250	5	0	0	2	0	0	0	5	0	0	2	0	0	0	46
57	19911A0251	0	AB	AB	AB	AB	AB	AB	0	AB	AB	AB	AB	AB	AB	0
58	19911A0252	5	2	2	1	4	4	3	5	2	2	1	4	4	3	12
59	19911A0253	5	2	2	2	4	4	3	5	2	2	1	4	4	3	48
60	19911A0254	5	2	2	2	4	4	3	5	2	2	2	4	4	3	0
61	19911A0255	5	1	1	2	4	4	3	5	1	1	1	4	4	3	45
62	19911A0256	5	1	1	2	3	3	2	5	1	1	1	3	3	2	52
63	19911A0257	5	2	2	1	5	5	4	5	2	2	2	5	5	4	28
64	19911A0258	5	2	2	2	5	5	4	5	2	2	1	5	5	4	48
65	19911A0259	5	2	2	1	5	5	4	5	2	2	2	5	5	4	49
66	19911A0260	5	2	2	1	5	5	4	5	2	2	2	5	5	4	49
67	19911A0261	5	1	1	1	2	2	2	5	1	1	0	2	2	2	12
68	19911A0262	5	1	1	1	3	3	2	5	1	1	2	3	3	2	26
69	19911A0263	5	1	1	2	4	4	3	5	1	1	2	4	4	3	51
70	19911A0264	5	2	2	1	5	5	4	5	2	2	1	5	5	4	54
71	19911A0265	5	0	0	1	1	1	1	5	0	0	0	1	1	1	26
72	19911A0266	5	1	1	1	3	3	3	5	1	1	1	3	3	3	31
73	19911A0267	5	1	1	2	2	2	2	5	1	1	0	2	2	2	26
74	19911A0268	5	1	1	1	4	4	3	5	1	1	2	4	4	3	27
75	19911A0269	5	1	1	2	4	4	3	5	1	1	2	4	4	3	36

76	19911A0270	5	2	2	1	5	5	4	5	2	2	2	5	5	4	11
77	19911A0271	5	2	2	1	5	5	4	5	2	2	1	5	5	4	45
78	19911A0272	5	0	0	2	0	0	0	5	0	0	2	0	0	0	49
79	19911A0273	5	2	2	2	5	5	4	5	2	2	2	5	5	4	26
80	19911A0274	5	2	2	2	4	4	3	5	2	2	1	4	4	3	45
81	19911A0275	5	2	2	1	5	5	4	5	2	2	1	5	5	4	28
82	20915A0201	5	2	2	1	5	5	4	5	2	2	1	5	5	4	50
83	20915A0202	5	1	1	1	1	1	1	5	1	1	0	1	1	1	48
84	20915A0203	5	2	2	1	4	4	3	5	2	2	1	4	4	3	45
85	20915A0204	5	1	1	2	3	3	2	5	1	1	2	3	3	2	58
86	20915A0205	5	2	2	2	5	5	4	5	2	2	1	5	5	4	29
87	20915A0206	5	0	0	2	1	1	1	5	0	0	0	1	1	1	48
88	20915A0207	5	1	1	1	4	4	3	5	1	1	2	4	4	3	54
89	20915A0208	5	1	1	2	3	3	2	5	1	1	1	3	3	2	45
90	20915A0209	5	2	2	1	5	5	4	5	2	2	1	5	5	4	39
91	20915A0210	5	1	1	1	3	3	3	5	1	1	2	3	3	3	51
92	20915A0211	5	2	2	2	5	5	4	5	2	2	2	5	5	4	48
93	20915A0212	5	2	2	2	4	4	3	5	2	2	1	4	4	3	62
94	20915A0213	5	2	2	2	5	5	4	5	2	2	0	5	5	4	48
95	20915A0214	5	2	2	2	5	5	4	5	2	2	1	5	5	4	50
96	20915A0215	5	2	2	2	5	5	4	5	2	2	1	5	5	4	38
97	20915A0216	5	2	2	2	5	5	4	5	2	2	2	5	5	4	35
98	20915A0217	5	2	2	1	4	4	3	5	2	2	1	4	4	3	61
99	20915A0218	5	2	2	2	4	4	3	5	2	2	2	4	4	3	46
100	20915A0219	5	1	1	1	2	2	1	5	1	1	0	2	2	1	48
101	20915A0220	5	1	1	2	3	3	3	5	2	2	1	4	4	3	14
102	20915A0221	5	2	2	2	5	5	4	5	2	2	1	5	5	4	49
103	20915A0222	5	2	2	2	4	4	3	5	2	2	2	4	4	3	62
104	20915A0223	5	2	2	2	5	5	4	5	2	2	2	5	5	4	26
105	20915A0224	5	2	2	2	5	5	4	5	2	2	1	5	5	4	56
106	20915A0225	5	2	2	1	4	4	3	5	2	2	2	4	4	3	39
Average marks		**4.858491**	**1.509901**	**1.509901**	**1.514851**	**3.774752**	**3.774752**	**3.019802**	**4.858491**	**1.511881**	**1.511881**	**1.29703**	**3.779703**	**3.779703**	**3.023762**	**37.650485**
No of students attemped		**106**	**101**	**101**	**101**	**101**	**101**	**101**	**106**	**101**	**101**	**101**	**101**	**101**	**101**	**103**
No of students scored 60% and		**103**	**61**	**17**	**101**	**82**	**82**	**61**	**103**	**17**	**17**	**91**	**82**	**82**	**62**	**59**
%of students scored 60%		97.169811	60.39604	60.39604	100	81.188119	81.188119	60.39604	97.169811	61.386139	61.386139	90.09901	81.188119	81.188119	61.386139	57.2815534
CO ATTAINMENT LEVEL		**3**	**2**	**2**	**3**	**3**	**3**	**2**	**3**	**2**	**2**	**3**	**3**	**3**	**2**	**1**

ASSESSMENT OF COs FOR THE COURSE

CO	Method	value	Average	Internal Exam	External Exam	Overall CO Attainment
CO 1	**MID I Q1**	2.0	2.75			
	MID I Q3	3.0				
	MID I Q4	3.0				
	ASM-I	3.0				
	MID I Q2	2.0				

CO 2	MID1 Q3	3.0	2.[illegible]	2.70	1.00	2.28
	MID I Q5	3.0				
	ASM-I	3.0				
CO 3	MID I Q6	2.0	2.75			
	MID 2 Q4	3.0				
	ASM-I	3.0				
	ASM-II	3.0				
CO 4	MID 2 Q1	2.0	2.75			
	MID 2 Q3	3.0				
	MID 2 Q5	3.0				
	ASM-II	3.0				
CO5	MID 2 Q2	2.0	2.50			
	MID 2 Q3	3.0				
	MID 2 Q6	2.0				
	ASM-II	3.0				

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Aziz Nagar Gate, C.B. Post, Hyderabad-500 075

DEPARTMENT OF ELECTRICAL AND ELECTRONICS ENGINEERING

BATCH 2019-2023

Course End Survey Analysis

II-I –A Sec(Academic Year 2020 -2021)					
Year/Sem:II-I	**Substantially High**	**Moderate**	**Low**	**Total**	**Attainment**
Network Analysis	44.4	8.2	0.4	53	2.82

	3	**2**	**1**	**Assessment**	**TOTAL**
CO1	44	8	1	**2.81**	**53**
CO2	45	8	0	**2.84**	53
CO3	43	10	0	**2.81**	53
CO4	48	5	0	**2.90**	53
CO5	42	10	1	**2.75**	53
	44.4	8.2	0.4	**2.82**	53

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Indirect Attainment(IDA) BACTH 2019-23 AY 2020-21

S.No	Roll.No	II EEE-II sem-A Sec				
		NA				
		CO1	CO2	CO3	CO4	CO5
1	19911A0201	3	3	3	3	3
2	19911A0202	3	3	3	3	3
3	19911A0203	3	3	3	3	3
4	19911A0205	3	3	3	3	3
5	19911A0206	3	3	3	3	2
6	19911A0207	3	3	3	3	3
7	19911A0208	3	3	3	3	3
8	19911A0209	3	3	3	3	3
9	19911A0210	3	3	2	3	3
10	19911A0211	3	3	3	3	3
11	19911A0212	3	3	3	3	2
12	19911A0213	3	2	3	2	3
13	19911A0214	3	3	2	3	3
14	19911A0215	3	3	3	3	3
15	19911A0216	2	3	3	3	3
16	19911A0217	3	2	2	3	3
17	19911A0218	3	3	3	2	2
18	19911A0219	3	3	3	3	3
19	19911A0220	3	3	3	3	2
20	19911A0221	3	3	3	2	3
21	19911A0222	2	3	3	3	3
22	19911A0223	3	3	3	3	3
23	19911A0224	3	3	2	3	3
24	19911A0225	3	3	3	3	3
25	19911A0226	3	3	3	3	2
26	19911A0227	3	2	2	3	3
27	19911A0228	2	3	3	3	3
28	19911A0229	3	2	3	3	3
29	19911A0230	3	3	3	2	3
30	19911A0231	3	3	3	3	3
31	19911A0232	2	3	3	3	3
32	19911A0233	3	3	3	3	3
33	19911A0234	3	3	3	3	3
34	19911A0235	3	2	3	3	2
35	19911A0236	2	3	2	3	3
36	19911A0237	3	3	2	3	2
37	19911A0238	3	2	3	3	3
38	19911A0239	3	3	3	3	3
39	19911A0240	3	3	3	2	2
40	19911A0241	3	3	3	3	3
41	20915A0201	3	3	3	3	3
42	20915A0202	1	3	3	3	3
43	20915A0203	3	2	2	3	2

44	20915A0204	2	3	3	3	3
45	20915A0205	3	3	3	3	3
46	20915A0206	3	3	3	3	3
47	20915A0207	3	3	3	3	1
48	20915A0208	3	3	3	3	3
49	20915A0209	3	3	2	3	3
50	20915A0210	3	3	3	3	2
51	20915A0211	3	3	3	3	3
52	20915A0212	2	2	2	3	2
53	20915A0213	3	3	3	3	3
		2.83019	2.84906	2.81132	2.90566	2.75472

Average=2.82

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DEPARTMENT OF ELECTRICAL AND ELECTRONICS ENGINEERING

Batch 2019-2023

COURSE CLOSURE REPORT

S.No	Parameters	Section	A SEC
		Course Name	Network Analysis
		Allotted Faculty	V.Vijaya Lakshmi
1	Quality of I/II-mid question papers(As per Blooms Taxonomy or not) submitted to the exam section		
2	No of students registered for the exam		53
3	No of students appeared for the exam		51
4	No of students passed		45
5	**Pass percentage**		88.23%
6	End exam result analysis (pass percentage > 90%)		0
7	End exam result analysis (pass percentage 80% to 90%)		3
8	End exam result analysis (pass percentage 70% to 80%)		6
9	End exam result analysis (pass percentage 60% to 70%)		12
10	End exam result analysis (pass percentage <60%)		32

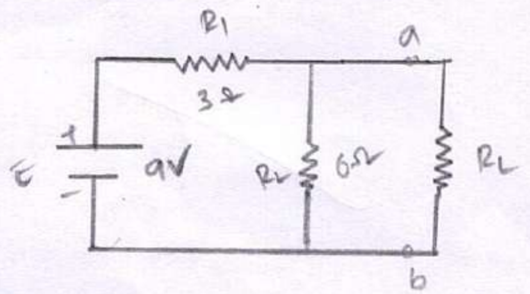
Faculty **HOD**

ASSIGNMENT-1

Ch. chetana
19911A0217

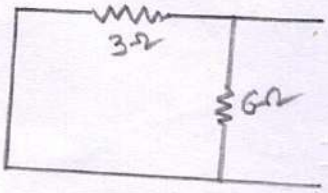
1. State Norton's Theorem and find Norton's Equivalent for the circuit shown.

In any linear bi-lateral network having number of active and passive elements, it can be replaced by single equivalent circuit consisting of equivalent current source (I_N) in parallel with equivalent resistance (R_N).



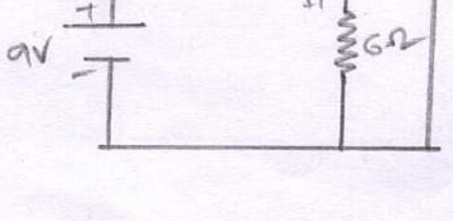
To find R_N:-

Open circuit the load terminal 'R_L' and short circuit the voltage source



$$R_N = \frac{3\times 6}{3+6} = \frac{18}{9} = 2\Omega$$

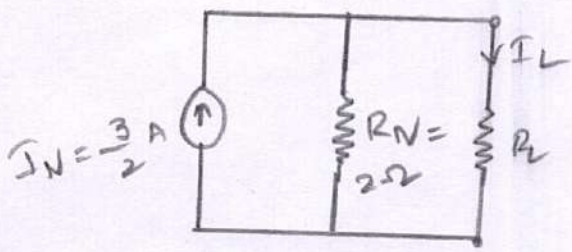
To find I_N:-



Due to short circuiting the load terminal, current does not flow through '6Ω' resistor.

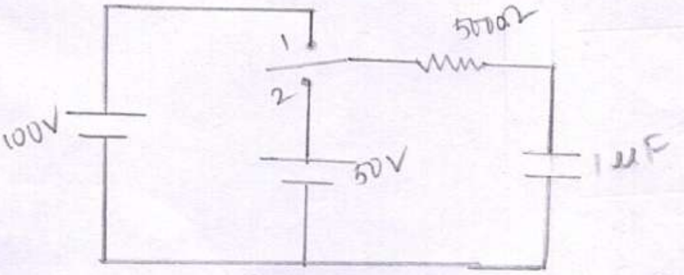
$$I_N = \frac{9}{6} = \frac{3}{2} A$$

Norton's equivalent circuit



$$\boxed{I_L = I_N \times \frac{R_N}{R_N + R_L}}$$

2. Switch is moved from position 1 to 2 at $t=0$ find the voltage $V_R(t)$ and $V_C(t)$ for $t=0$



$V(0^+) = 100\,V$

Apply KVL to the loop

$$5000\,i + \frac{1}{10^{-6}}\int i\,dt + 50 = 0$$

$$5000\,i + 10^{6}\int i\,dt = -50$$

differentiate on both sides

$$5000\frac{di}{dt} + 10^6 i = 0$$

$$\frac{di}{dt} + \frac{10^6}{5000} i = 0$$

$$\frac{di}{dt} + 200 i = 0$$

$$\Rightarrow i(t) = C_1 e^{-200t} \quad \text{---} ①$$

Apply KVL to the loop ②

$$50 + 5000 i(0^+) = 0$$

$$i(0^+) = \frac{-150}{5000} = -0.03 A$$

At $t=0$, $i(0^+) = -0.03$

$$-0.03 = C_1 e^0$$

$$\Rightarrow C_1 = -0.03$$

Substitute C_1 in eq ①

$$i(t) = -0.03 e^{-200t}$$

$$\rightarrow v_c(t) = \frac{1}{c}\int i\,dt$$
$$= \frac{1}{10^{-6}}\int -0.03 e^{-200t}\,dt$$

differentiate

$$v_c(t) = 10^6 \times \frac{-0.03}{-200} \cdot e^{-200t} + C_2$$

$$= 150 e^{-200t} + C_2$$

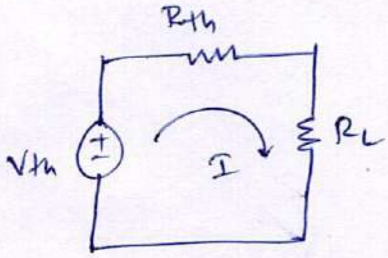
At $t=0^+$, $v_c(0^+) = 100V$

$$\Rightarrow 100 = 150 e^0 + C_2$$
$$C_2 = -50$$
$$v_c(t) = -50 + 150 e^{-200t}$$
$$v_c(t) = V_c(\infty) + (V_c(0) - V(\infty)) e^{-t/\tau}, \quad \tau = RC$$

3. State and Prove the maximum power transfer theorem for both AC and DC excitation.

In an active network maximum power transfer to the load takes place when the load resistance is equal to equivalent resistance of the network as viewed from the terminals of the load.

Proof for DC



$$I = \frac{V_{th}}{R_{th}+R_L}$$

$$P = I^2 R_L$$

$$P = \left(\frac{V_{th}}{R_{th}+R_L}\right)^2 R_L$$

$$P = \frac{V_{th}^2}{(R_{th}+R_L)^2} R_L$$

$$\frac{dP}{dR_L} = 0$$

$$d\left(\frac{u}{v}\right) = \frac{v\,du - u\,dv}{v^2}$$

$$\frac{(R_{th}+R_L)^2 \times V_{th}^2 - V_{th}^2 R_L \times 2(R_{th}+R_L)\times 1}{(R_{th}+R_L)^4} = 0$$

$$= (R_{th}^2 + R_L^2 + 2R_{th}R_L)V_{th}^2 - V_{th}^2(2R_LR_{th} + 2R_L^2) = 0$$

$$R_{th}^2 + R_L^2 + 2R_{th}R_L - 2R_LR_{th} - 2R_L^2 = 0$$

$$R_{th}^2 + R_L^2 - 2R_L^2 = 0$$

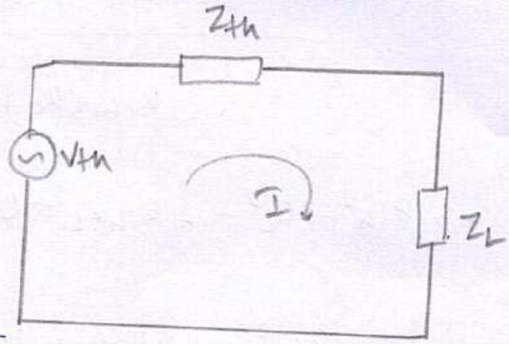
$$\boxed{R_{th} = R_L}$$

$$P_{max} = \frac{V_{th}^2}{(R_L+R_L)^2} R_L$$

$$= \frac{V_{th}^2}{(2R_L)^2} R_L \Rightarrow \frac{V_{th}^2}{4R_L^2} \times R_L$$

$$P_{max} = \frac{V_{th}^2}{4R_L}$$

Proof:- For AC



$$Z_{th} = R_{th} + jX_{th}$$

$$Z_L = R_L + jX_L$$

$$I = \frac{V_{th}}{Z_{th}+Z_L} = \frac{V_{th}}{(R_{th}+R_L)+j(X_{th}+X_L)}$$

$$I = \frac{V_{th}}{\sqrt{(R_{th}+R_L)^2+(X_{th}+X_L)^2}}$$

$$P = I^2 R_L$$

$$= \left[\frac{V_{th}^2}{\sqrt{(R_{th}+R_L)^2+(X_{th}+X_L)^2}}\right]^2 \times R_L$$

$$P = \frac{V_{th}^2}{(R_{th}+R_L)^2+(X_{th}+X_L)^2} \times R_L$$

$$\frac{dP}{dX_L} = 0$$

$$\frac{(R_{th}+R_L)^2+(X_{th}+X_L)^2 \times 0 - V_{th}^2 R_L \times 2(X_{th}+X_L) \times 1}{((R_{th}+R_L)^2+(X_{th}+X_L)^2)^2} = 0$$

$$\cancel{X}_{th}^2 R_L \times 2(X_{th}+X_L)=0$$

$$X_{th}+X_L=0$$

$$\boxed{X_L=-X_{th}}$$

$$\frac{dP}{dR_L}=0$$

$$\frac{(R_{th}+R_L)^2+(X_{th}+X_L)^2 V_{th}^2 - V_{th}^2 R_L \times 2(R_{th}+R_L)\times 1}{(R_{th}+R_L)^2}$$

$$\Rightarrow R_{th}^2+R_L^2+2R_{th}R_L+0-2R_L R_{th}-2R_L^2=0$$

$$R_{th}^2-R_L^2=0$$

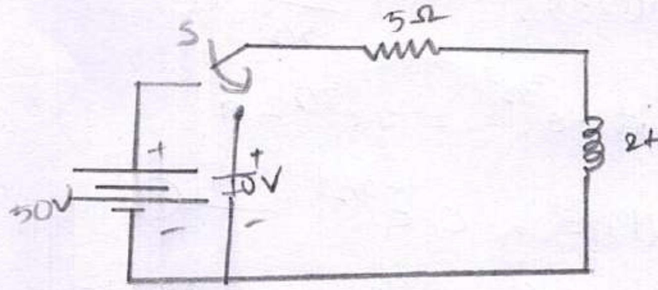
$$R_{th}=R_L$$

$$\boxed{Z_L=Z_{th}}$$

$$\text{Power (max)}=\frac{V_{th}^2}{(R_L+R_L)^2}R_L$$

$$P_{max}=\frac{V_{th}^2}{4R_L}$$

4. In the circuit shown in figure, switch S is in position 1 for a long time and brought to position 2 in time .. Determine the circuit current.



sol:-

$$i(0^-) = \frac{50}{5} = 10A$$

$$i(0^+) = 10A$$

Apply KVL to the loop,

$$-10 + 5i(t) + 2\frac{di(t)}{dt} = 0$$

$$5i(t) + 2\frac{di(t)}{dt} = 10$$

$$\frac{di(t)}{dt} + \frac{5}{2}i(t) = \frac{10}{2}$$

$$\frac{di(t)}{dt} + \frac{5}{2}i(t) = 5$$

$$e^{5/2t}\left[\frac{di(t)}{dt} + \frac{5}{2}i(t)\right] = 5e^{5/2t}$$

$$e^{5/2}\frac{di(t)}{dt} + \frac{5}{2}i(t)e^{5/2t} = 5e^{5/2t}$$

$$\frac{d}{dt}\left(i(t)\cdot e^{5/2t}\right] = 5e^{5/2t}$$

$$d\left(i(t)\cdot e^{5/2t}\right] = 5e^{5/2t}\,dt$$

integrate on both sides

$$\int d\left(idt\, e^{5/2t}\right] = 5\int e^{5/2t}\,dt$$

$$i(t)\cdot e^{5/2t} = 5\int e^{5/2t}\,dt + C_1$$

$$i(t)\cdot e^{5/2t} = 5\,\frac{e^{5/2}}{5/2} + C$$

$$i(t)\cdot e^{5/2t} = 2e^{5/2} + 9$$

$$i(t) = 2 + C_1 e^{-5/2t} \quad \text{—— (1)}$$

Apply initial conditions ; At $t=0$; $i(0^+) = 10A$

$$i(0^+) = 2 + C_1 e^{-5/2t}$$

$$10 = 2 + C_1 e^0$$

$$C_1 = 10 - 2$$

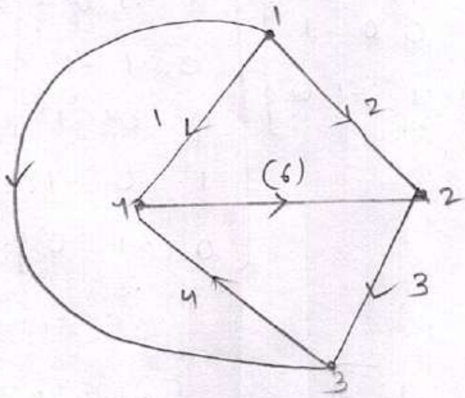
$$C_1 = 8$$

substituting $C_1 = 8$ in eqn (1), we get

$$i(t) = 2 + 8e^{-5/2t}$$

$$\boxed{i(t) = 2 + 8e^{-5/2t}}$$

5. For the following graph find the number of possible trees.



sol:-

Number of possible trees = det $(A \times A^T)$

Incidence matrix

$$A = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \begin{bmatrix} 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & -1 & 1 & 0 & 0 & -1 \\ 0 & 0 & -1 & 1 & -1 & 0 \\ -1 & 0 & 0 & -1 & 0 & 1 \end{bmatrix} \end{array}$$

Reduced incidence matrix

$$A = \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \begin{bmatrix} 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & -1 & 1 & 0 & 0 & -1 \\ 0 & 0 & -1 & 1 & -1 & 0 \end{bmatrix} \end{array}$$

$$A^T = \begin{bmatrix} 1 & 0 & 0 \\ 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \\ 1 & 0 & -1 \\ 0 & -1 & 0 \end{bmatrix}$$

$$\det(A \times A^T) = \begin{bmatrix} 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & -1 & 1 & 0 & 0 & -1 \\ 0 & 0 & -1 & 1 & -1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \\ 1 & 0 & -1 \\ 0 & -1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1+1+1 & -1 & -1 \\ -1 & 1+1+1 & -1 \\ -1 & -1 & 1+1+1 \end{bmatrix} = \begin{bmatrix} 3 & -1 & -1 \\ -1 & 3 & -1 \\ -1 & -1 & 3 \end{bmatrix}$$

$$\det(A \times A^T) = 3 \begin{vmatrix} 3 & -1 \\ -1 & 3 \end{vmatrix} - (-1) \begin{vmatrix} -1 & -1 \\ -1 & 3 \end{vmatrix} -1 \begin{vmatrix} -1 & 3 \\ -1 & -1 \end{vmatrix}$$

$$= 3(9-1) + 1(-3-1) - 1(1+3)$$

$$= 3(8) + 1(-4) - 4$$

$$= 24 - 4 - 4$$

$$= 16$$

The number of possible trees are "16"

B. Sravan
19911A0210

① Derive Conditions for reciprocity and Symmetry for Z, ABCD parameters.

Z- parameters:–

Condition for Symmetry:

$V_1 = Z_{11} I_1 + Z_{12} I_2$

$V_2 = Z_{21} I_1 + Z_{22} I_2$

O.C port ②, $I_2 = 0$

$V_1 = Z_{11} I_1$

$\frac{V_1}{I_1} = Z_{11}$

$V_1 = V_s$

$\frac{V_s}{I_1} = Z_1$

Open Circuit port ①, $I_1 = 0$

$V_2 = Z_{22} I_2$

$\frac{V_2}{I_2} = Z_{22}$

$V_2 = V_s$

$\frac{V_s}{I_2} = Z_{22}$

$\frac{V_s}{I_1} = \frac{V_s}{I_2}$

$\boxed{Z_{11} = Z_{22}} \rightarrow$ Symmetry

Condition for Reciprocity:

$V_1 = Z_{11} I_1 + Z_{12} I_2$

$V_2 = Z_{21} I_1 + Z_{22} I_2$

S.C port ② $V_2 = 0$

$V_1 = Z_{11} I_1 - Z_{12} I_2'$

$0 = Z_{21} I_1 - Z_{22} I_2'$

$Z_{21} I_1 = Z_{22} I_2'$

$I_1 = \frac{Z_{22}}{Z_{21}} I_2'$

$V_1 = Z_{11} \times \frac{Z_{22}}{Z_{21}}$,

$I_2' - Z_{12} I_2'' = Z_{11} 2$

$\frac{V_1}{I_2} = \frac{\Delta Z}{\Delta Z_{21}}$, $V_1 = V_s$

$\frac{V_s}{I_2} = \frac{\Delta Z}{Z_{21}}$

S.c port ① $V_1 = 0$

$0 = -Z_{11} I_1' + Z_{12} I_2$

$V_2 = -Z_{21} I_1' + Z_{22} I_2$

$Z_{12} I_2 = Z_{11} I_1'$

$I_2 = Z_{11} I_1'$

$\boxed{Z_{12} = Z_{21}} \rightarrow$ Reciprocity

ABCD - parameters

Condition for Symmetry n/w

$V_1 = AV_2 - BI_2$

$I_1 = CV_2 - DI_2$

O.C port ②, $I_2 = 0$

$V_1 = AV_2$

$I_1 = CV_2$

$\frac{V_1}{I_1} = \frac{AV_2}{CV_2} = \frac{A}{C}$

let $V_1 = V_s$

$\frac{V_s}{I_1} = \frac{A}{C}$

O.C port ① $I_1 = 0$

$V_1 = AV_2 - BI_2$

$0 = CV_2 - DI_2$

$CV_2 = DI_2$

$\frac{V_2}{I_2} = \frac{D}{C}$

$V_2 = V_s$

$\frac{V_s}{I_2} = \frac{D}{C}$

$\frac{V_s}{I_1} = \frac{V_s}{I_2} = \frac{A}{C} = \frac{D}{C}$

$\boxed{A = D}$ → Symmetry

Condition for reciprocal n/w

Short circuit port ② $V_2 = 0$

$V_1 = -B(-I_2') = BI_2'$

$\frac{V_1}{I_2'} = \frac{B}{1} \rightarrow V_1 = V_s = \frac{V_2}{I_2'} = \frac{B}{1}$

Short circuit port ① $V_1 = 0$

$0 = AV_2 - BI_2 \Rightarrow AV_2 = BI_2$

$I_2 = \frac{A}{B} V_2$

$-I_1' = CV_2 - DI_2$

$= CV_2 - \frac{DA}{B} V_2$

$I_1' = \frac{BC - AD}{B} V_2$

$\frac{V_2}{I_1'} = \frac{B}{AD - BC}$.

$\boxed{V_2 = V_s}$

$\frac{V_s}{I_2'} = \frac{B}{AD - BC}$

$\frac{V_s}{I_2'} = \frac{V_s}{I_1'}$

$\boxed{AD - BC = 1}$ → reciprocal.

(8) A three phase balanced delta connected load of $(4-j8)\Omega$ is connected across a 400V, 3-ϕ balanced supply. Determine the phase currents and line current. Assume the phase of sequence to be RYB also calculate the power drawn by load.

Sol

Given data

$Z = 4-j8$

$V_L = V_{ph} = 400V$

I_{ph} & $I_L = ?$

$I_{ph} = \frac{V_{ph}}{R_{ph}}$

$= \frac{400}{4}$

$I_{ph} = 100\ A$

$I_L = \sqrt{3} I_{ph}$

$= \sqrt{3}[100]$

$I_L = 173.20\ A$

$\cos\phi = \frac{R}{|Z|}$

$= \frac{4}{\sqrt{4^2+8^2}}$

$= \frac{4}{\sqrt{16+64}}$

$= \frac{4}{\sqrt{80}}$

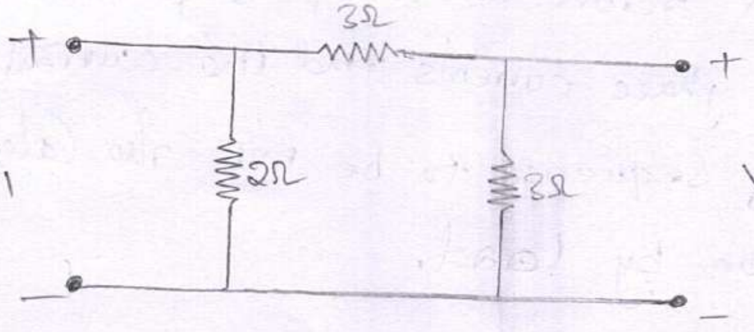
$= \frac{4}{8.9}$

$\cos\phi = 0.4$

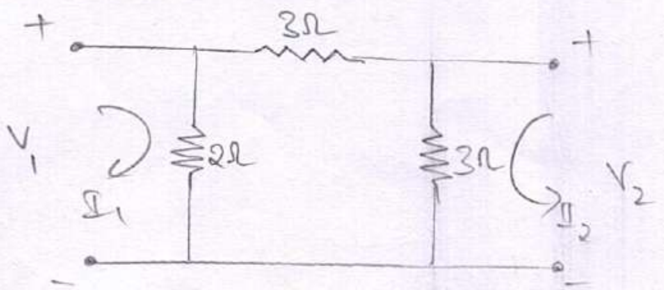
$P = \sqrt{3}\ V_L I_L \cos\phi$

$= \sqrt{3}[400][173.20][0.4] = 48026.3\ W$

(3) Find the z-parameters for the following network from basics.



Sol Given Circuit is



Apply K.V.L to the loop I_1

$$2I_1 - 2I_3 = V_1 \quad \text{—— (1)}$$

Apply K.V.L to the loop I_2

$$3I_2 + 3I_3 = V_2 \quad \text{—— (2)}$$

Apply K.V.L to the loop I_3

$$8I_3 - 2I_1 + 3I_2 = 0 \quad \text{—— (3)}$$

from equation (3)

$$I_3 = \frac{2I_1 - 3I_2}{8} \quad \text{—— (4)}$$

Put eqn ④ in ①

$$2I_1 - 2I_3 = V_1$$

$$2I_1 - 2\left[\frac{2I_1 - 3I_2}{8}\right] = V_1$$

$$2I_1 - \frac{4I_1 - 6I_2}{8} = V_1$$

$$V_1 = \frac{8I_1 - 4I_1 + 6I_2}{8}$$

$$V_1 = \frac{I_1}{2} + \frac{3}{4}I_2$$

Put eqn ④ in ②

$$V_2 = 3I_2 + 3I_3$$

$$V_2 = 3I_2 + 3\left[\frac{2I_1 - 3I_2}{8}\right]$$

$$V_2 = 3I_2 + \frac{6I_1 - 9I_2}{8}$$

$$= \frac{24I_2 + 6I_1 - 9I_2}{8}$$

$$= \frac{15I_2}{8} + \frac{6I_1}{8}$$

$$V_2 = \frac{3}{4}I_1 + \frac{15}{8}I_2$$

$$V_1 = \frac{1}{2}I_1 + \frac{3}{4}I_2$$

$$= \frac{3}{4}I_1 + \frac{15}{8}I_2$$

Compare the above eqn with z-parameter Conditions.

$$V_1 = I_1 Z_{11} + I_2 Z_{12}$$

$$V_2 = I_1 Z_{21} + I_2 Z_{22}$$

then

$$Z_{11} = \frac{1}{2}, \quad Z_{12} = \frac{3}{4}$$

$$Z_{21} = \frac{3}{4}, \quad Z_{22} = \frac{15}{8}$$

(4) Obtain power equation using 2-wattmeter method for Balanced and Unbalanced Systems.

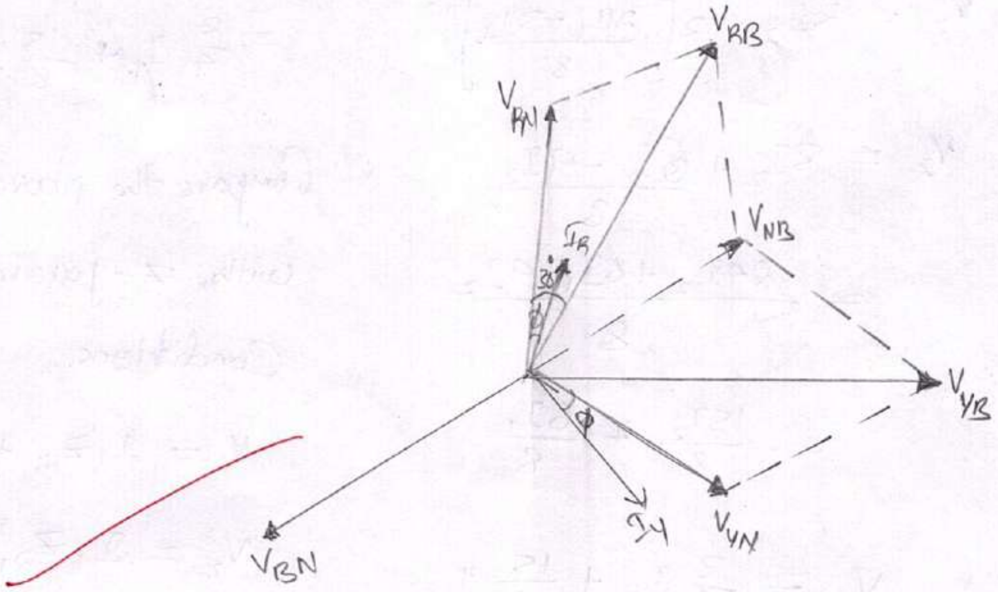
<u>Measurement of Active power by Two-wattmeter method.</u>

fig shows a balanced star-connected load and this load may be assumed to be inductive, let V_{RN}, V_{YN} and V_{BN} be the three phase voltages and I_R, I_y and I_B be the phase currents. The phase currents will lag behind their respective phase voltages by angle ϕ.

Current through current coil of $W_1 = I_R$

Voltage across voltage coil $W_1 = V_{RB} = V_{RN} + V_{NB}$
$= V_{RN} - V_{BN}$

fig shows the phasor diagram of balanced star-connected inductive load.



from the phasor diagram, it is clear that the phase angle b/w V_{RB} and I_R is $(30° - \phi)$ $W_1 = V_{RB} I_R \cos(30° - \phi)$

Current through current coil of $W_2 = I_y$

Voltage across voltage coil of $W_2 = V_{YB} = V_{YN} + V_{NB} = V_{YN} - V_{BN}$

from the phasor diagram, it is clear that phase angle b/w V_{YB} and I_y is $[30° + \phi]$.

$$W_2 = V_{YB} I_y \cos(30° + \phi)$$

But,
$$I_2 = I_y = I_L$$

$$V_{RB} = V_{YB} = V_L$$

$$W_1 = V_L I_L \cos[30° - \phi]$$

$$W_2 = V_L I_L \cos(30° + \phi)$$

$$W_1 + W_2 = V_L I_L \cos(30° + \phi) + \cos(30° - \phi) = V_L I_L (2 \cos 30° \cos\phi) = \sqrt{3} V_L I_L \cos\phi$$

Thus, the sum of two wattmeter readings gives three-phase power.

Measurement of Reactive power by Two-Wattmeter method.

We knows that

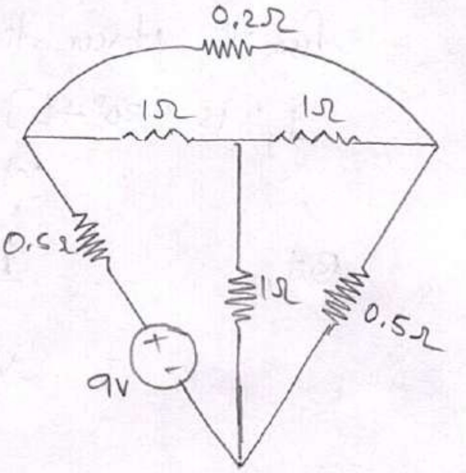
$$W_1 - W_2 = V_L I_L \sin\phi$$

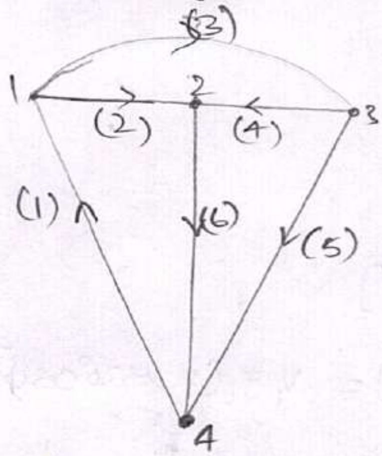
The total reactive power in a three-phase system is given by

$$Q = \sqrt{3} V_L I_L \sin\phi = \sqrt{3} [W_1 - W_2]$$

Thus, total reactive power of a three-phase system is obtained by multiplying the difference of two-wattmeter reading by $\sqrt{3}$.

(5) Write down the incidence matrix and Cutset matrices for the circuit shown in the figure.





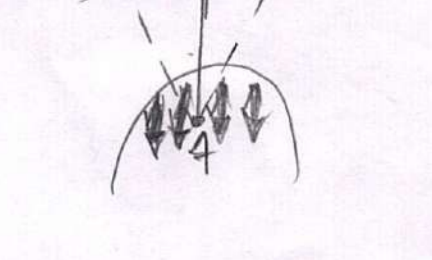
incidence Matrix.

$$A = \begin{array}{c|cccccc} & \multicolumn{6}{c}{\text{No of branches} \rightarrow} \\ \text{No of nodes} \downarrow & 1 & 2 & 3 & 4 & 5 & 6 \\ \hline 1 & -1 & 1 & 1 & 0 & 0 & 0 \\ 2 & 0 & -1 & 0 & -1 & 0 & 1 \\ 3 & 0 & 0 & -1 & 1 & 1 & 0 \\ 4 & 1 & 0 & 0 & 0 & -1 & -1 \end{array}$$

$\Rightarrow$ Reduced incidence Matrix

$$A = \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \begin{bmatrix} -1 & 1 & 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 & 0 & 1 \\ 0 & 0 & -1 & -1 & 1 & 0 \end{bmatrix} \end{array}$$

Cut set



C-1 : $\{1, 2, 3\}$

C-2 : $\{1, 5, 6\}$

C-3 : $\{3, 4, 5\}$

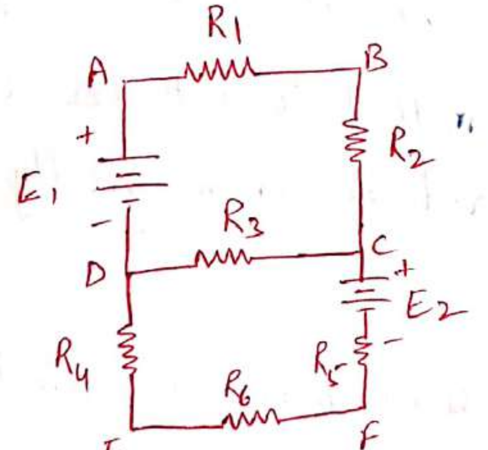
Network Analysis:

UNIT-I

Prepared by
V. Vijaya Lakshmi
EEE Dept.
VJIT

Network Theorems (DC & AC), Mesh and Nodal Analysis

Mesh:- Mesh (or loop) is a set of branches forming a closed path in a network in such a way that if one branch is removed then remaining branches do not form a closed path. A loop also can be defined as a closed path which originates from a particular node, terminating at the same node, travelling through various other nodes without travelling through any node twice. In the fig. paths A-B-C-D-A, A-B-C-F-E-D-A, D-C-F-E-D etc are loops of the network.



An Electrical Network

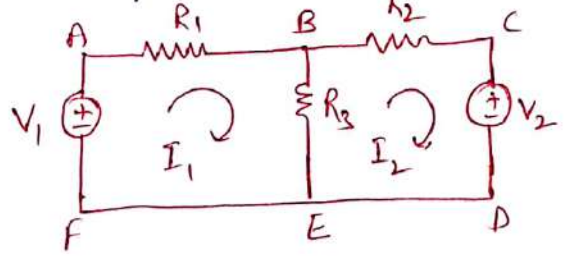
Node:- A point at which two or more elements are joined together is called a node. The junction points are also the nodes of the network. In the network shown in the Fig. A, B, C, D, E and F are the nodes of the network.

Loop Analysis or Mesh Analysis:-

This method of analysis is specially useful for the circuits that have many nodes and loops. The difference between application of Kirchoff's laws and loop analysis is, in loop analysis instead of branch currents, the loop currents are considered for writing the equations. The another difference is each branch of the network may carry more than one current. The total branch current must be decided by the algebraic sum of all currents through that branch. While in analysis using Kirchoff's laws, each branch current carries only one current.

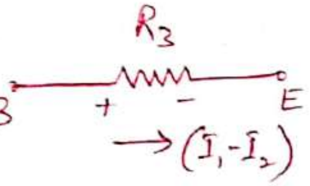
The advantage of this method is that for complex network the number of unknowns reduces which greatly simplifies calculation work.

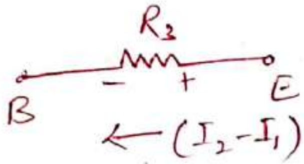
Consider following network shown in Fig. There are two loops. So assuming two loop currents as I_1 and I_2



While assuming loop currents, consider the loops such that each element of the network will be included atleast once in any of the loops.

Now Branch B-E carries two currents I_1 from B to E and I_2 from E to B. So net current through branch B-E will $(I_1 - I_2)$ and corresponding drop across R_3 must be as shown below in Fig.



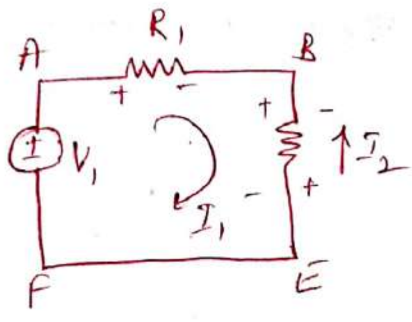


Consider loop A-B-E-F-A

For branch B-E, polarities of voltage drops will be B +ve, E -ve for current I_1 while E +ve, B -ve for current I_2 flowing through R_3.

Now while writing loop equations assume main loop current as positive and remaining loop current must be treated as negative for common branches.

Writing loop equations for the network shown in the Fig



For loop A-B-E-F-A

$$-I_1 R_1 - I_1 R_3 + I_2 R_3 + V_1 = 0$$

For loop B-C-D-E-B

$$-I_2 R_2 - V_2 - I_2 R_3 + I_1 R_3 = 0$$

By solving above simultaneous equations any unknown branch current can be determined

1. While assuming loop currents make sure that atleast one loop current links with every element.

2. No two loops should be identical.

3. Choose minimum number of loop currents.

4. If current in a particular branch is required, then try to choose loop current in such a way that only one loop current links with that branch.

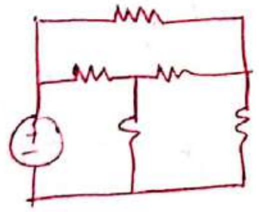
→ If a network has large no. of voltage sources it is useful to use Mesh analysis.

$$\boxed{\text{KVL + Ohm's law = Mesh analysis}}$$

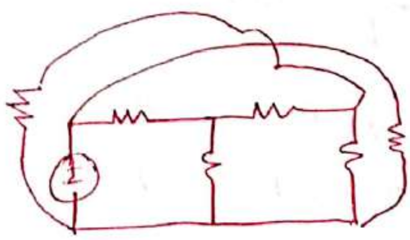
→ Mesh analysis is only applicable for planar network
For non planar circuits mesh analysis is not applicable.

** → A Circuit is said to be planar if it can be drawn on a planar surface without crossovers.

** → A non planar circuit can't be drawn on a plane surface without a crossover.



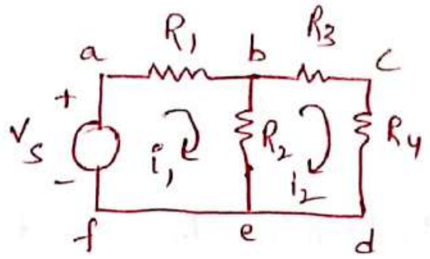
Planar



Non planar

Eg:-

Check:-



→ Check whether the circuit is planar or not.

→ Select mesh currents

→ Write KVL for every loop & solve it

for mesh (1) $V_s = i_1R_1 + R_2(i_1 - i_2)$

(2) $R_3 i_2 + R_4 i_2 + R_2(i_2 - i_1) = 0$

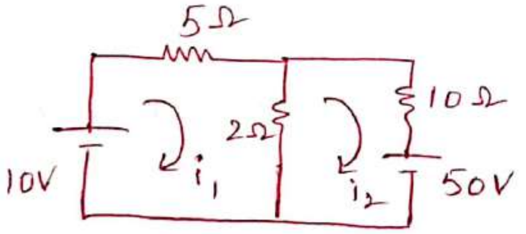
No. of Mesh equations = No. of branches - (No. of nodes - 1)

$$\boxed{M = B - (N-1)}$$

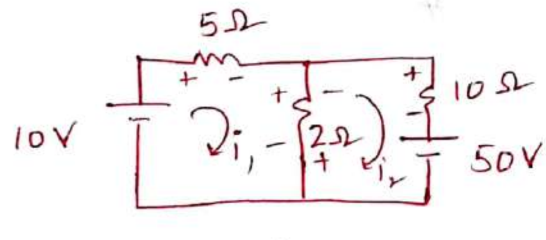
In the above ckt

$$M = 5 - (4-1) = 2$$

Problem 1:- Write mesh currents equations in the circuit shown and determine the currents.



Sol:-



Apply KVL to mesh (1)

$$10 - 5i_1 - 2(i_1 - i_2) = 0 \Rightarrow -7i_1 + 2i_2 = -10 \rightarrow (1)$$

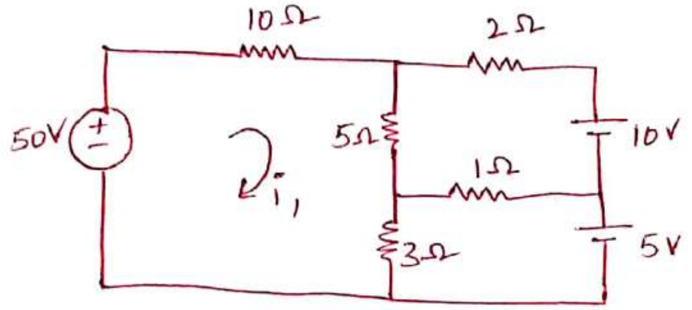
Apply KVL to mesh (2)

$$-10i_2 - 50 - 2(i_2 - i_1) \Rightarrow 2i_1 - 12i_2 = 50 \rightarrow (2)$$

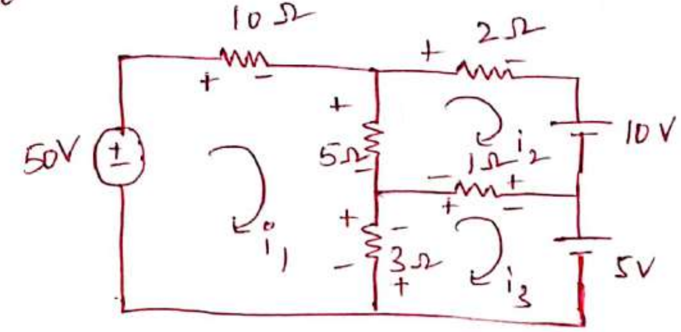
$i_1 = 0.25A$

$i_2 = -4.125A$

→ Problem 2:- Determine the mesh currents I_1 in the circuit shown



Sol:



Apply KVL to loop ①

$\cancel{50 - 10i_1 - 5i_2 - 3}$

$50 - 10i_1 - 5(i_1 - i_2) - 3(i_1 - i_3) = 0 \Rightarrow -18i_1 + 5i_2 + 3i_3 = -50 \rightarrow ①$

Apply KVL to loop ②

$-2i_2 - 10 - 1(i_2 - i_3) - 5(i_2 - i_1) = 0 \Rightarrow 5i_1 - 8i_2 + i_3 = 10 \rightarrow ②$

Apply KVL to loop ③

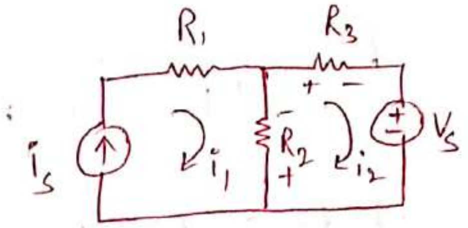
$-3(i_3 - i_1) - 1(i_3 - i_2) - 5 = 0 \Rightarrow 3i_1 + i_2 - 4i_3 = 5 \rightarrow ③$

$i_1 = 3.3A$

$i_2 = 0.99A$

$i_3 = 1.47A$

Mesh Current Analysis with Current Sources:-



→ Mesh current i_1 is equal to i_s

i.e., $i_1 = i_s$

→ Write KVL for second loop

$$-R_3 i_2 - V_s - R_2(i_2 - i_1) = 0$$

$$V_s = i_1 R_2 - i_2(R_2 + R_3)$$

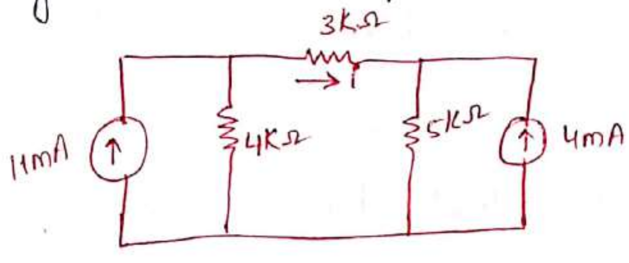
Since $i_1 = i_s$

$$\frac{V_s - i_s R_2}{R_2 + R_3} = -i_2 \Rightarrow i_2 = \frac{-V_s + i_s R_2}{R_2 + R_3}$$

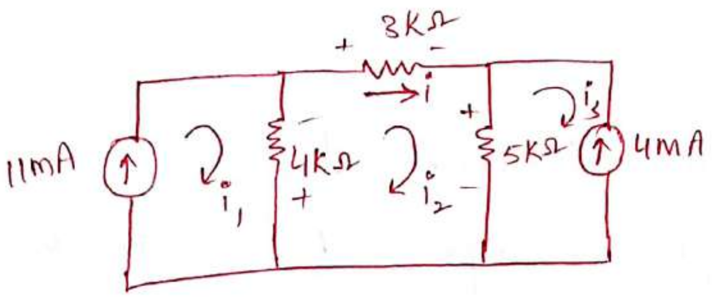
** → Presence of Current Sources reduces the no of mesh equations in mesh analysis.

Problem 1:-

Using mesh analysis find 'i' in the circuit shown in fig.



Sol:



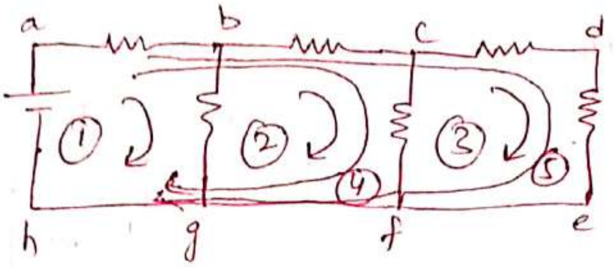
From the circuit $i_1 = 11\text{mA}$, $i_3 = -4\text{mA}$

Apply KVL to mesh 2

$$-3000 i_2 - 5000(i_2 - i_3) - 4000(i_2 - i_1) = 0$$

$$4000 i_1 - 12000 i_2 + 5000 i_3 = 0 \Rightarrow 4000 \times 11 \times 10^{-3} - 12000 i_2 + 5000 \times -4 \times 10^{-3} = 0$$

$\therefore i_2 = 2\text{mA} \Rightarrow \boxed{i = 2\text{mA}}$



ab, bc, cd, dc, cf, bg, ah are branches

①, ②, ③ are mesh equations → $\underbrace{a, b, c, d}_{\text{Common Node}}$ $(\overbrace{e/f/g/h}^{\text{Reference node}})$ are nodes /junctions.

→ Mesh is defined as a loop which doesnot contain any other loop.

→ loops → ①, ②, ③, ④, ⑤ are loops

→ Mesh is always a loop
But every loop not a mesh

$$\text{Mesh equations} = M = [B-(N-1)] = 7-(5-1) = 7-4 = 3$$

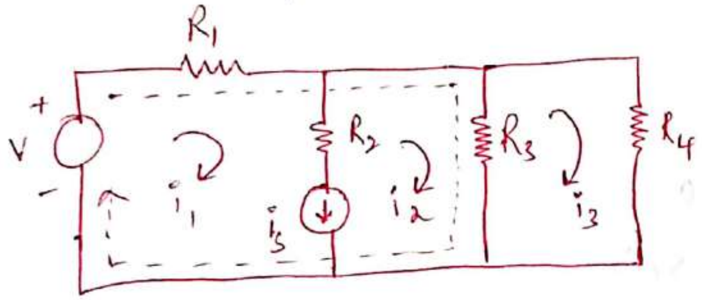
Super Mesh:- A Supermesh occurs when a current source is contained between two essential meshes. It is a larger mesh created from two meshes that has an independent or dependent current source as a common element.

Super Mesh Analysis:-

When a current source is common to two meshes then we use the concept of super mesh to analyze the circuit using mesh current method.

A Supermesh encloses more than one mesh. for each common current source between two meshes, the no. of meshes reduces by one, thus reducing the no. of Independent mesh equations by one.

Eg: In the fig.. i_s is the current source common to mesh ① & ②. Now we can create supermesh shown in dotted line as in fig that consists of the interior of mesh ① & ②.



Now we can apply KVL for super mesh

$$R_1 i_1 + R_3(i_2 - i_3) = V \Rightarrow R_1 i_1 + R_3 i_2 - R_3 i_3 = V \rightarrow ①$$

Consider mesh ③

$$-R_3(i_3 - i_2) - R_4 i_3 = 0 \Rightarrow R_3(i_3 - i_2) + R_4 i_3 = 0 \rightarrow ②$$

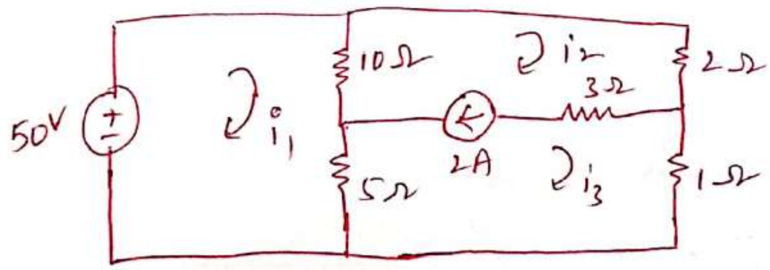
Finally the current i_s from current source is equal to difference between two meshes currents i.e.,

$$i_1 - i_2 = i_s \rightarrow ③$$

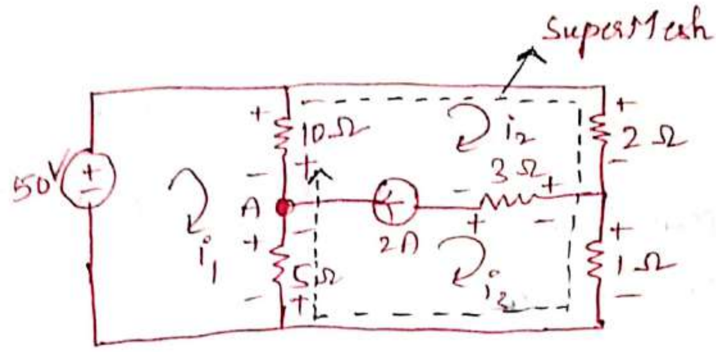
from ①, ② & ③ we calculate i_s.

$$\boxed{\text{Super Mesh analysis} = \text{Ohm's Law} + \text{KVL} + \text{KCL}}$$

Problem:- Determine current in 5Ω resistor in the network given in figure.



Sol:



Apply KVL to loop ①

$$50 - 10(i_1 - i_2) - 5(i_1 - i_3) = 0$$

$$-15i_1 + 10i_2 + 5i_3 = -50 \longrightarrow ①$$

Apply KVL to super mesh (1 & 2 meshes)

$$-10(i_2 - i_1) - 2i_2 - i_3 - 5(i_3 - i_1) = 0$$

$$15i_1 - 12i_2 - 6i_3 = 0 \longrightarrow ②$$

Apply KCL at node A

$$i_2 - i_3 = 2 \longrightarrow ③$$

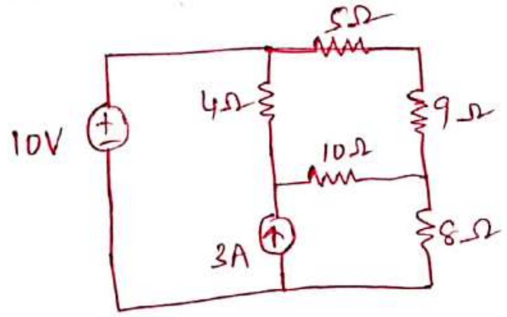
$i_1 = 20A$

$i_2 = 17.33A$

$i_3 = 15.33A$

Current in 5Ω resistor $= (i_1 - i_3) = (20 - 15.33)A = 4.67A \downarrow$

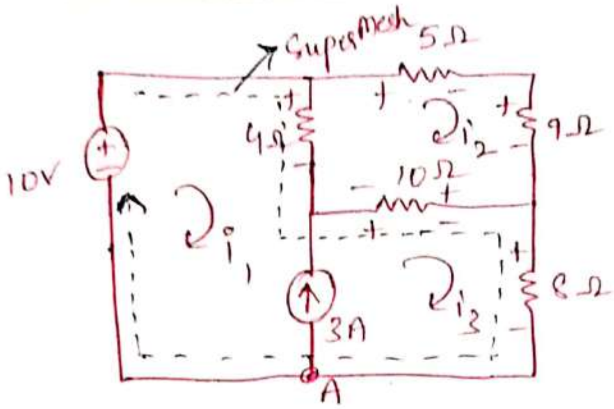
Problem 2:- Find mesh current i_1 in the circuit shown.



Sol:

Assume loop currents

Apply KVL to mesh ① & ③



$$10 - 4(i_1 - i_2) - 10(i_3 - i_2) - 8i_3 = 0$$

$$-4i_1 + 14i_2 - 18i_3 = -10 \rightarrow (1)$$

Apply KVL to loop (2)

$$-5i_2 - 9i_2 - 10(i_2 - i_3) - 4(i_2 - i_1) = 0$$

$$4i_1 - 28i_2 + 10i_3 = 0 \rightarrow (2)$$

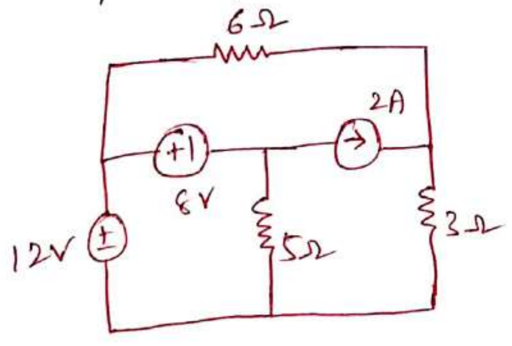
Apply KCL to ~~loop (3)~~ node A

$$i_3 - i_1 = 3 \rightarrow (3)$$

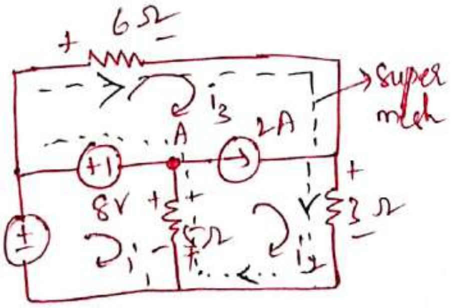
Solve (1), (2) and (3)

$$i_1 = -1.933A,\ i_2 = 0.104A,\ i_3 = 1.06A$$

Problem 3:- Find voltage across 3Ω resistor by using mesh analysis (supermesh problem)



Sol:



Apply KVL to loop (1)

$$12 - 8 - 5(i_1 - i_2) = 0$$

$$-5i_1 + 5i_2 = -4 \rightarrow (1)$$

Apply KVL to super mesh

$$-6i_3 - 3i_2 - 5(i_2 - i_1) = 0$$

$$5i_1 - 8i_2 - 6i_3 = 0 \rightarrow (2)$$

$$+8$$

$$5i_1 - 8i_2 - 6i_3 = -8 \rightarrow (2)$$

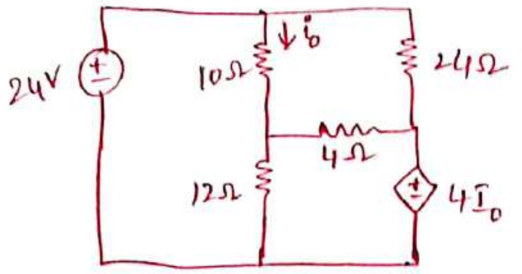
Apply KCL at node A

$$i_2 - i_3 = 2A \rightarrow (3)$$

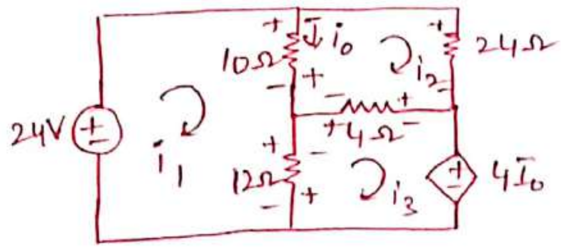
$$i_1 = 3.46A, \quad i_2 = 2.66A, \quad i_3 = 0.66A$$

Dependent Sources Mesh Method Problems:-

Problem 1:- Find the current i_0 for the circuit shown in the figure.



Sol:-



$$i_0 = (i_1 - i_2)$$

Apply KVL to loop (1)

$$24 - 10(i_1 - i_2) - 12(i_1 - i_3) = 0$$

$$-22i_1 + 10i_2 + 12i_3 = -24 \rightarrow (1)$$

Apply KVL to loop (2)

$$-24i_2 - 4(i_2 - i_3) - 10(i_2 - i_1) = 0$$

$$10i_1 - 38i_2 + 4i_3 = 0 \rightarrow (2)$$

Apply KVL to loop (3)

$$-4(i_3 - i_2) - 4i_0 - 12(i_3 - i_1) = 0$$

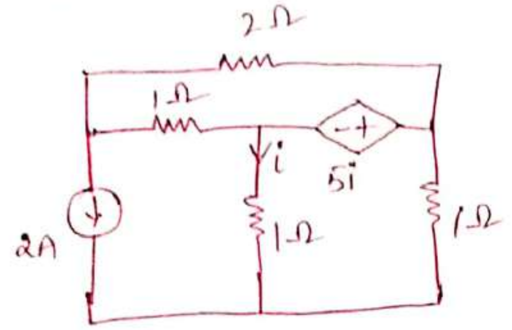
$$-4i_3 + 4i_2 - 4(i_1 - i_2) - 12i_3 + 12i_1 = 0$$

$$8i_1 + 8i_2 - 16i_3 = 0 \rightarrow (3)$$

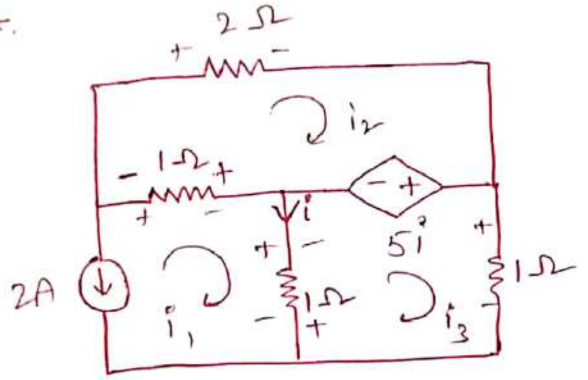
$$i_1 = 2.25A, \quad i_2 = 0.75A, \quad i_3 = 1.5A$$

$$i_0 = i_1 - i_2 = 2.25 - 0.75 = 1.5A$$

Problem 2: Using Mesh analysis find the magnitude of current in dependent source and current through 2Ω resistor.



Sol:



Apply KVL to loop 2

$-2i_2 - 5i - 1(i_2 - i_1) = 0$

$-2i_2 - 5(i_1 - i_3) - 1(i_2 - i_1) = 0$

$-4i_1 - 3i_2 + 5i_3 = 0 \rightarrow (1)$

$i_1 = -2A$

$i = (i_1 - i_3)$

Apply KVL to loop 3

$5i - i_3 - (i_3 - i_1) = 0$

$5(i_1 - i_3) - 2i_3 + i_1 = 0$

$6i_1 - 7i_3 = 0 \rightarrow (2)$

$6 \times -2 - 7i_3 = 0$

$\Rightarrow -7i_3 = 12 \Rightarrow i_3 = \frac{-12}{7} = -1.71A$

Substitute i_3 and i_1 in (1)

$-4 \times -2 - 3i_2 + 5 \times -1.71 = 0$

$-3i_2 = -8 + 8.55$

$= 0.55$

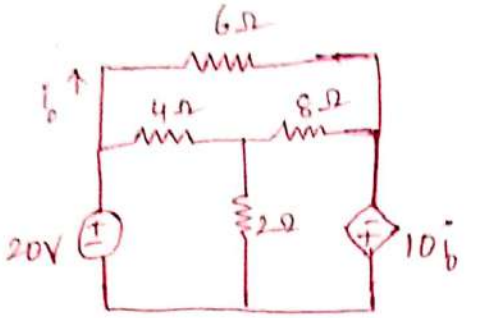
$i_2 = \frac{-0.55}{3} = -0.18 A.$

$\therefore i_1 = -2A, i_2 = -0.18A, i_3 = -1.71A.$

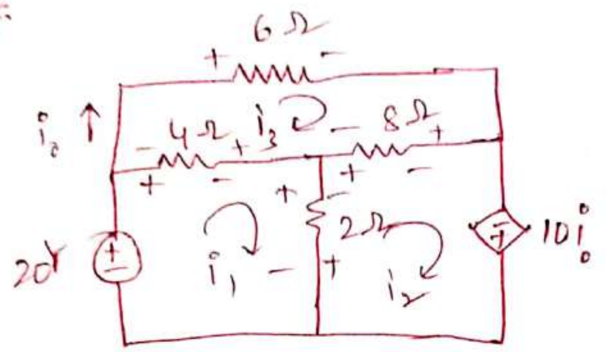
$i = i_1 - i_3 = -2 + 1.71 = -0.29A.$

Magnitude of current source $= 5i = 5 \times 0.29 = 1.45V$

Problems:- Find the current i_o in the circuit shown in fig.



Sol:



$$i_o = i_3$$

Apply KVL to loop ①

$$20 - 4(i_1 - i_3) - 2(i_1 - i_2) = 0$$

$$-6i_1 + 2i_2 + 4i_3 = -20 \rightarrow ①$$

Apply KVL to loop ②

$$-8(i_2 - i_3) + 10i_o - 2(i_2 - i_1) = 0$$

$$2i_1 - 10i_2 + 8i_3 + 10i_3 = 0$$

$$2i_1 - 10i_2 + 18i_3 = 0 \longrightarrow ②$$

Apply KVL to loop ③

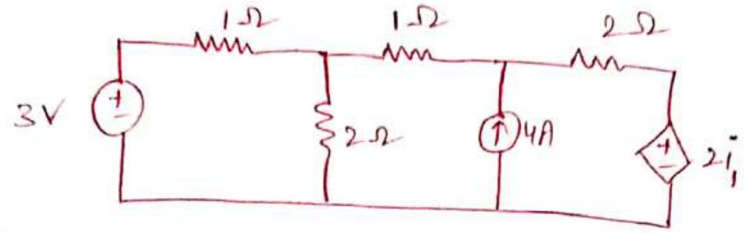
$$-6i_3 - 8(i_3 - i_2) - 4(i_3 - i_1) = 0$$

$$4i_1 + 8i_2 - 18i_3 = 0 \longrightarrow ③$$

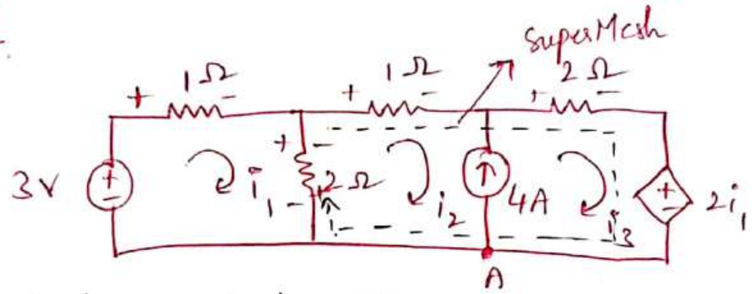
$$\boxed{\begin{aligned} i_1 &= -3.214\,A \\ i_2 &= -9.64\,A \\ i_3 &= -5\,A \end{aligned}}$$

Dependent Source Super Mesh Problems:-

Problem 1: Find the loop currents i_1, i_2 and i_3 in the network of by mesh analysis.



Sol:



Apply KVL to loop ①

$$3 - 1i_1 - 2(i_1 - i_2) = 0$$

$$-3i_1 + 2i_2 = -3 \longrightarrow ①$$

Loops ② and ③ forms a super mesh.

So Apply KVL to loops ② & ③

$$-i_2 - 2i_3 - 2i_1 - 2(i_2 - i_1) = 0$$

$$-3i_2 - 2i_3 = 0 \longrightarrow ②$$

Apply KCL at node A.

$$i_3 - i_2 = 4 \longrightarrow ③$$

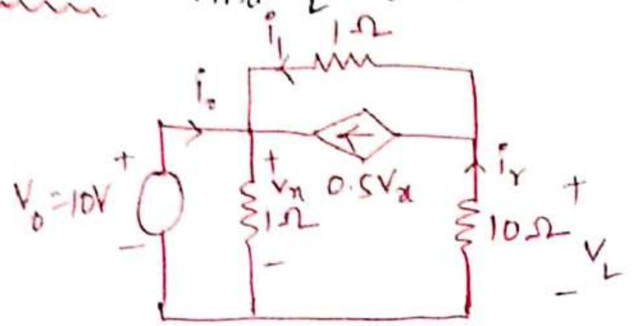
Solve ①, ② & ③

$$i_1 = -0.06A$$

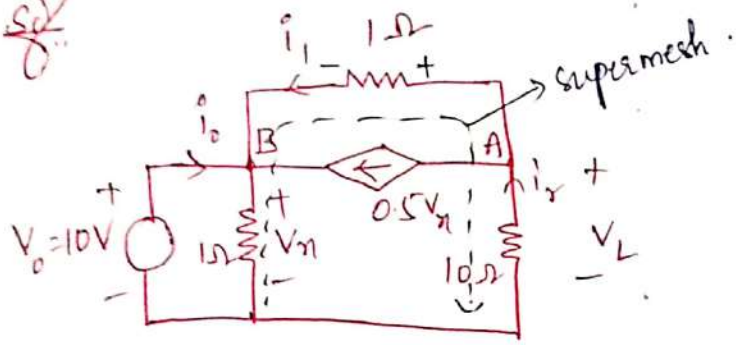
$$i_2 = -1.6A$$

$$i_3 = 2.4A$$

Problem :- Find V_L using KVL



Sol:-



$V_x = 10V.$

$i_r = \dfrac{-V_L}{10}$

From fig Apply KCL at node A.

$$i_r = 0.5V_x + i_1 \Rightarrow i_1 = i_r - 0.5V_x$$

At B apply KCL

$$i_0 + 0.5V_x + i_1 = \frac{V_x}{1}$$

Apply KVL to Super mesh.

$$+V_x + i_1 - V_L = 0$$

$$V_x + (i_r - 0.5V_x) - V_L = 0$$

$$V_x + \left(\frac{-V_L}{10} - 0.5V_x\right) - V_L = 0.$$

$$10 - \frac{V_L}{10} - 0.5 \times 10 - V_L = 0$$

$$5 - 0.1V_L - V_L = 0$$

$$-1.1V_L = -5$$

$$\boxed{V_L = 4.54\,V}$$

Nodal Analysis:-

This method is mainly based on Kirchoff's Current Law (KCL). This method uses the analysis of different nodes of the network. Every junction point in a network where two or more branches meet is called a "node".

If network has more current sources we use nodal analysis.

$$\boxed{\text{KCL + Ohm's Law = Nodal analysis}}$$

→ In general circuit in a 'N' node circuit, one of the nodes is chosen as reference node or datum node, then it is possible to write (N-1) node equations by assuming (N-1) node voltages.

→ In general circuit reference node we assume at zero potential or ground.

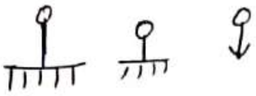
→ The node voltage is the voltage of a given node with respect to one particular node, called reference node which is assumed at zero potential.

→ Select a node as a reference node. Assign voltages to other nodes as $V_1, V_2 \ldots V_{n-1}$ to remaining $(n-1)$ nodes. The voltages are referenced with respect to the reference node.

→ Apply KCL to each of the $(n-1)$ non reference nodes. Use ohm's law to express the branch currents in terms of node voltages

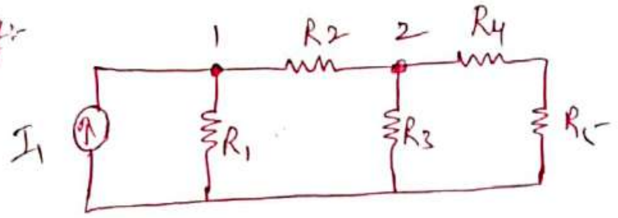
→ Solve the resulting simultaneous equations to obtain the unknown node voltages.

The reference node is commonly called as ground since it is assumed to have zero potential.

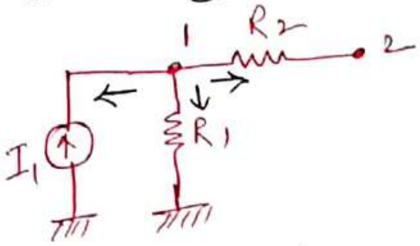
The node voltage is the voltage of a given node with respect to one particular node, called the reference node, which we assume at zero potential 

Current flows from higher potential to lower potential in a resistor.

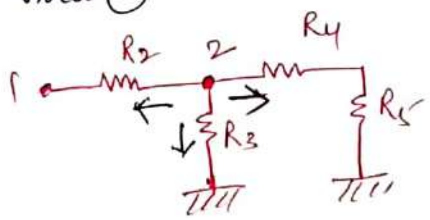
Eg:-



At node ①



At node ②



Apply KCL at node ①

$$\frac{V_1}{R_1} + \frac{V_1 - V_2}{R_2} - I_1 = 0$$

$$\frac{V_1}{R_1} + \frac{V_1 - V_2}{R_2} = I_1 \rightarrow ①$$

$$\frac{V_2 - V_1}{R_2} + \frac{V_2}{R_3} + \frac{V_2}{R_4 + R_5} = 0 \rightarrow ②$$

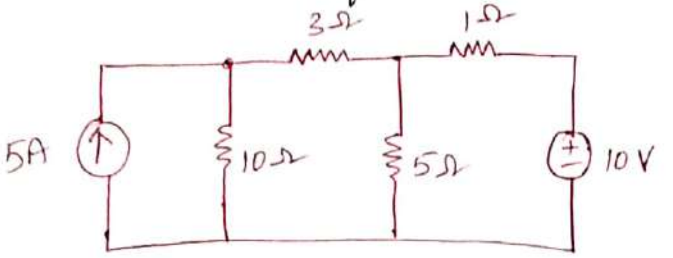
Rearranging the above equations

$$V_1\left(\frac{1}{R_1} + \frac{1}{R_2}\right) - V_2\left(\frac{1}{R_2}\right) = I_1$$

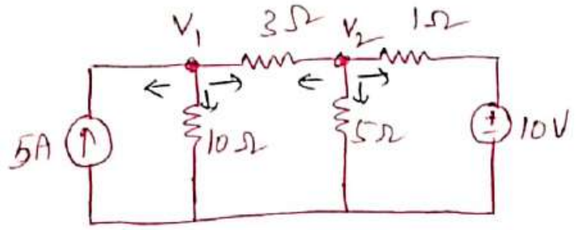
$$V_1\left(\frac{-1}{R_2}\right) + V_2\left(\frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4 + R_5}\right) = 0$$

By solving the above equations, we obtain V_1 and V_2 voltages at each node.

Problem :- Write the node equations and determine the current in each branch of the network shown in figure.



Sol:-



Apply KCL at node ①

$$-5 + \frac{V_1}{10} + \frac{V_1 - V_2}{3} = 0$$

$$0.1V_1 + 0.33V_1 - 0.33V_2 = 5$$

$$0.43V_1 - 0.33V_2 = 5 \rightarrow ①$$

Apply KCL to node ②

$$\frac{V_2 - V_1}{3} + \frac{V_2}{5} + \frac{V_2 - 10}{1} = 0$$

$$0.33V_2 - 0.33V_1 + 0.2V_2 + V_2 - 10 = 0$$

$$-0.33V_1 + 1.53V_2 = 10 \rightarrow ②$$

$$V_1 = 19.94\,V, \quad V_2 = 10.83\,V$$

Current flowing through 10Ω $= \frac{V_1}{10} = \frac{19.94}{10} = 1.99$ Amps ↓

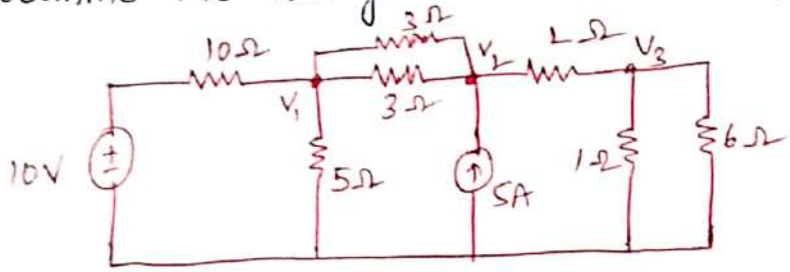
Current flowing through 3Ω $= \frac{V_1 - V_2}{3} = \frac{19.94 - 10.83}{3} = 3.03A$ from node 1 to 2

Current flowing through 5Ω $= \frac{V_2}{5} = \frac{10.83}{5} = 2.16A$

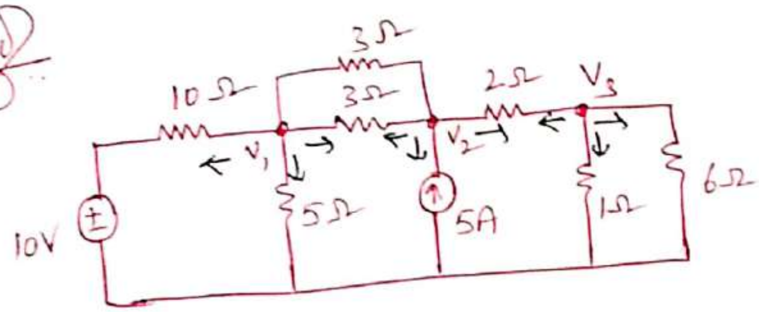
Current flowing through 1Ω $= \frac{V_2 - 10}{1} = \frac{10.83 - 10}{1} = \cancel{9.94A}\ 0.83A$ from node 2 to 10V source.

Problem 2:-

Determine the voltages at each node for the circuit



Sol:



Apply KCL at node ①

$$\frac{V_1-10}{10}+\frac{V_1}{5}+\frac{V_1-V_2}{1.5}=0$$

$$0.1V_1+0.2V_1+0.66V_1-0.66V_2=1$$

$$0.96V_1-0.66V_2=1 \rightarrow ①$$

Apply KCL at node ②

$$-5+\frac{V_2-V_1}{1.5}+\frac{V_2-V_3}{2}=0$$

$$0.66V_2-0.66V_1+0.5V_2-0.5V_3=5$$

$$-0.66V_1+1.16V_2-0.5V_3=5 \rightarrow ②$$

Apply KCL at node ③

$$\frac{V_3-V_2}{2}+\frac{V_3-0}{0.85}=0$$

$$0.5V_3-0.5V_2+1.17V_3=0$$

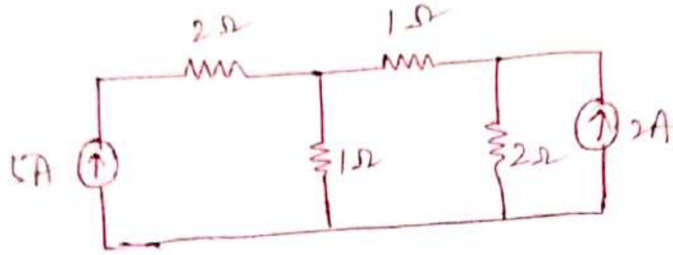
$$-0.5V_2+1.67V_3=0 \longrightarrow ③$$

$V_1=8.06V$

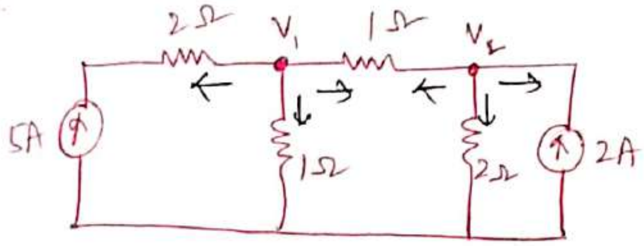
$V_2=10.21V$

$V_3=3.05V$

Problem 3:- Using nodal analysis, find the current in the resistors.



Sol:



Apply KCL at node ①

$$-5 + \frac{V_1}{1} + \frac{V_1 - V_2}{1} = 0$$

$$2V_1 - V_2 = 5 \rightarrow ①$$

Apply KCL at node ②

$$\frac{V_2 - V_1}{1} + \frac{V_2}{2} - 2 = 0$$

$$-V_1 + 1.5V_2 = 2 \rightarrow ②$$

$$V_1 = 4.75V, \quad V_2 = 4.5V$$

Current flowing through 2Ω = 5A from current source 5A to node 1

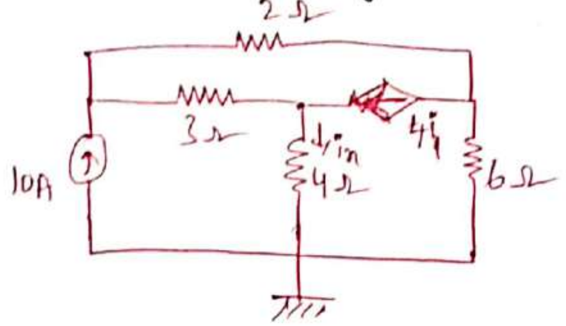
Current flowing through 1Ω = $\frac{V_1}{1} = \frac{4.75}{1} = 4.75A$

Current flowing through 1Ω = $\frac{V_1 - V_2}{1} = \frac{4.75 - 4.5}{1} = 0.25A$ from node 1 to 2

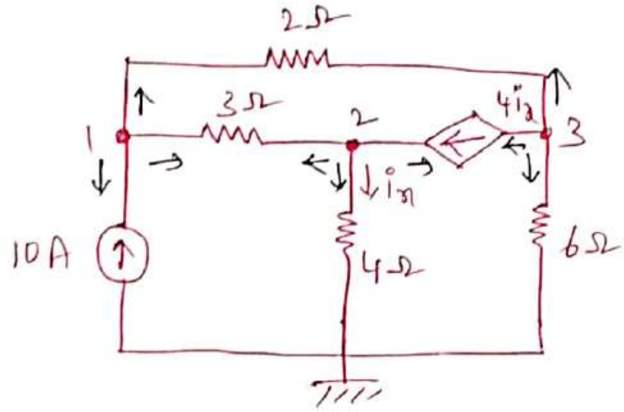
Current flowing through 2Ω = $\frac{V_2}{2} = \frac{4.5}{2} = 2.25A$ from node 2 to reference node

Dependent Source Nodal Analysis Problems:-

Problem:- Find node voltages at three non reference nodes in the circuit.



Sol:



Apply KCL at node ①

$$-10 + \frac{V_1 - V_2}{3} + \frac{V_1 - V_3}{2} = 0$$

$$0.333V_1 - 0.333V_2 + 0.5V_1 - 0.5V_3 = 10$$

$$0.833V_1 - 0.333V_2 - 0.5V_3 = 10 \rightarrow ①$$

Apply KCL at node ②

$$\frac{V_2 - V_1}{3} + i_n - 4i_a = 0 \quad , \quad i_n = \frac{V_2}{4}$$

$$\frac{V_2 - V_1}{3} + \frac{V_2}{4} - 4\left(\frac{V_2}{4}\right) = 0$$

$$0.33V_2 - 0.333V_1 + 0.25V_2 - V_2 = 0$$

$$-0.333V_1 - 0.42V_2 = 0 \rightarrow ②$$

Apply KCL at node ③

$$\frac{V_3}{6} + \frac{V_3 - V_1}{2} + 4i_n = 0$$

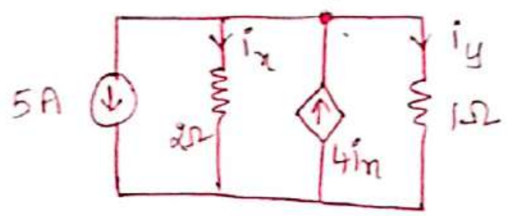
$$0.166V_3 + 0.5V_3 - 0.5V_1 + 4\left(\frac{V_2}{4}\right) = 0$$

$$0.166V_3 + 0.5V_3 - 0.5V_1 + V_2 = 0$$

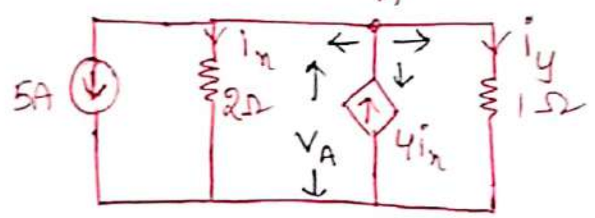
$$-0.5V_1 + V_2 + 0.666V_3 = 0$$

$$V_1 = 80.62V \;,\; V_2 = -63.92 = -64V \;,\; V_3 = 156.51V$$

Problem 2:- obtain i_x, i_y and numerical value of current dependent source.



Sol:-



Apply KCL at node A.

$$5 + i_x - 4i_x + i_y = 0$$

Assume node voltage at A to be V_A.

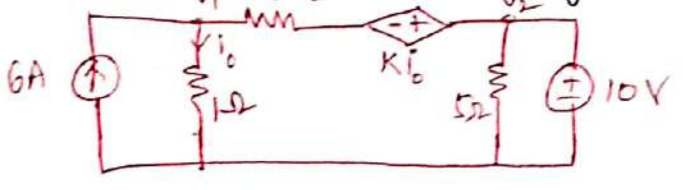
$$5 + \frac{V_A}{2} - 4\left(\frac{V_A}{2}\right) + \frac{V_A}{1} = 0$$

$$-\frac{V_A}{2} = -5 \Rightarrow V_A = 10V$$

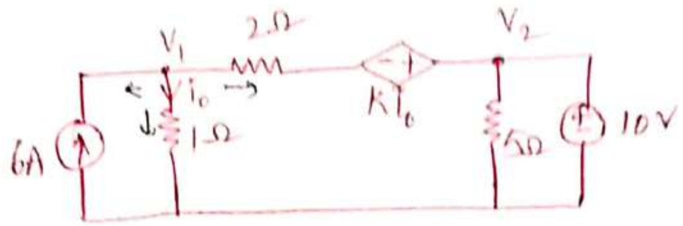
$$i_x = \frac{10}{2} = 5A$$

∴ Numerical value of current dependent source $= 4i_x = 4\times5$
$= 20A$.

Problem 3:- If power loss in 1Ω resistor is 25W, find the value of K in the dependent source using nodal method.



Sol



Apply KCL at node ①, Given $V_2 = 10V$

$$-6 + i_0 + \frac{V_1 + Ki_0 - V_2}{2} = 0$$

$$-6 + \frac{V_1}{1} + \frac{V_1 + Ki_0 - 10}{2} = 0$$

$$\frac{2V_1 + V_1 + Ki_0 - 10}{2} = 6$$

$$3V_1 + Ki_0 - 10 = 12$$

$$3V_1 + K\left(\frac{V_1}{1}\right) - 10 = 12$$

$$(K+3)V_1 - 10 = 12 \longrightarrow ①$$

Given

Power loss in 1Ω resistor $= i_0^2 \times 1 = \left(\frac{V_1}{1}\right)^2 \times 1 = 25$

$$V_1^2 = 25$$

$$V_1 = 5V$$

Substitute V_1 in ①

$$(K+3)5 - 10 = 12$$

$$5K + 15 = 22$$

$$5K = 22 - 15 = 7$$

$$K = 7/5 = 1.4$$

$$\therefore \boxed{K = 1.4}$$

Super Node:- Whenever a voltage source (Independent or Dependent) is connected between the two non reference nodes, then these two nodes form a generalized node called "Super Node". A Super node can be regarded as a surface enclosing the voltage source and its two nodes.

Super Node Analysis:-

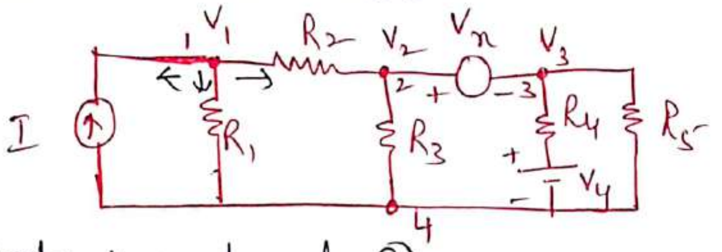
Suppose any of the branches in the network has a voltage source, then it is slightly difficult to apply nodal analysis

→ One way to overcome this difficulty is to apply the supernode technique.

→ In this method, the two adjacent nodes that are connected by a voltage source are reduced to a single node & then the equations are formed by applying KCL.

$$\boxed{\text{Super Node analysis} = \text{Ohm's law} + \text{KVL} + \text{KCL}}$$

Consider the circuit below



Node 1, 2, 3 are non reference node
Node 4 is reference node

Apply KCL at node ①

$$-I + \frac{V_1}{R_1} + \frac{V_1 - V_2}{R_2} = 0$$

$$\Rightarrow I = V_1\left(\frac{1}{R_1} + \frac{1}{R_2}\right) - V_2\left(\frac{1}{R_2}\right) \rightarrow ①$$

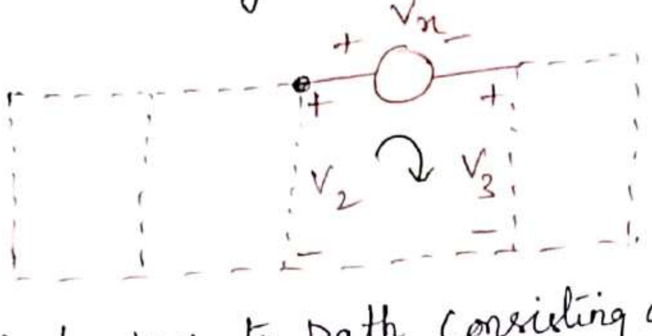
→ V_x is between nodes ② & ③ it is slightly difficult to find out the current. The supernode technique can be conveniently applied in this case.

Accordingly we can write combined equation for nodes ② & ③

$$\frac{V_2 - V_1}{R_2} + \frac{V_2}{R_3} + \frac{V_3 - V_4}{R_4} + \frac{V_3}{R_5} = 0 \longrightarrow ②$$

Since V_x is in between two non reference nodes we apply KCL & KVL to determine the node voltages.

A Supernode may be regarded as a closed surface enclosing the voltage source and its two nodes.



Apply KVL to path consisting of V_x, V_1 & V_2

$$-V_x - V_3 + V_2 = 0$$

$$V_2 - V_3 = V_x \longrightarrow (3)$$

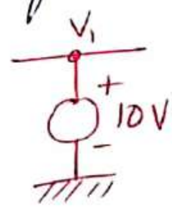
By solving equations (1), (2) and (3)

V_1, V_2 and V_3 can be obtained.

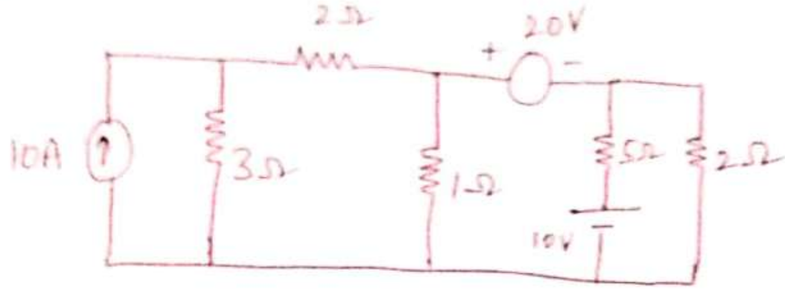
Note the following properties of Super Node.

1. The voltage source inside the supernode provides a constraint equation needed to solve for node voltages.
2. A Supernode has no voltage of its own.
3. A Supernode requires the application of both KVL and KCL.

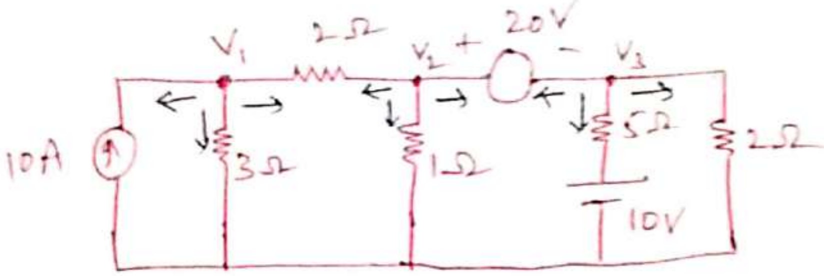
Note:- If a voltage source is connected between the reference node and non reference node we set the voltage at the non reference node equal to the voltage of the voltage source. For eg... $V_1 = 10V$



Problem:- Determine the current in 5Ω resistor for the circuit 1.26 shown below.



Sol



Since 20V voltage source is in between two non reference nodes Nodes ② & ③ form a Super Node.

Apply KCL to node ①

$$-10 + \frac{V_1}{3} + \frac{V_1 - V_2}{2} = 0$$

$$V_1\left(\frac{1}{3}+\frac{1}{2}\right) - V_2\left(\frac{1}{2}\right) = 10$$

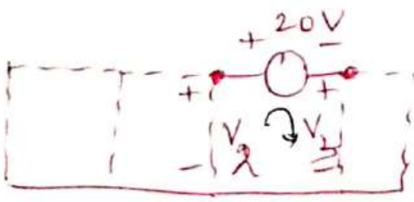
$$0.833V_1 - 0.5V_2 = 10 \longrightarrow ①$$

Apply KCL to SuperNode.

$$\frac{V_2 - V_1}{2} + \frac{V_2}{1} + \frac{V_3 - 10}{5} + \frac{V_3}{2} = 0$$

$$V_1\left(\frac{-1}{2}\right) + V_2\left(\frac{1}{2}+1\right) + V_3\left(\frac{1}{5}+\frac{1}{2}\right) = 2$$

$$-0.5V_1 + 1.5V_2 + 0.7V_3 = 2 \longrightarrow ②$$



Apply KVL to the path Consisting of super node.

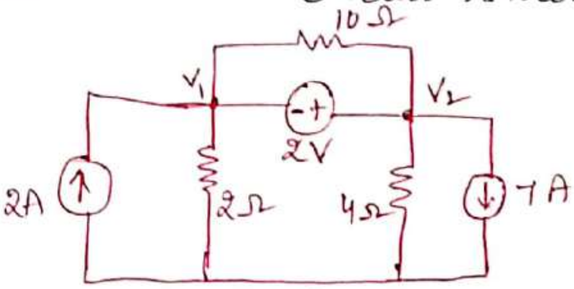
$$V_2 - 20 - V_3 = 0 \Rightarrow V_2 - V_3 = 20 \longrightarrow ③$$

Solve (1), (2) & (3) equations

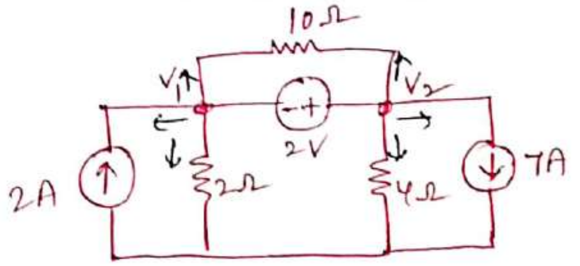
$V_1 = 18.95V$, $V_2 = 11.58V$, $V_3 = -8.41V$

Current in 5Ω resistor $= \frac{V_3 - 10}{5} = \frac{-8.41-10}{5} = -3.68$ Amps from node 3 to source 10V.

Problem 2:- For the circuit shown, find node voltages.



Sol:



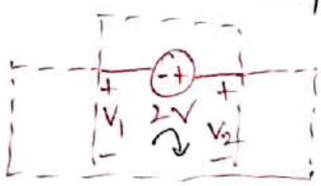
Nodes (1) & (2) form a super node since they have a voltage source b/w two non reference nodes.

Apply KCL to (1) & (2) Nodes

$$-2 + \frac{V_1}{2} + \frac{V_1 - V_2}{10} + \frac{V_2 - V_1}{10} + \frac{V_2}{4} + 7 = 0$$

$$V_1\left(\frac{1}{2} + \frac{1}{10} - \frac{1}{10}\right) + V_2\left(\frac{-1}{10} + \frac{1}{10} + \frac{1}{4}\right) = -5$$

$$0.5V_1 + 0.25V_2 = -5 \rightarrow (1)$$



Apply KVL to the path containing voltage source b/w two non reference nodes.

$$V_1 + 2 - V_2 = 0$$

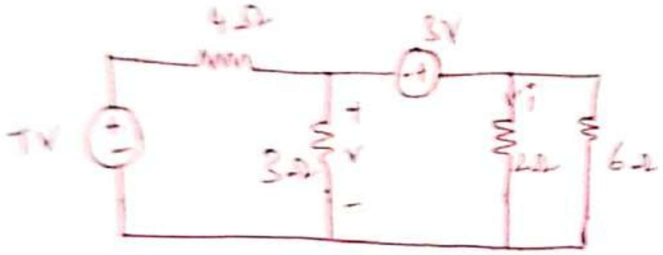
$$V_1 - V_2 = -2 \longrightarrow (1)$$

Solve (1) & (2) equations to obtain V_1 & V_2

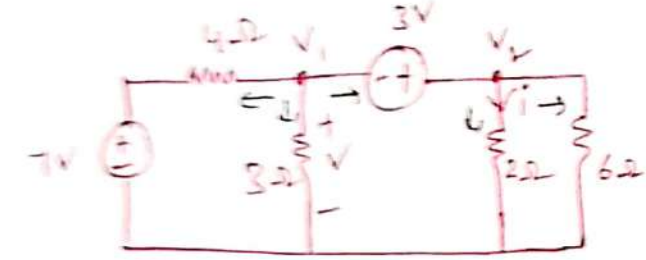
$$V_1 = -7.33V$$

$$V_2 = -5.33V$$

Problem 2 - Find v & i in the Circuit.



Sol



Node 1 and 2 form a super node.

Apply KCL to SuperNode

$$\frac{V_1-7}{4}+\frac{V_1}{3}+\frac{V_2}{2}+\frac{V_2}{6}=0$$

$$V_1\left(\frac{1}{4}+\frac{1}{3}\right)+V_2\left(\frac{1}{2}+\frac{1}{6}\right)=\frac{7}{4}$$

$$0.58V_1+0.66V_2=1.75\rightarrow (1)$$

Apply KVL to path Containing voltage source between nodes

$$V_1+3-V_2=0$$

$$V_1-V_2=-3\rightarrow (2)$$

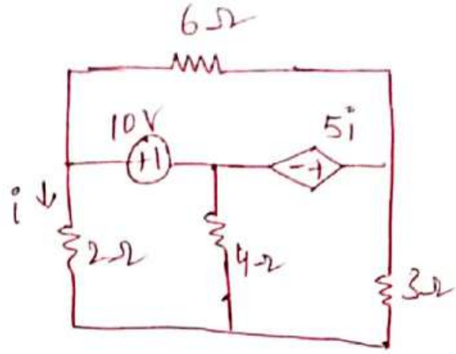
$$V_1=-0.185V,\quad V_2=2.814V$$

$$V=V_1=-0.185V$$

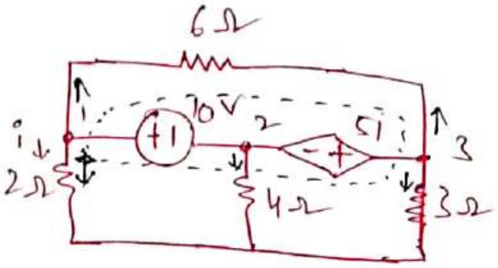
$$i=\frac{V_2}{2}=\frac{2.814}{2}=1.407\text{ Amps}.$$

Dependent sources Super Node analysis Problems:-

Problem 1:- Find V_1, V_2 and V_3 in the Circuit shown using nodal analysis.



Sol:



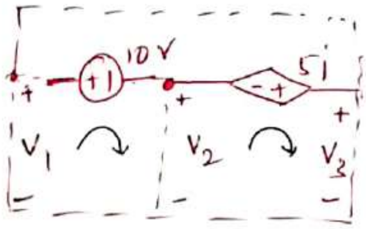
Nodes 1, 2, 3 form super Node.

Apply KCL to Super Node.

$$\frac{V_1}{2} + \frac{V_1 - V_3}{6} + \frac{V_2}{4} + \frac{V_3 - V_1}{6} + \frac{V_3}{3} = 0$$

$$V_1\left(\frac{1}{2} - \frac{1}{6} + \frac{1}{6}\right) + V_2\left(\frac{1}{4}\right) + V_3\left(\frac{-1}{6} + \frac{1}{6} + \frac{1}{3}\right) = 0$$

$$0.5V_1 + 0.25V_2 + 0.333V_3 = 0 \longrightarrow (1)$$



Apply KVL to loop (1)

$$V_1 - 10 - V_2 = 0$$

$$V_1 - V_2 = 10 \longrightarrow (2)$$

Apply KVL to loop (2)

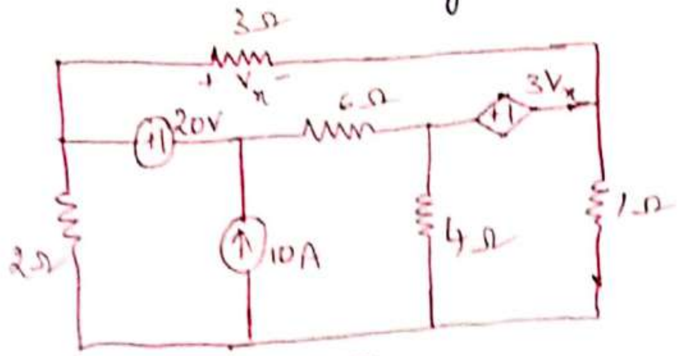
$$V_2 + 5i - V_3 = 0$$

$$V_2 - V_3 = -5i \Rightarrow V_2 - V_3 = 5\left(\frac{-V_1}{2}\right)$$

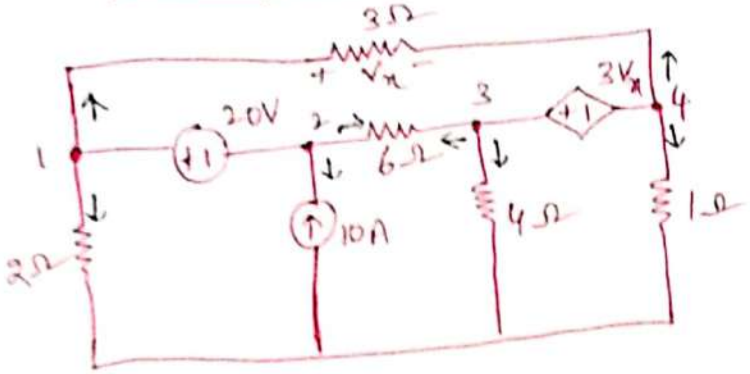
$$V_2 - V_3 + 2.5V_1 = 0 \longrightarrow (3)$$

$V_1 = 3.04V$, $V_2 = -6.95V$, $V_3 = 0.65V$

Problem 2 :- Find node voltages in the circuit



Sol



Nodes ① & ② , ③ & ④ form super nodes

Apply KCL to ① & ② $V_x = V_1 - V_4$

$$\frac{V_1}{2} + \frac{V_1 - V_4}{3} + \frac{V_2 - V_3}{6} - 10 = 0$$

$$V_1\left(\frac{1}{2} + \frac{1}{3}\right) + V_2\left(\frac{1}{6}\right) - V_3\left(\frac{1}{6}\right) - V_4\left(\frac{1}{3}\right) = 10$$

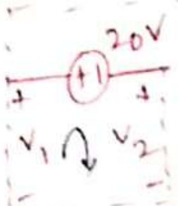
$$0.83V_1 + 0.16V_2 - 0.16V_3 - 0.33V_4 = 10 \rightarrow ①$$

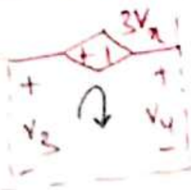
Apply KCL to ③ & ④

$$\frac{V_3 - V_2}{6} + \frac{V_3}{4} + \frac{V_4}{1} + \frac{V_4 - V_1}{3} = 0$$

$$V_1\left(\frac{-1}{3}\right) + V_2\left(\frac{-1}{6}\right) + V_3\left(\frac{1}{6} + \frac{1}{4}\right) + V_4\left(1 + \frac{1}{3}\right) = 0$$

$$-0.33V_1 - 0.16V_2 + 0.41V_3 + 1.33V_4 = 0 \rightarrow ②$$





Apply KVL to loop ①

$V_1 - 20 - V_2 = 0 \Rightarrow V_1 - V_2 = 20 \rightarrow ③$

Apply KVL to loop ③

$V_3 - 3V_x - V_4 = 0 \Rightarrow V_3 - 3(V_1 - V_4) - V_4 = 0$

$-3V_1 + V_3 + 2V_4 = 0 \rightarrow ④$

From (4) $V_3 = 3V_1 - 2V_4$

Substitute in (1), (2) & (3).

From (1) $0.83V_1 + 0.16V_2 - 0.16(3V_1 - 2V_4) - 0.33V_4 = 10$

$0.35V_1 + 0.16V_2 - 0.01V_4 = 10 \rightarrow (5)$

From (2) $-0.33V_1 - 0.16V_2 + 0.41(3V_1 - 2V_4) + 1.33V_4 = 0$

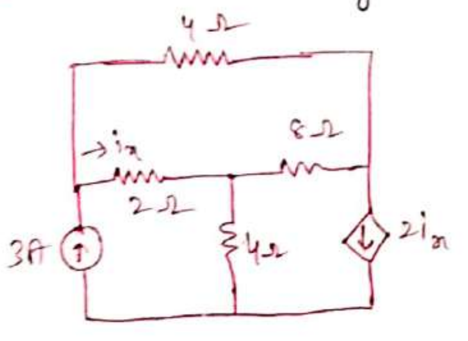
$0.9V_1 - 0.16V_2 + 0.51V_4 = 0 \rightarrow (6)$

From (3) $V_1 - V_2 = 20 \rightarrow (7)$

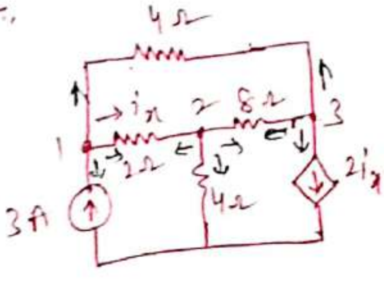
solve (5), (6) & (7) $V_1 = 25.04$, $V_2 = 5.04V$, $V_3 = 3V_1 - 2V_4$

$V_4 = -42.61V$, $V_3 = 160.34V$ =

Problem 3:- Determine voltages at the nodes. (Nodal analysis problem)



Sol.



Apply KCL at node (1)

$$-3 + \frac{V_1 - V_2}{2} + \frac{V_1 - V_3}{4} = 0$$

$$V_1\left(\frac{1}{2} + \frac{1}{4}\right) + V_2\left(\frac{-1}{2}\right) + V_3\left(\frac{-1}{4}\right) = 3$$

$$0.75V_1 - 0.5V_2 - 0.25V_3 = 3 \rightarrow (1)$$

Apply KCL at node (2)

$$\frac{V_2 - V_1}{2} + \frac{V_2}{4} + \frac{V_2 - V_3}{8}$$

$$V_1\left(\frac{-1}{2}\right) + V_2\left(\frac{1}{2} + \frac{1}{4} + \frac{1}{8}\right) + V_3\left(\frac{-1}{8}\right) = 0$$

$$-0.5V_1 + 0.875V_2 - 0.125V_3 = 0 \rightarrow (2)$$

$$\boxed{V_1 = 4.8V,\ V_2 = 2.4V,\ V_3 - 2.4V}$$

Apply KCL at node (3)

$$2i_x + \frac{V_3 - V_2}{8} + \frac{V_3 - V_1}{4} = 0$$

$$2\left(\frac{V_1 - V_2}{2}\right) + \frac{V_3 - V_2}{8} + \frac{V_3 - V_1}{4} = 0$$

$$V_1\left(1 - \frac{1}{4}\right) + V_2\left(-1 - \frac{1}{8}\right) + V_3\left(\frac{1}{8} + \frac{1}{4}\right) = 0$$

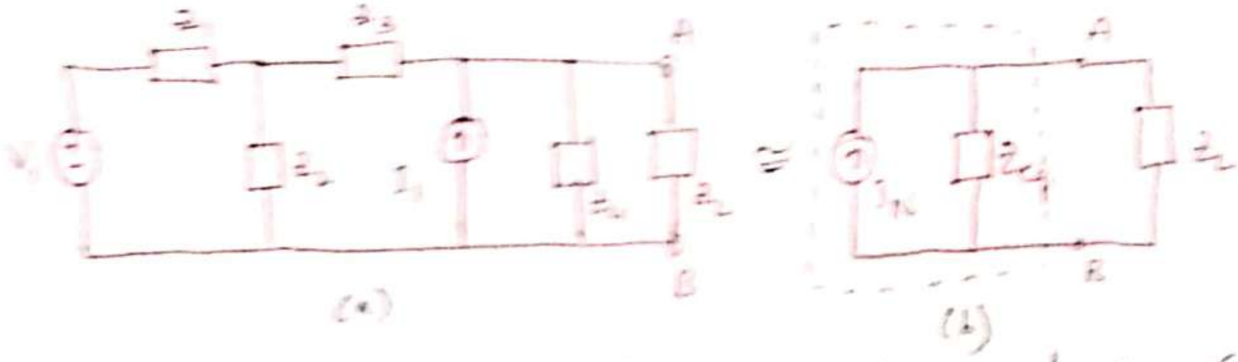
$$0.75V_1 - 1.125V_2 + 0.375V_3 = 0 \rightarrow (3)$$

Theorems:-

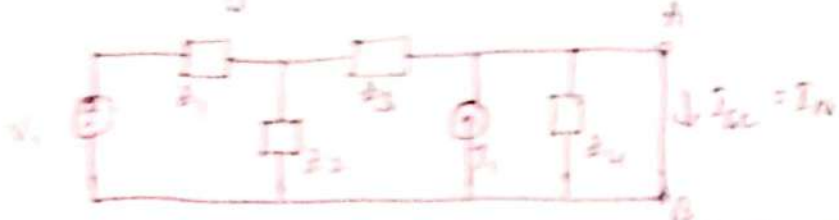
Norton's Theorem:-

Statement:- Any combination of linear bilateral circuit elements and active sources, regardless of connection or complexity, connected to a given load Z_L, can be replaced by a simple two terminal network, consisting of a single current source of I_N amperes and a single impedance Z_{eq} in parallel with it, across the two terminals of the load Z_L. The I_N is the short-circuit current flowing through the short circuited path, replaced instead of Z_L. It is also called Norton's current. The Z_{eq} is the equivalent impedance of the given network as viewed through the load terminals, with Z_L removed and all the active sources are replaced by their internal impedances. If internal impedances are unknown then the independent voltage sources must be replaced by short circuit while the independent current sources must be replaced by open circuit, while calculating Z_{eq}.

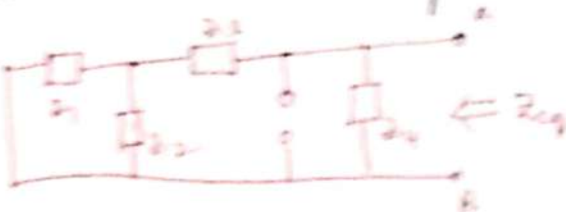
Explanation of Norton's theorem:- Consider a network shown in fig (a) below. The terminals A-B are load terminals where load impedance Z_L is connected. According to Norton's theorem, the entire network can be replaced by a current source I_N, and an equivalent impedance Z_{eq} in parallel with it, across the load terminals A-B as shown in fig (b)



For obtaining current I_N, short the load terminals A-B. Calculate the current through the short circuited path by using any of the network simplification techniques, This is Norton's current I_N. It is shown in Fig below



While the equivalent impedance Z_{eq} is to be obtained by the same procedure as in case of Thevenin's theorem



When the circuit is replaced by Norton's equivalent across the load terminals, then the load current can be easily obtained by using current division in a parallel circuit as,

$$I_L = I_N \times \frac{Z_{eq}}{Z_L + Z_{eq}}$$

This theorem is also called dual of Thevenin's theorem. This is because, if the thevenin's equivalent voltage source is converted to an equivalent current source, the Norton's equivalent is obtained. this is shown in fig

From source transformation we can write

$$I_N = \frac{V_{th}}{Z_{eq}}$$

$$\text{or } Z_{eq} = \frac{V_{th}}{I_N}$$

Steps to Apply Norton's theorem:-

Step 1:- Short the branch, through which the current is to be Calculated by removing the impedance between the terminals

Step 2:- obtain the current through this short circuited branch, using any of the network simplification techniques. This current is nothing but Norton's current I_N.

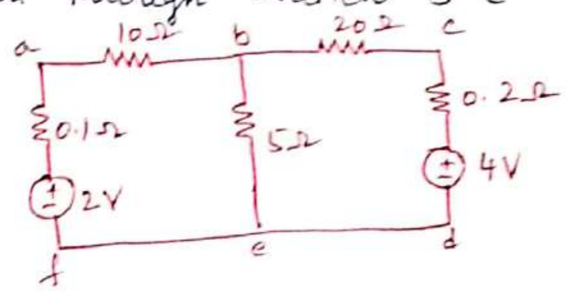
Step 3:- Calculate the equivalent impedance Z_{eq}, as viewed through the two terminals of interest by removing the branch impedance and making all the independent sources inactive

Step 4:- Draw the Norton's equivalent across the terminals of interest, showing a current source I_N with the impedance Z_{eq} parallel with it. Reconnect the branch impedance now. Let it be Z_L. The current through the branch of interest is,

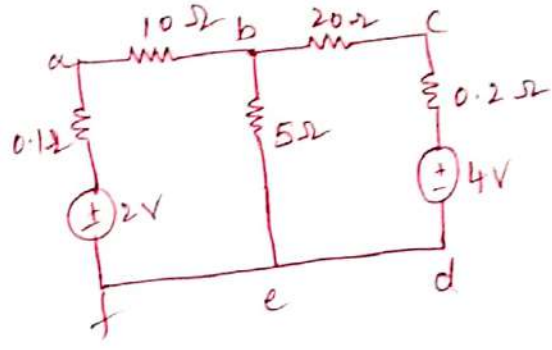
$$I = I_N \times \frac{Z_{eq}}{Z_{eq}+Z_L}$$

[Note:- If dependent sources are present in the circuit then $\boxed{Z_{eq} = \frac{V_{th}}{I_N}}$]

Problem 1:- Find the current through branch 'b-e' using Norton's theorem.

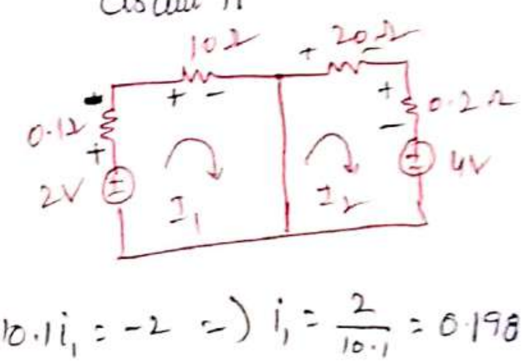


Sol:-



Step:- 1

Remove 5Ω resistor and short circuit it



Step 2:- Apply KVL to loop ①

$$2 - 0.1i_1 - 10i_1 = 0 \Rightarrow -10.1 i_1 = -2 \Rightarrow i_1 = \frac{2}{10.1} = 0.198 \text{ Amps}$$

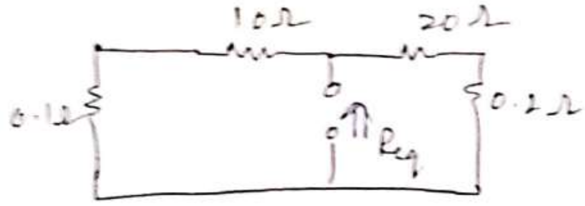
Apply KVL to loop ②

$$-20i_2 - 0.2i_2 - 4 = 0$$

$$-20.2i_2 = 4 \Rightarrow i_2 = \frac{-4}{20.2} = -0.198 \text{ amps}$$

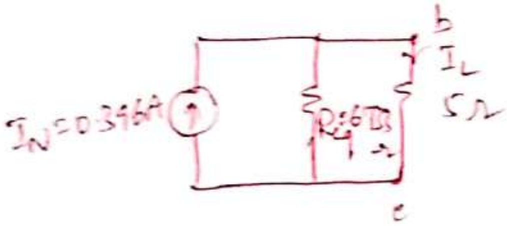
$$I_N = I_1 - I_2 = 0.198 - (-0.198) = 0.396 \text{ amps}.$$

Step 3:- Calculate equivalent impedance.



$$R_{eq} = (10+0.1) // (20+0.2)$$
$$= 10.1 // 20.2$$
$$= 6.733\,\Omega.$$

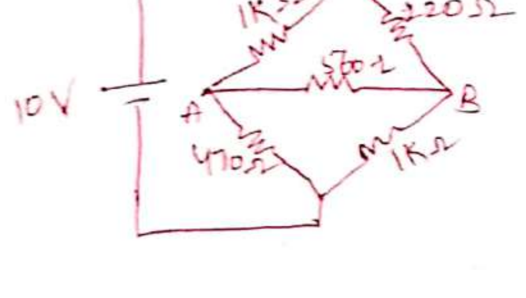
Step 4:- Draw Norton's equivalent circuit & find current through branch 'b-c'.



$$I_L = I_N \times \frac{R_{eq}}{R_{eq}+5}$$
$$= 0.396 \times \frac{6.733}{6.733+5} = 0.227 \text{ Amps}$$

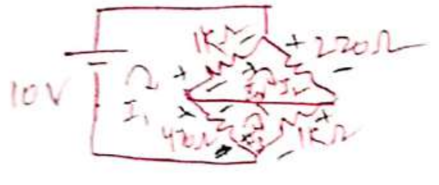
∴ Current flowing through 5Ω resistor = 0.227 amps

Problem 2:- Find current through 560Ω resistor using Norton's theorem



Sol:

Step 1:- Remove 560Ω and short circuit A-B terminals



Step 2:- Find I_N

Apply KVL to loop (1)

$$10 - 1000(i_1 - i_2) - 470(i_1 - i_3) = 0$$

$$-1470 i_1 + 1000 i_2 + 470 i_3 = -10 \rightarrow (1)$$

Apply KVL to loop (2)

$$-220 i_2 - 1000(i_2 - i_1) = 0$$

$$1000 i_1 - 1220 i_2 = 0 \rightarrow (2)$$

Apply KVL to loop (3)

$$-1000 i_3 - 470(i_3 - i_1) = 0$$

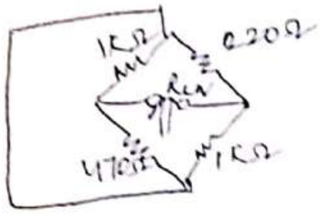
$$470 i_1 - 1470 i_3 = 0 \rightarrow (3)$$

$i_1 = 19.99\,mA$, $i_2 = 16.39\,mA$, $i_3 = 6.39\,mA$.

$i_N = i_2 - i_3 = 16.39\,mA - 6.39\,mA = 10\,mA$.

$\therefore i_N = 10\,mA$

Step 3:- To find R_{eq}

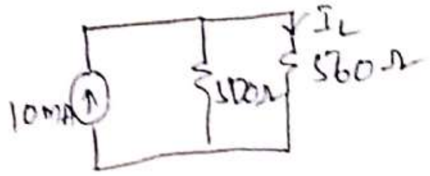


$$R_{eq} = (470 \,/\!/\, 1K\Omega) + (220 \,/\!/\, 1K\Omega)$$

$$= 319.72 + 180.32$$

$$= 500\,\Omega$$

Step 4:- To find I_L, draw Norton's equivalent Circuit.

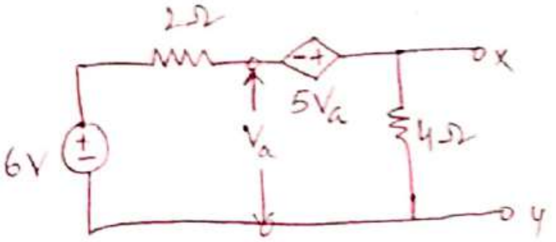


$$I_L = 10 \times 10^{-3} \times \frac{500}{500 + 560}$$

$$= 4.716\,mA.$$

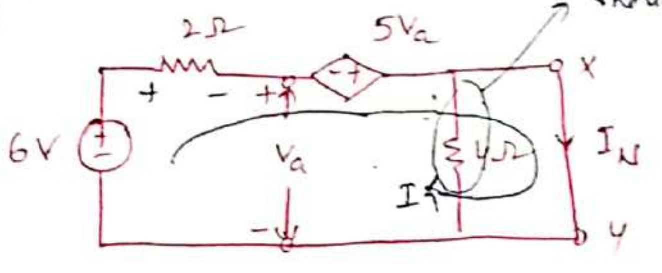
$\therefore$ Current flowing through $560\,\Omega = 4.7\,mA$

Problems: Find Norton's equivalent of the network shown in fig. at x-y terminals. [Dependent source problem]



Sol

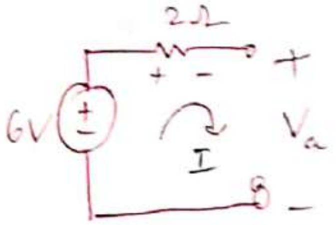
shorted, so no current flow



Step-1 short circuit x y terminals.

So 4Ω resistor is shorted and so it is bipassed.

$I = I_N$



$6 - 2I - V_a = 0$

$\Rightarrow V_a = 6 - 2I$

Apply KVL to the loop

$$6 - 2I + 5V_a = 0$$

$$6 - 2I + 5(6 - 2I) = 0$$

$$6 - 2I + 30 - 10I = 0$$

$$-12I + 36 = 0$$

$$-12I = -36$$

$$\Rightarrow I = \frac{36}{12} = 3$$

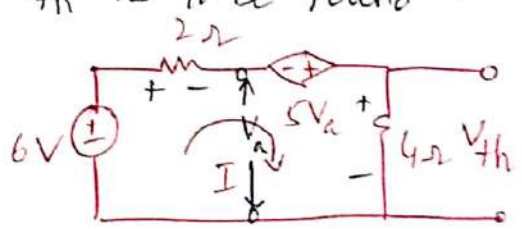
$\therefore I = 3A.$

$\Rightarrow I_N = 3$ Amps

Step 2:- To find R_{eq}.

If we have dependent sources in the network $R_{eq} = \frac{V_{th}}{I_N}$

So V_{th} is to be found.



$$6 - 2I + 5V_a - 4I = 0.$$

$$6 - 2I + 5(6 - 2I) - 4I = 0$$

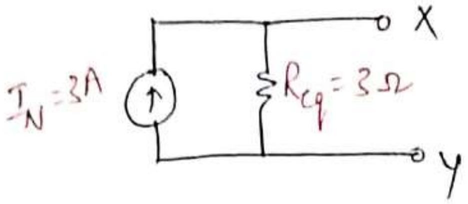
$$6 - 2I + 30 - 10I - 4I = 0$$

$$-16I = -36 \Rightarrow I = \frac{36}{16} = 2.25A$$

$$V_{th} = 4 \times I = 4 \times 2.25 = 9V$$

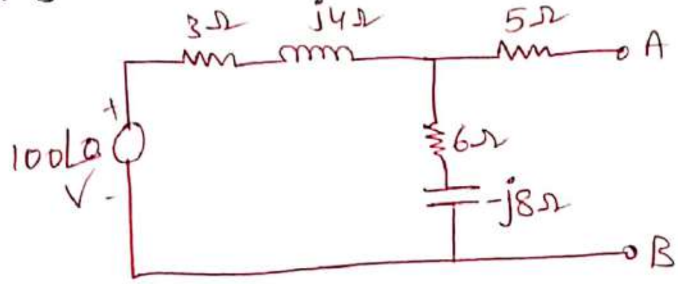
$$\therefore R_{eq} = \frac{V_{th}}{I_N} = \frac{9}{3} = 3\Omega$$

Step3:- Draw Norton's equivalent circuit.

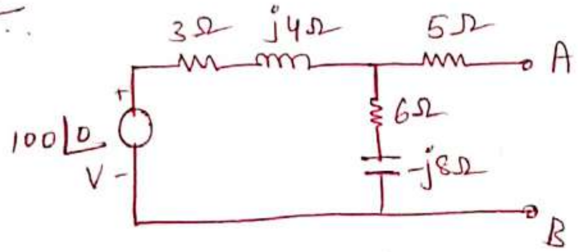


AC Excitation Norton Theorem Problems:-

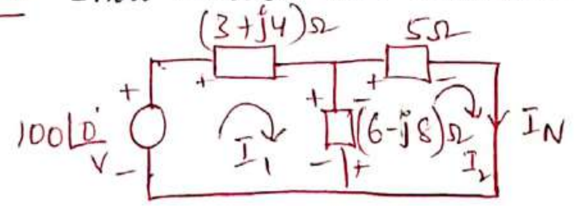
Problem1:- Obtain Norton equivalent circuit with respect to terminals A and B.



Sol:-



step1:- Short circuit AB terminals.



Step2:- To find I_N $\qquad I_2 = I_N$

Apply KVL to loop ①

$$100 - (3+j4)I_1 - (6-j8)(I_1 - I_2) = 0$$

$$[-(3+j4) - (6-j8)]I_1 + (6-j8)I_2 = -100$$

$$(-9+j4)I_1 + (6-j8)I_2 = -100 \rightarrow ①$$

$$(-9+j4)I_1 = -100 - (6-j8)I_2$$

$$I_1 = \frac{-100 - (6-j8)I_2}{-9+j4} \rightarrow ②$$

Apply KVL to loop ②

$$-5I_2-(6-j8)(I_2-I_1)=0$$

$$-11I_2+j8I_2+6I_1-j8I_1=0$$

$$I_2(-11+j8)=(-6+j8)I_1 \rightarrow ②$$

Substitute I_1 in eq ②

$$I_2(-11+j8)=(-6+j8)\left(\frac{-100-(6-j8)I_2}{-9+j4}\right)$$

$$\cancel{\frac{I}{2} = \frac{(-6+j8)(-106+j8)}{(-9+j4)(-11+j8)}} = \cancel{7.927+0.35i}$$

$$I_2(-11+j8)=\frac{600-j800+(36-j48-j48+(-64))I_2}{(-9+j4)}$$

$$I_2(-11+j8)(-9+j4)=600-j800+(-28-j96)I_2$$

$$I_2(133.9\angle -66.65)+(28+j96)I_2=600-j800$$

$$I_2(133.9\angle -66.65)+100\angle 81.93\,I_2=600-j800$$

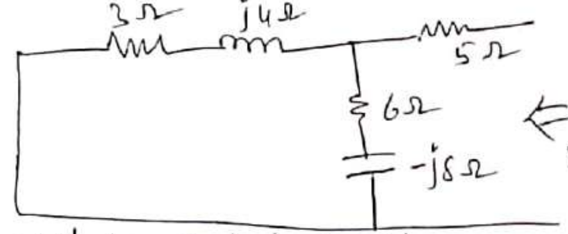
$$I_2(97.05\angle -13.17)=600-j800$$

$$I_2=10.3\angle -41.27^\circ \text{ A}.$$

$$\therefore I_N=10.3\angle -41.27^\circ \text{ A}.$$

Step 3:- To find Z_{eq}.

Short circuit voltage source.

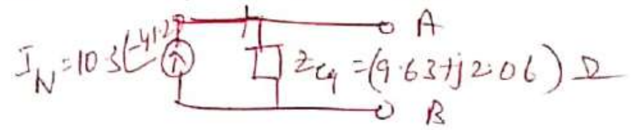


$$Z_{eq}=\frac{(3+j4)\times(6-j8)}{3+j4+6-j8}+5$$

$$=4.63+2.06i+5$$

$$=(9.63+j2.06)\Omega$$

Step 4:- To draw Norton's equivalent circuit.



Maximum Power Transfer theorem:-

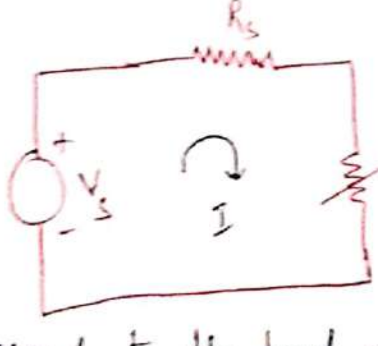
The maximum power transfer theorem can be stated as

Statement- In an active network, maximum power transfer to the load takes place when the load resistance is equal to equivalent resistance of the network as viewed from the terminals of the load (For DC Excitation)

In an active network, maximum power transfer to the load takes place when the load impedance is the complex conjugate of an equivalent impedance of the network as viewed from the terminals of the load. (For AC Excitation)

Explanation of Maximum Power transfer theorem to DC Excitation:-

Many circuits basically consist of sources supplying voltage, current or power to the load; for example a radio speaker system or a microphone supplying the input signals to voltage pre-amplifiers. Sometimes it is necessary to transfer maximum voltage, current or power from source to the load. In the simple resistive circuit shown in Fig. R_s is the source resistance. Our aim is to find the necessary conditions so that the power delivered by the source to load is maximum.



It is a fact that more voltage is delivered to the load when the load resistance is high as compared to resistance of source. On the other hand, maximum current is transferred to the load when load resistance is small compared to source resistance.

For many applications, an important consideration is the maximum power transfer to the load; for example power transfer is desirable from the output amplifier to the speaker of an audio sound system. The maximum power transfer theorem states that maximum power is delivered from source to a load when the load resistance is equal to the source resistance

For the circuit shown above

$$I = \frac{V_S}{R_S + R_L}$$

Power delivered to load R_L is $P = I^2 R_L = \frac{V_S^2}{(R_S+R_L)^2} R_L$

To determine the value of R_L for maximum power to be transferred to the load, we have to set the first derivative of the above equation with respect to R_L i.e, when $\frac{dP}{dR_L}$ is zero

$$\frac{dP}{dR_L} = \frac{d}{dR_L}\left[\frac{V_S^2 R_L}{(R_S+R_L)^2}\right] = \frac{(R_S+R_L)^2 V_S^2 - V_S^2 R_L(2(R_S+R_L))}{(R_S+R_L)^4} = 0$$

$$\Rightarrow V_S^2 (R_S+R_L)^2 = V_S^2 \times 2R_L (\cancel{R_S+R_L})$$

$$V_S^2 R_S + V_S^2 R_L = 2V_S^2 R_L \Rightarrow V_S^2 R_S = 2V_S^2 R_L - V_S^2 R_L \quad [\because V_S = V_{th}]$$
$$= V_S^2 R_L$$

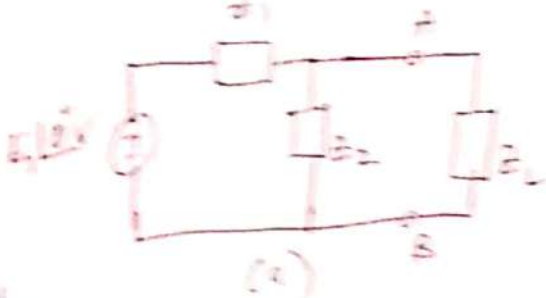
$$\therefore \boxed{R_S = R_L}, \quad P_{max} = \frac{V_S^2}{(R_S+R_L)^2} R_L = \frac{V_S^2}{4R_L^2} \times R_L = \frac{V_S^2}{4R_L} = \frac{V_{th}^2}{4R_L}$$

So, Maximum power is transferred to the load when load resistance is equal to source resistance.

Explanation of Maximum power Transfer Theorem for AC Excitation:-

Consider a network shown in Fig (a)

Let Z_{eq} be the equivalent impedance of the network as viewed from the terminals A-B and replacing all the independent sources by their internal impedances, as shown in Fig. (b)

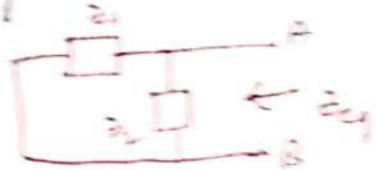


Let Z_{eq} be represented as

$$Z_{eq} = R + jX$$

Then maximum power will be transferred to the load, if Z_L is Complex Conjugate of Z_{eq}

The Complex conjugate is mathematically denoted as Z_{eq}^*

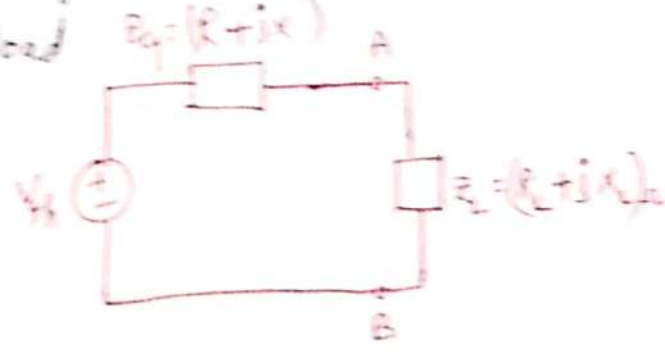


$$\text{So } Z_L = Z_{eq}^* = R - jX$$

Thus for maximum power transfer to the load, the resistance of load and resistance part of Z_{eq} must be same while the reactance of the load and Z_{eq} must also be same in magnitude but opposite in sign. So if Z_{eq} reactance is inductive, Z_L must be capacitive and vice versa

Proof of Maximum Power Transfer theorem :-

Let the given network is replaced by its Thevenin's equivalent across the load terminals as shown in Fig



Let $Z_{eq} = (R + jX)\,\Omega$

$Z_L = (R_L + jX_L)\,\Omega$

$$I = \frac{V_{th}}{Z_{eq} + Z_L} = \frac{V_{th}}{(R + jX) + (R_L + jX_L)}$$

The power delivered to load is $P_L = I^2 R_L$

$$I = \frac{V_{th}}{\sqrt{(R+R_L)^2 + (X+X_L)^2}} \Rightarrow \cancel{P_L = \frac{V_{th}^2}{\sqrt{(R+R_L)^2 + (X+X_L)^2}}}$$

$$P_L = \frac{V_{th}^2}{(R+R_L)^2+(X+X_L)^2} \times R_L$$

Now for load impedance Z_L, both R_L and X_L are variable and are to be decided such that power will be maximum. Hence according to maximum theorem we can write that for the maximum power transfer, w.r.t variable X_L and fixed R_L

$$\frac{dP}{dX_L} = 0$$

$$\frac{d}{dX_L}\left(\frac{V_{th}^2 R_L}{(R+R_L)^2+(X+X_L)^2}\right) = 0$$

$$\frac{\cancel{\left((R+R_L)^2+(X+X_L)^2\right) \times V_{th}^2} - (V_{th}^2 R_L)(2(X+X_L))}{\left((R+R_L)^2+(X+X_L)^2\right)^2} = 0$$

$$\boxed{\begin{gathered}\cancel{X\, V_{th}^2\left((R+R_L)^2+(X+X_L)^2\right)} = \cancel{V_{th}^2 \times (2R_L)(X+X_L)} \\ R^2+R_L^2+2RR_L+X^2+X_L^2+2XX_L = 2R_L\end{gathered}}$$

$$-2V_{th}^2 R_L (X+X_L) = 0$$

$$X = -X_L$$

$$\therefore \boxed{X_L = -X}$$

Thus load reactance must be same in magnitude of the reactance of Z_{eq} but opposite in sign.

Similarly power transfer will be maximum w.r.to variable R_L and fixed X_L when,

$$\frac{dP_L}{dR_L} = 0$$

$$\frac{d}{dR_L}\left[\frac{V_{th}^2 R_L}{(R+R_L)^2+(X+X_L)^2}\right] = 0 \Rightarrow$$ Substitute $X_L = -X$ as already derived.

$\left[\because X+X_L = X-X=0\right]$

$$\frac{V_{th}^2 R_L(2(R+R_L)) - (R+R_L)^2 V_{th}^2}{(R+R_L)^4} = 0$$

$$2\cancel{V_{th}^2}R_L\cancel{(R+R_L)} = (R+R_L)^{\cancel{2}}\cancel{V_{th}^2}$$

$$2R_L = R+R_L$$

$$2R_L - R_L = R$$

$$\Rightarrow R_L = R$$

$$\boxed{R = R_L}$$

Thus the resistance of the load must be same as that of equivalent impedance of the network. Thus when Z_L is the complex conjugate of Z_{eq}, the power transfer to the load is maximum and is given by

$$P_{max} = I^2 R_L = \frac{V_{th}^2}{(2R_L)^2} R_L = \frac{V_{th}^2}{4R_L} \qquad \text{as } I = \frac{V_{th}^2}{2R_L}$$

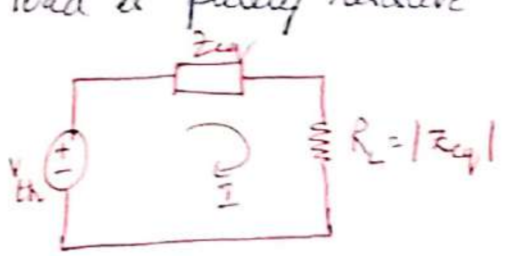
$$\boxed{\therefore P_{max} = \frac{V_{th}^2}{4R_L}}$$

where V_{th} = Thevenin's Voltage as circuit is replaced by its Thevenin equivalent

Corollary:- If pure resistance is to be connected as load for maximum power transfer then its value must be equal to the absolute magnitude of Z_{eq}.

$R_L = |Z_{eq}|$ for P_{max} when load is purely resistive

$$I = \frac{V_{th}}{Z_{eq}+R_L}$$

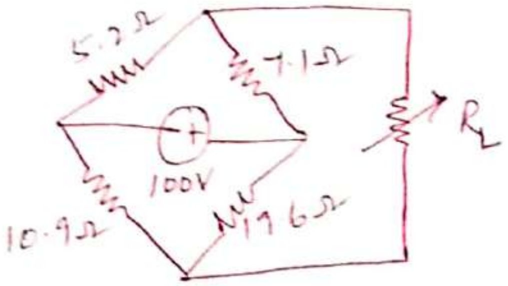


and hence maximum power delivered to the load is given by

$$P_{max} = I^2 R_L = \left[\frac{V_{th}}{Z_{eq}+R_L}\right]^2 R_L \text{ and it is not given by } \frac{V_{th}^2}{4R_L}$$

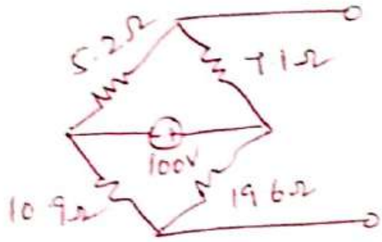
Problems-

Problem 1:- For the circuit, find the value of R_L that will receive maximum power. Determine this maximum power.

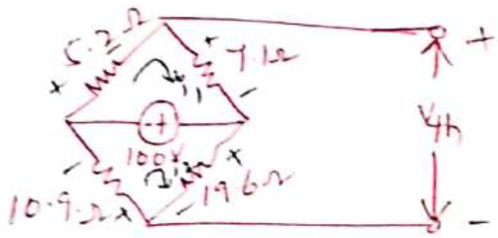


Sol:-

Step 1:- Remove R_L



Step 2:- Find V_{th} between the removed terminals



Apply KVL to loop ①

$-5.2i_1 - 7.1i_1 - 100 = 0$

$-12.3i_1 = 100$

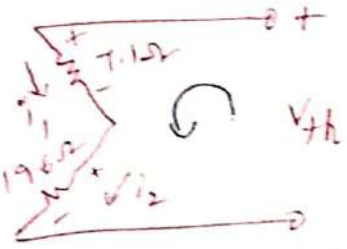
$i_1 = \frac{-100}{12.3} = -8.13A$

Apply KVL to loop ②

$-19.6i_2 - 10.9i_2 + 100 = 0$

$-30.5i_2 = -100$

$\Rightarrow i_2 = \frac{100}{30.5} = 3.28A$



Apply KVL to the path to find V_{th}

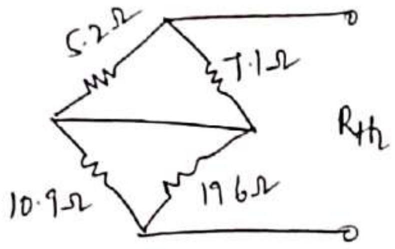
$V_{th} - 7.1i_1 - 19.6i_2 = 0$

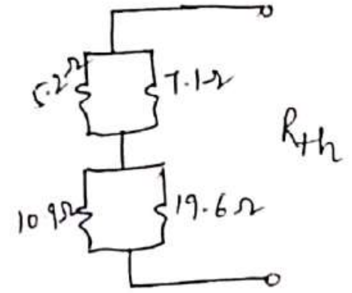
$V_{th} - 7.1 \times -8.13 - 19.6 \times 3.28 = 0$

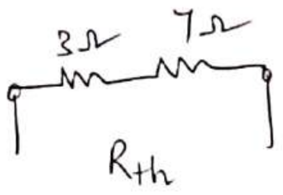
$V_{th} = 64.262 - 57.723$

$= \cancel{6.36V}\ 6.54V$

Step 3:- To find R_{th}

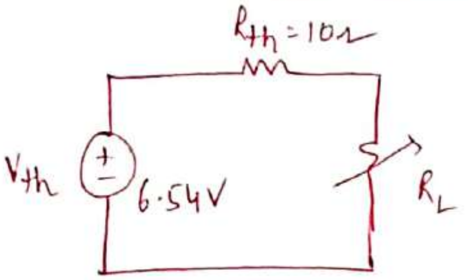






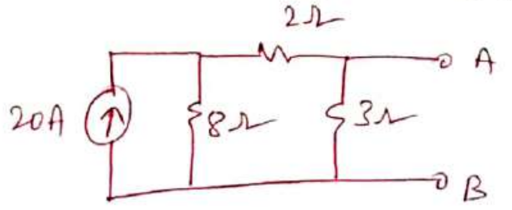
$R_{th} = 10\Omega$

$$\therefore \boxed{R_L = R_{th} = 10\Omega}$$

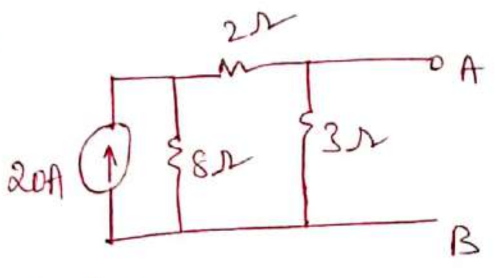


$$P_{max} = \frac{V_{th}^2}{4R_L} = \frac{6.54^2}{4 \times 10} = 1.069\,W$$

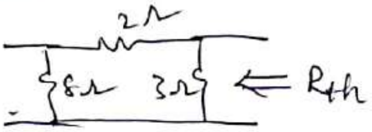
Problem 2:- Determine the value of load resistance to be connected between A-B to absorb maximum power. What is maximum power



Sol:-



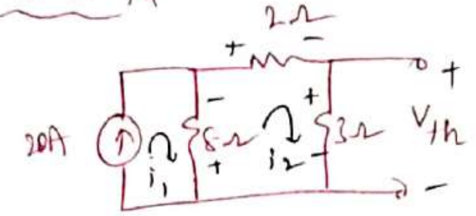
To find R_{th}:- open circuit current source.



$$R_{th} = (8+2) \,//\, 3 = \frac{10 \times 3}{10+3} = 2.307\Omega$$

$$\boxed{R_L = R_{th} = 2.307\Omega}$$

To find V_{th}:-



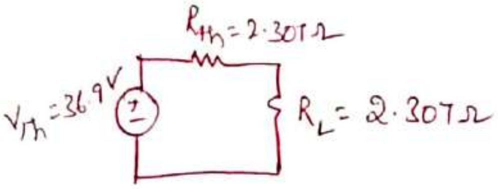
$i_1 = 20A$

Apply KVL to loop ②

$$-2i_2 - 3i_2 - 8(i_2 - i_1) = 0$$
$$-13i_2 + 8i_1 = 0$$
$$-13i_2 + 160 = 0$$
$$-13i_2 = -160$$

$i_2 = \frac{160}{13} = \cancel{1.23A}\ 12.3A$

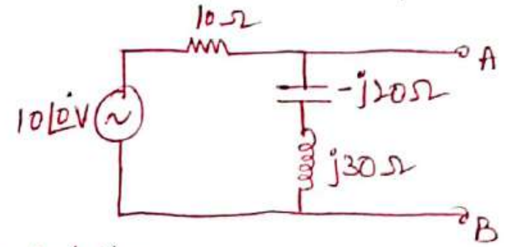
$V_{th} = 3i_2 = \cancel{3 \times 1.23 = 3.69V}$
$= 3 \times 12.3 = 36.9V$



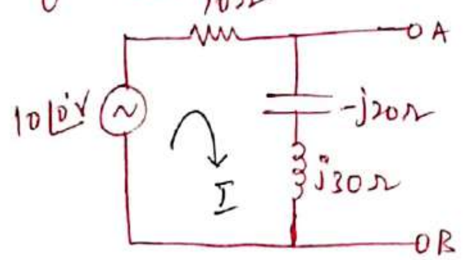
$$P_{max} = \frac{V_{th}^2}{4R_L} = \frac{36.9^2}{4 \times 2.307} = 147.55W$$

Problem 3:- (AC Excitation)

Find the load impedance required to be connected across the terminals A-B for the maximum power transfer, in the network shown. Also find maximum power delivered to the load.



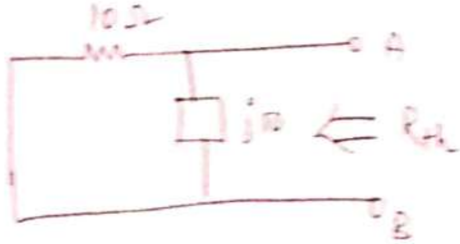
Sol:- To find V_{th}:-



$V_{th} = I \times (-j20 + j30)$
$= 0.707\angle -45° \times (j10)$
$= 7.07\angle 45°\ V$

$$I = \frac{10\angle 0°}{10 - j20 + j30} = \frac{10\angle 0°}{10 + j10} = 0.707\angle -45°$$

To find R_{th} :-



$$\Rightarrow Z_{th} = 10 // j10 = \frac{10 \times j10}{10+j10} = \frac{j100}{10+j10} = 7.07 \angle 45^\circ \ \Omega$$

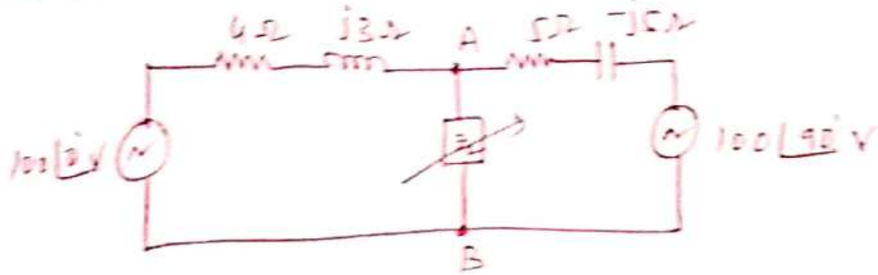
$$= 5+5i$$

$$\therefore \ Z_L = 5+5i$$

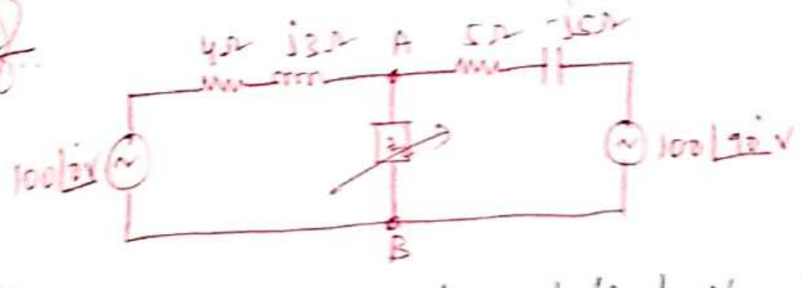
$$R_L = 5\Omega$$

$$P_{max} = \frac{V_{th}^2}{4R_L} = \frac{7.07^2}{4\times5} = 2.499 = 2.5W$$

Problem 4:- Find Z_L for maximum power and value of P_{max}

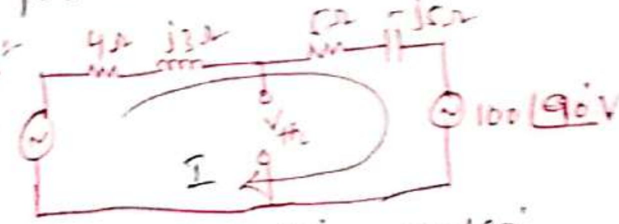


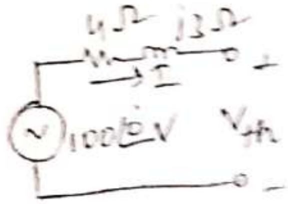
Sol:-



Step 1:- open AB terminals and find V_{th} between A & B terminals

To find V_{th}:-





$$I = \frac{100\angle 0^\circ - 100\angle 90^\circ}{9-j2}$$

$$= 15.33 \angle -32.47^\circ$$

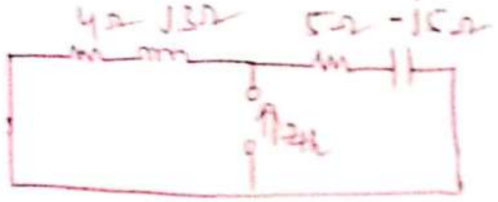
$$100\angle 0^\circ - (4+j3)I - V_{th} = 0$$

$$V_{th} = 100\angle 0^\circ - (4+j3)(15.33\angle -32.47^\circ)$$

$$= 24.29 \angle -14^\circ$$

$\therefore V_{th} = 24.29 \angle -14^{\circ}$ V

To find Z_{th}:-



$$Z_{th} = (4+j3) \parallel (5-j5) = \frac{(4+j3)(5-j5)}{9-j2} = 3.83 \angle 4.39^{\circ} \, \Omega$$

$$= (3.82 + j0.29) \, \Omega$$

$\therefore R_L = 3.82 \, \Omega \qquad \Rightarrow Z_L = Z_{th}^{*} = 3.82 - j0.29$

$$P_{max} = \frac{V_{th}^2}{4R_L} = \frac{24.29^2}{4 \times 3.82} = 38.61 \, W$$

Reciprocity Theorem:-

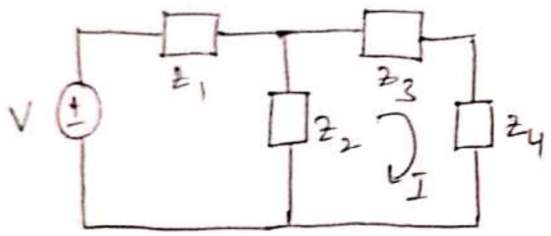
Reciprocity theorem states that In any linear network consisting of linear and bilateral elements and active sources, the ratio of voltage V introduced in one loop to the current I in other loop is same as the ratio obtained if the positions of V and I are interchanged in the network. While calculating the ratio, the sources other than one which is considered to obtain the ratio, must be replaced by their internal resistances (or impedances)

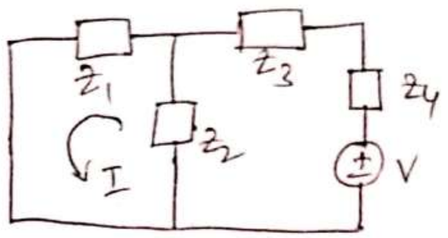
Explanation:-

Consider the network shown

V is the voltage introduced in loop 1 while I is the current in loop 2. The ratio of voltage V to I is $\frac{V}{I}$

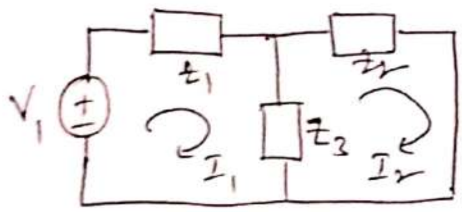
Reciprocity theorem states that the ratio $\frac{V}{I}$ remains same, if the positions of V and I are interchanged in the network, as shown in fig.





In other words, the V and I are mutually transferable. The ratio $\frac{V}{I}$ is called transfer impedance where V is voltage introduced in loop 1 and I is the response due to V in loop 2

Proof of Reciprocity Theorem:-



Consider the network shown in fig.
Let us calculate the ratio $\frac{V_1}{I_2}$
Applying KVL to two loops

$$-I_1Z_1 - I_1Z_3 + I_2Z_3 + V_1 = 0$$

$$I_1(Z_1+Z_3) - I_2Z_3 = V_1 \rightarrow (1)$$

Apply KVL to second loop

$$-Z_2I_2 - Z_3(I_2-I_1) = 0$$

$$(Z_3+Z_2)I_2 = Z_3I_1$$

$$I_1 = \left(\frac{Z_2+Z_3}{Z_3}\right)I_2$$

Substitute I_1 in (1).

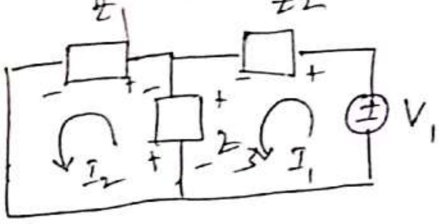
$$\left(\frac{Z_2+Z_3}{Z_3}\right) \times (Z_1+Z_3)I_2 - I_2Z_3 = V_1$$

$$\left[\frac{(Z_1+Z_3)(Z_2+Z_3)}{Z_3} - Z_3\right] I_2 = V_1$$

$$\frac{Z_1Z_2+Z_3Z_2+Z_1Z_3+\cancel{Z_3^2}-\cancel{Z_3^2}}{Z_3} I_2 = V_1$$

$$\frac{V_1}{I_2} = \frac{Z_1Z_2+Z_2Z_3+Z_3Z_3}{Z_3} \rightarrow (A)$$

Let us interchange the positions of V_1 and I_2 as shown in fig.



Apply KVL to loop ①.

$$V_1 - Z_2I_1 - Z_3(I_1-I_2) = 0$$

$$V_1 - I_1(Z_2+Z_3) + Z_3I_2 = 0 \rightarrow (1)$$

Apply KVL to loop ②

$$-Z_1I_2 - Z_3(I_2-I_1) = 0$$

$$Z_3I_1 - I_2(Z_1+Z_3) = 0$$

$$Z_3I_1 = I_2(Z_1+Z_3)$$

$$I_1 = \frac{Z_1+Z_3}{Z_3} I_2$$

Substitute I_1 in ①

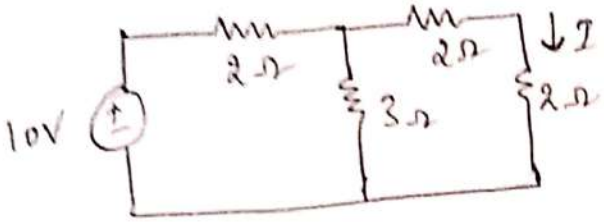
$$V_1 - \frac{(Z_1+Z_3)}{Z_3}(Z_2+Z_3) I_2 + Z_3I_2 = 0$$

$$\frac{V_1}{I_2} = \frac{(Z_1+Z_3)(Z_2+Z_3)}{Z_3} - Z_3 = \frac{Z_1Z_2+Z_2Z_3+Z_3Z_1+\cancel{Z_3^2}-\cancel{Z_3^2}}{Z_3}$$

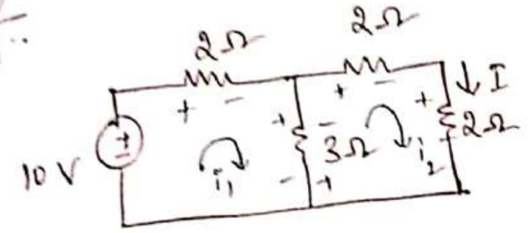
$$\therefore \frac{V_1}{I_2} = \frac{Z_1Z_2+Z_2Z_3+Z_3Z_1}{Z_3} \rightarrow (B)$$

Both ratio's (A) & (B) are same. So reciprocity theorem is verified.

Problem :- Verify reciprocity theorem for the voltage V and current I in the network shown.



Sol:-



Apply KVL to loop ①

$10 - 2i_1 - 3(i_1 - i_2) = 0$

$-5i_1 + 3i_2 = -10 \rightarrow ①$

Apply KVL to loop ②

$-2i_2 - 2i_2 - 3(i_2 - i_1) = 0$

$3i_1 - 7i_2 = 0 \rightarrow ②$

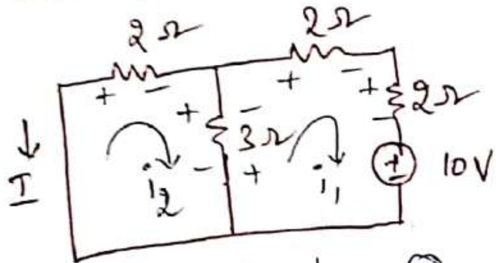
$\Rightarrow$ $i_1 = 2.69$ A

$i_2 = 1.15$ A

$\therefore I = i_2 = 1.15$ A

$\therefore \frac{V}{I} = \frac{10}{1.15} = 8.69 \rightarrow (A)$

Interchange the voltage source to second loop and find current in ~~second loo~~ first loop



Apply KVL to loop ①

$-2i_1 - 2i_1 - 3(i_1 - i_2) - 10 = 0$

$-7i_1 + 3i_2 = 10 \rightarrow ①$

Apply KVL to loop ②

$-2i_2 - 3(i_2 - i_1) = 0$

$3i_1 - 5i_2 = 0 \rightarrow ②$

$i_1 = -1.92$ A

$i_2 = -1.15$ A.

$I = -i_2 = 1.15$ A.

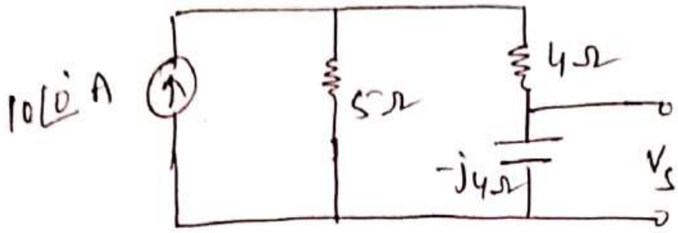
$\therefore \frac{V}{I} = \frac{10}{1.15} = 8.69 \rightarrow (B)$

Since ratio's A & B are same

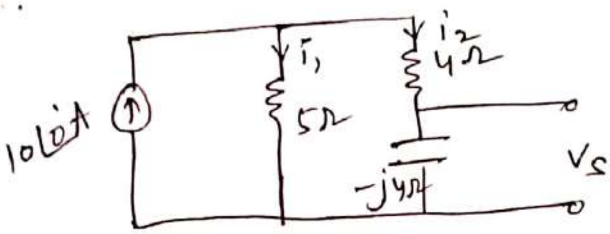
Reciprocity theorem is verified.

Problem 2:- (AC Excitation)

Verify reciprocity theorem for the network shown in fig.



Sol.

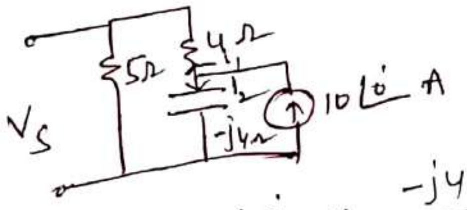


$$i_2 = 10\angle 0^\circ \times \frac{5}{5+4-j4} = 5.07\angle 23.96 \text{ A}$$

$$V_s = -j4 \times 5.07\angle 23.96 = 20.28\angle -66.04^\circ \text{ V}$$

$$\frac{V_s}{I} = \frac{20.28\angle -66.04}{10\angle 0^\circ} = 2.02\angle -66.04 \rightarrow \text{(A)}$$

Now interchange the positions of V_s and I



$$i_1 = 10\angle 0^\circ \times \frac{-j4}{-j4+4+5} = 4.06\angle -66.03^\circ \text{ A}.$$

$$V_s = 5i_1 = 5 \times 4.06\angle -66.03^\circ = 20.3\angle -66.04^\circ \text{ V}$$

$$\frac{V_s}{I} = \frac{20.3\angle -66.04}{10} = 2.03\angle -66.04^\circ \rightarrow \text{(B)}$$

The ratio's A & B are same

So reciprocity theorem is verified.

Millimann's Theorem:- Milliman's theorem states that

If n voltages sources $V_1, V_2 \dots V_n$ having internal resistances (or impedances) $Z_1, Z_2 \dots Z_n$ respectively are in parallel, then these sources may be replaced by a single voltage source of voltage V_m having a series impedance Z_m where V_m and Z_m are given by

$$V_m = \frac{V_1G_1 + V_2G_2 + V_3G_3 + \dots V_nG_n}{G_1 + G_2 + G_3 \dots G_n} = \frac{\sum_{K=1}^{n} V_K G_K}{\sum_{K=1}^{n} G_K}$$

where G_1, G_2 are conductances corresponding to resistances $R_1, R_2 \dots R_n$

$$G_1 = \frac{1}{R_1}, \; G_2 = \frac{1}{R_2} \;\dots\; G_n = \frac{1}{R_n}$$

and $$V_m = \frac{V_1Y_1 + V_2Y_2 + V_3Y_3 + \dots V_nY_n}{Y_1 + Y_2 + Y_3 + \dots Y_n} = \frac{\sum_{K=1}^{n} V_K Y_K}{\sum_{K=1}^{n} Y_K}$$ (For ac excitation)

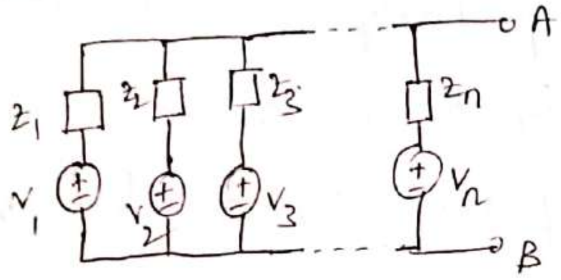
where $Y_1, Y_2 \dots Y_n$ are admittances corresponding to Impedances $Z_1, Z_2 \dots Z_n$

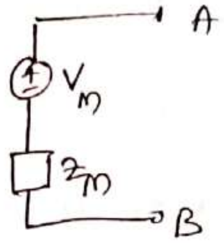
$$Y_1 = \frac{1}{Z_1}, \; Y_2 = \frac{1}{Z_2} \;\dots\; Y_n = \frac{1}{Z_n}$$

$$\cancel{G_m =} \; R_m = \frac{1}{G_1 + G_2 + \dots G_n} = \frac{1}{\sum_{K=1}^{n} G_K}$$

$$Z_m = \frac{1}{Y_1 + Y_2 + \dots Y_n} = \frac{1}{\sum_{K=1}^{n} Y_K}$$

Explanation:- Consider the n voltage sources $V_1, V_2 \cdots V_n$ having series impedances $Z_1, Z_2 \cdots Z_n$ connected in parallel as shown.





$$Y = \frac{1}{Z}$$

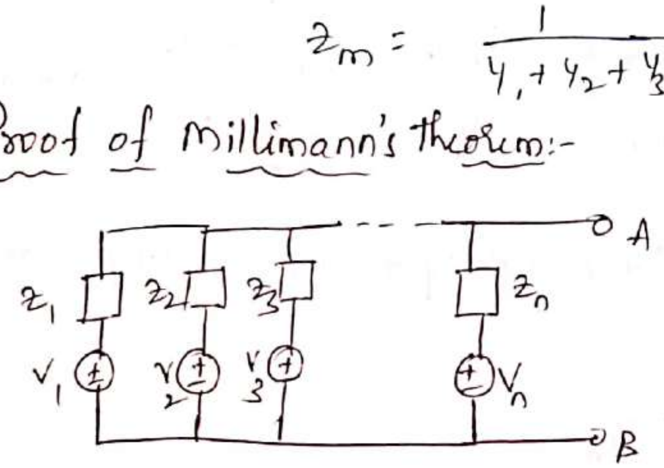
$$Y_1 = \frac{1}{Z_1}, \; Y_2 = \frac{1}{Z_2} \cdots Y_n = \frac{1}{Z_n}$$

Then according to millimann's theorem, all voltage sources can be combined to get a single voltage source V_m with a series impedance Z_m as shown in fig...

$$V_m = \frac{V_1 Y_1 + V_2 Y_2 + \cdots V_n Y_n}{Y_1 + Y_2 + \cdots Y_n}$$

$$Z_m = \frac{1}{Y_1 + Y_2 + Y_3 + \cdots Y_n}$$

Proof of millimann's theorem:-



Consider n voltage sources in parallel as shown in fig. Let us convert each voltage source into equivalent current source. For source 1, $I_1 = \frac{V_1}{Z_1} = V_1 Y_1$

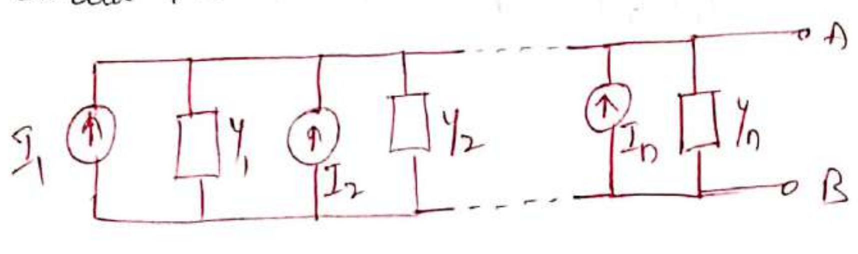
$\therefore$ as $Y_1 = \frac{1}{Z_1}$

Similarly for remaining sources, we can write

$I_2 = V_2 Y_2$, $I_3 = V_3 Y_3 \cdots I_n = V_n Y_n$

where $Y_1, Y_2 \cdots Y_n$ are admittances to be connected in parallel

Hence circuit reduces to

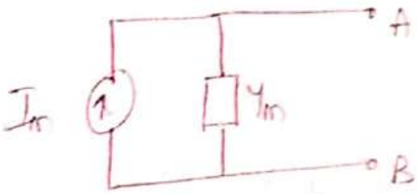


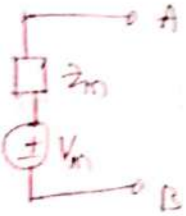
Hence the effective current source across the terminals A-B is

$$I_m = I_1 + I_2 + \cdots I_n \quad \rightarrow (1)$$

$$Y_m = Y_1 + Y_2 + \cdots Y_n \quad \rightarrow (2)$$

This is because admittances in parallel get added to each other. Hence circuit reduces to as shown





Converting this equivalent current source into the voltage source, we get

$$V_m = \frac{I_m}{Y_m}$$

$$Z_m = \frac{1}{Y_m}$$

$$V_m = I_m Z_m$$

Substituting I_m and Y_m from equations (1) and (2)

$$V_m = (I_1 + I_2 + \cdots I_n) \cdot \frac{1}{(Y_1 + Y_2 + \cdots Y_n)}$$

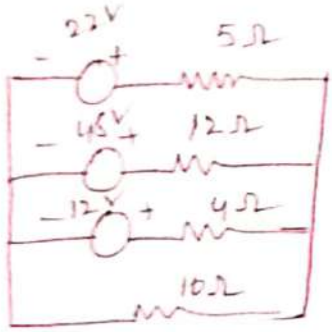
$$I_1 = \frac{V_1}{Z_1} = V_1 Y_1, \; I_2 = V_2 Y_2, \; \cdots I_n = V_n Y_n$$

$$\therefore \quad V_m = \frac{V_1 Y_1 + V_2 Y_2 + \cdots V_n Y_n}{Y_1 + Y_2 + \cdots Y_n}$$

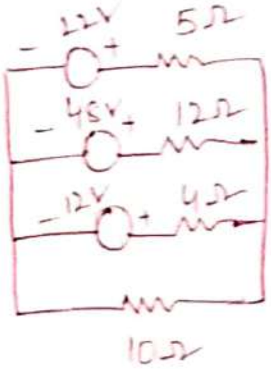
$$Z_m = \frac{1}{Y_m} = \frac{1}{Y_1 + Y_2 + \cdots Y_n}$$

Thus milliman's theorem is proved.

Problem :- Use Milliman's theorem to find the current through 10Ω resistance in the circuit.



Sol:-



From given network we can write,

$V_1 = 12V,\ V_2 = 48V,\ V_3 = 22V$

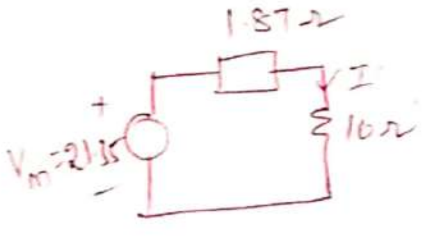
$Z_1 = 4\Omega,\ Z_2 = 12\Omega,\ Z_3 = 5\Omega$

$Y_1 = \frac{1}{4} = 0.25\mho,\ Y_2 = \frac{1}{12} = 0.083\mho,\ Y_3 = \frac{1}{5} = 0.2\mho$

$$V_m = \frac{V_1Y_1 + V_2Y_2 + V_3Y_3}{Y_1+Y_2+Y_3} = \frac{12\times0.25 + 48\times0.083 + 22\times0.2}{0.25+0.083+0.2}$$

$$= \cancel{24}\ 21.35V$$

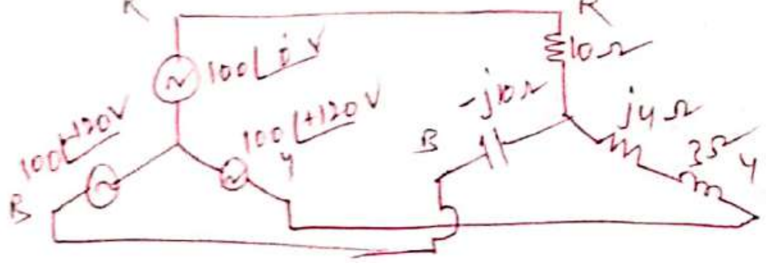
$$Z_m = \frac{1}{Y_1+Y_2+Y_3} = \frac{1}{0.25+0.083+0.2} = 1.87\Omega$$



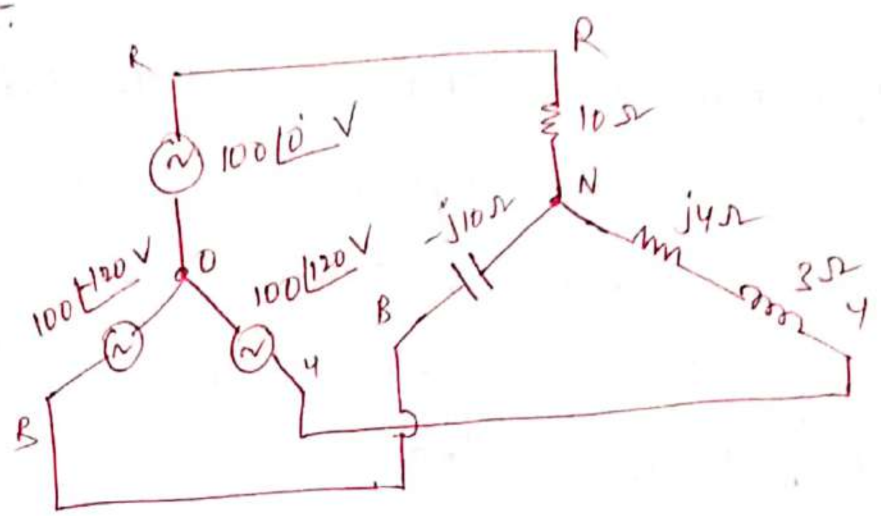
$$I = \frac{21.35}{1.87+10} = 1.79A$$

Problem 2 :- AC Excitation

Using Milliman's theorem find the neutral shift voltage V_{ON}



Sol:



Given that

$V_{RO} = 100\angle 0^\circ$ V $\qquad Z_{RN} = 10\,\Omega$

$V_{YO} = 100\angle 120^\circ$ V $\qquad Z_{YN} = (3+j4)\,\Omega$

$V_{BO} = 100\angle -120^\circ$ V $\qquad Z_{BN} = -j10\,\Omega$

$$Y_{RN} = \frac{1}{10} = 0.1\,\mho, \quad Y_{YN} = \frac{1}{3+j4} = 0.2\angle -53.13^\circ\,\mho$$

$$Y_{BN} = \frac{1}{-j10} = 0.1\angle 90^\circ\,\mho$$

$$V_{NO} = \frac{V_{RO}Y_{RN} + V_{YO}Y_{YN} + V_{BO}Y_{BN}}{Y_{RN}+Y_{YN}+Y_{BN}} = \frac{100\angle 0^\circ \times 0.1 + 100\angle 120^\circ \times 0.2\angle -53.13^\circ + 0.1\angle 90^\circ \times 100\angle -120^\circ}{0.1 + 0.2\angle -53.13^\circ + 0.1\angle 90^\circ}$$

$$= \frac{10\angle 0^\circ + 20\angle 66.87^\circ + 10\angle -30^\circ}{0.1 + 0.2\angle -53.13^\circ + 0.1\angle 90^\circ}$$

$$= 130.27\angle 42.05\ \text{V}$$

$$Z_{NO} = \frac{1}{Y_{RN}+Y_{YN}+Y_{BN}} = \frac{1}{0.1+0.2\angle -53.13 + 0.1\angle 90^\circ} = 4.385\angle 15.25^\circ$$

$$= (4.23 + j1.15)\,\Omega$$

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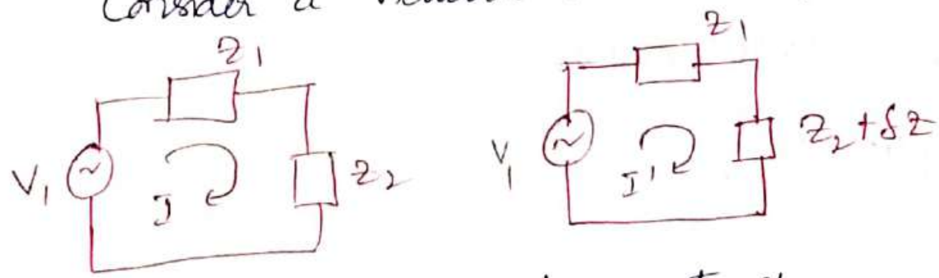
Compensation Theorem:-

In circuit analysis many times it is required to study the effect of change in resistance (or) impedance in one of its branches on the corresponding voltages and currents of the network. The compensation theorem provides a very simple way for studying such effects. The statement is as follows.

Statement:- In any linear network consisting of linear and bilateral resistances (or) impedances and active sources, if the impedance Z of the branch carrying current I increases by δZ, then the increment or decrement of voltage or current in each branch of the network is that voltage or current that would be produced by an opposing voltage source of value $V_c (= I \cdot \delta Z$ or $I\,\delta R)$ introduced in the altered branch after replacing original sources by their internal impedances.

Explanation:-

Consider a network shown in fig.



V_1 is voltage applied to network, I is the current flowing through Z_1 & Z_2. Consider that impedance Z_2 increases by δZ. Due to this, the current in the circuit changes to I' as shown in fig.

· Then the effect of change in impedance is the change in current which is given by

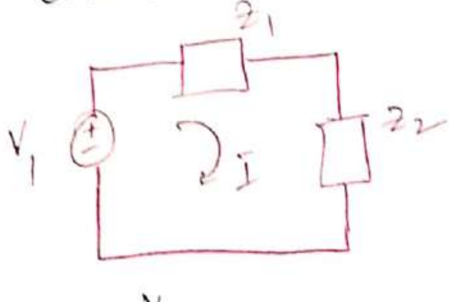
$$\delta I = I - I'$$

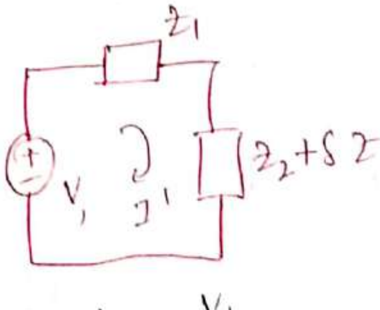
Now this current can be directly calculated by using the compensation theorem. First modify the branch of which impedance is changed, by connecting a voltage source V_c of value $I \cdot \delta Z$. The new voltage source must be connected in the branch with proper polarity. Then replace original active source V by its internal impedance as shown in Fig.

The voltage source introduced in modified branch, V_c is called compensation source with value $I \cdot \delta Z$ where I is current through impedance before impedance of branch is changed and δZ is change in impedance.

Proof of Compensation Theorem:-

Consider a network shown in Fig.





$$I = \frac{V_1}{Z_1+Z_2} \qquad I' = \frac{V_1}{Z_1+Z_2+\delta Z}$$

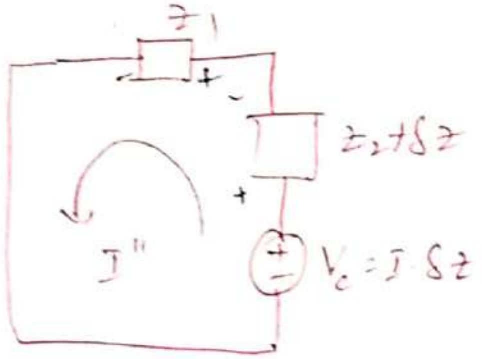
$$\delta I = I - I' = \frac{V_1}{Z_1+Z_2} - \frac{V_1}{Z_1+Z_2+\delta Z} = V_1\left[\frac{1}{Z_1+Z_2} - \frac{1}{Z_1+Z_2+\delta Z}\right]$$

$$= V_1\left[\frac{\cancel{Z_1}+\cancel{Z_2}+\delta Z - \cancel{Z_1} - \cancel{Z_2}}{(Z_1+Z_2)(Z_1+Z_2+\delta Z)}\right] = \frac{V_1}{Z_1+Z_2} \cdot \frac{\delta Z}{Z_1+Z_2+\delta Z}$$

$$\delta I = \frac{I \cdot \delta Z}{Z_1 + Z_2 + \delta Z} = \frac{V_c}{Z_1 + Z_2 + \delta Z} \rightarrow (1)$$

$\therefore$ Compensating voltage $V_c = I \cdot \delta Z$

Now Consider that the branch is modified as shown in Fig and also original voltage source is short circuited. Let the current in circuit be I''



Apply KVL to loop

$$V_c - (Z_2 + \delta Z) I'' - Z_1 I'' = 0$$

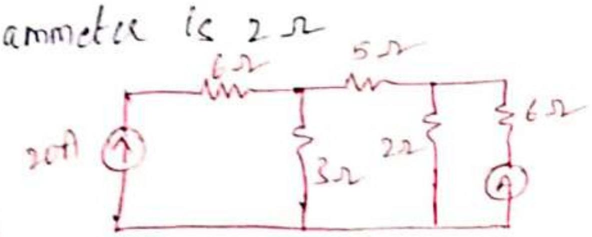
$$V_c - I''(Z_2 + \delta Z + Z_1) = 0$$

$$I'' = \frac{V_c}{Z_1 + Z_2 + \delta Z} \rightarrow (2)$$

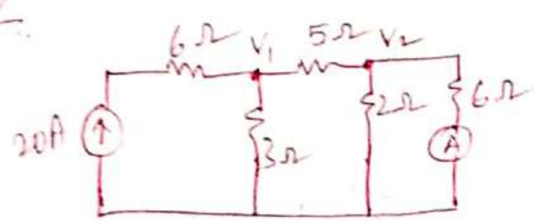
equations (1) = (2) $\Rightarrow \delta I = I''$

Thus Compensation theorem is proved.

Problems:- Using Compensation theorem, determine the ammeter reading where it is connected to 6Ω resistor in Fig. The internal resistance of ammeter is 2Ω



Sol



Apply KCL at node (1)

$$-20 + \frac{V_1}{3} + \frac{V_1 - V_2}{5} = 0$$

$$V_1\left(\frac{1}{3} + \frac{1}{5}\right) - V_2\left(\frac{1}{5}\right) = 20 \Rightarrow 0.53V_1 - 0.2V_2 = 20$$

Apply KCL at node (2)

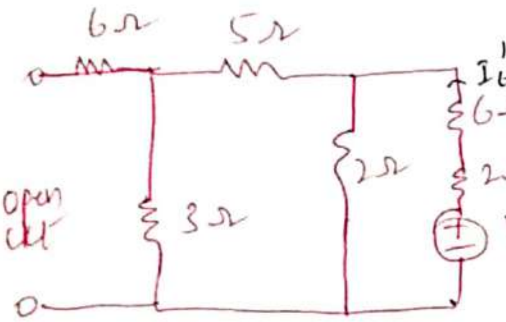
$$\frac{V_2 - V_1}{5} + \frac{V_2}{2} + \frac{V_2}{6} = 0$$

$$-V_1\left(\frac{1}{5}\right) + V_2\left(\frac{1}{5} + \frac{1}{2} + \frac{1}{6}\right) = 0 \Rightarrow -0.2V_1 + 0.86V_2 = 0$$

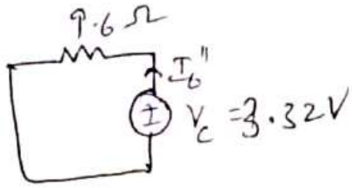
$V_1 = \cancel{25.52V}$ 41.51V

$V_2 = \cancel{5.93V}$ 10V

$$I_6 = \frac{10}{6} = 1.66A.$$



$V_c = I_6 \cdot \delta R$
$= 1.66 \times 2$
$= 3.32V$

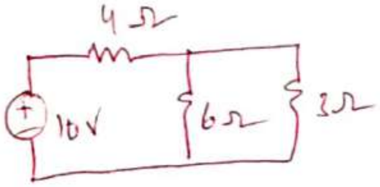


$$I_6'' = \frac{3.32}{9.6} = 0.34A$$

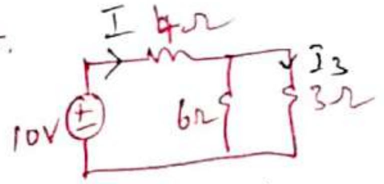
$R_{eq} = (5+3//2)+8$
$= 9.6\Omega$

Ammeter reading $= I_6 - I_6'' = 1.66 - 0.34 = 1.31A$

Problem 2:- Determine current flowing through ammeter having 1Ω resistance in series with 3Ω

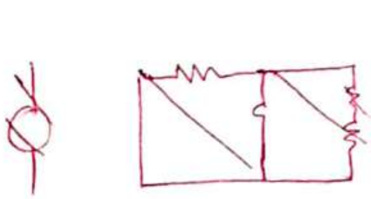


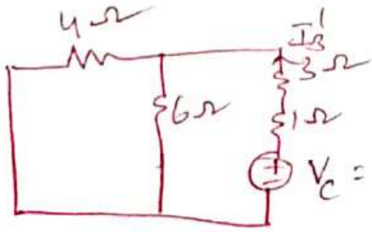
Sol:-



$$I = \frac{10}{4+6//3} = \frac{10}{4+2} = \frac{10}{6} = 1.66A$$

$$I_3 = I \times \frac{6}{6+3} = 1.66 \times \frac{6}{9} = 1.11A$$





$V_c = I_3 \times \Delta R$
$= 1.11 \times 1 = 1.11V$

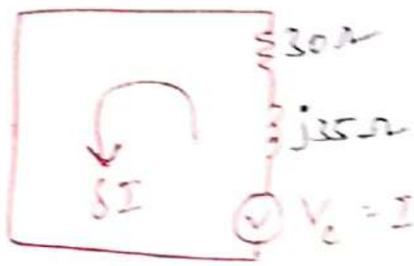
$$I_3' = \frac{V_c}{4//6+4} = \frac{1.11}{6.4} = 0.17A$$

Ammeter reading $= I_3 - I_3' = 1.11 - 0.17 = 0.93A$

Problems – A.C. Excitation: Calculate change in current in the network shown in Fig by using Compensation theorem when the reactance has changed to $j35\,\Omega$

Sol:

$$I = \frac{100\angle 45^\circ}{30+j40} = 2\angle -8.13^\circ \text{ A}$$



$$\delta Z = j40 - j35 = j5\ \Omega$$

$$V_c = I \cdot \delta Z$$

$$= 2\angle -8.13 \times j5$$

$$= 10\angle 81.87^\circ$$

$$\delta I = \frac{V_c}{30+j35} = \frac{10\angle 81.87^\circ}{30+j35} = 0.216\angle 32.47^\circ \text{ A.}$$

$\therefore$ Change in current $= 0.216\angle 32.47^\circ$ A.

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UNIT-II

DC and AC Transient Analysis

Transient response of R-L, R-C, R-L-C Circuits (series and Parallel) for DC excitation - Initial Conditions - Solution method using differential equation and Laplace transforms.

Transient response of R-L, R-C, R-L-C circuits (series and parallel) for sinusoidal excitation - Initial Conditions - Solution method using differential equation and Laplace transforms.

DC Transient Response of an RL Circuit:-

(1)

Consider a circuit consisting of a resistance and inductance as shown in fig. The inductor in the circuit is intially uncharged and is in series with the resistor. When the switch 'S' is closed we can find the complete solution of the current. Application of Kirchoff's voltage law to the circuit results in the following differential equation

$$V = Ri + L\frac{di}{dt}$$

$$V = L\left[\frac{di}{dt} + \frac{R}{L}i\right]$$

$$\frac{di}{dt} + \frac{R}{L}i = \frac{V}{L}$$

$$e^{R/L\,t}\frac{di}{dt} + \frac{R}{L}e^{R/L\,t}i = \frac{V}{L}e^{R/L\,t}$$

$$\frac{d}{dt}\left(ie^{R/L\,t}\right) = \frac{V}{L}e^{R/L\,t}$$

$$d\left(ie^{R/L\,t}\right) = \frac{V}{L}e^{R/L\,t}dt$$

$$\int d\left(ie^{R/L\,t}\right) = \int \frac{V}{L}e^{R/L\,t}dt$$

$$ie^{R/L\,t} = \frac{V}{\cancel{L}}e^{R/L\,t} \times \frac{\cancel{L}}{R} + c$$

$$i(t) = \frac{V}{R}e^{R/L\,t} \times e^{-R/L\,t} + ce^{-R/L\,t}$$

$$i(t) = \frac{V}{R} + ce^{-R/L\,t}$$

At $t = 0^+$ $i(0) = 0$

$$0 = \frac{V}{R} + c \Rightarrow c = -\frac{V}{R}$$

$$i(t) = \frac{V}{R} - \frac{V}{R}e^{-R/L\,t}$$

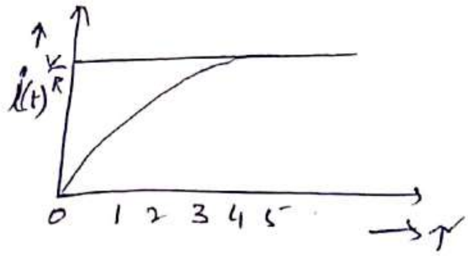
$$i(t) = \frac{V}{R}\left(1 - e^{-R/L\,t}\right) \rightarrow (1)$$

eq (1) consists of two parts

$\frac{V}{R}$ & $-\frac{V}{R}e^{-R/L\,t}$

$\frac{V}{R}$ ↓ steady state part

$-\frac{V}{R}e^{-R/L\,t}$ ↓ transient part



$$\boxed{\tau = \frac{L}{R}\ \text{sec}}$$

transient part

$$i = -\frac{V}{R}e^{-R/L\,t}$$

$$i(\tau) = -\frac{V}{R}e^{-t/\tau} = -0.368\frac{V}{R}$$

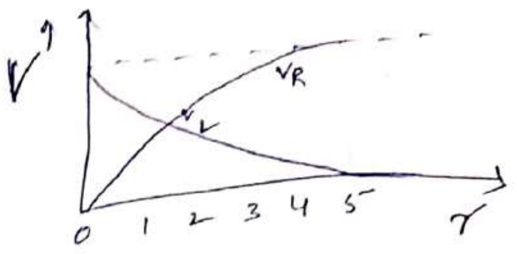
$$i(2\tau) = -\frac{V}{R}e^{-2} = -0.135\frac{V}{R}$$

$$i(3\tau) = -\frac{V}{R}e^{-3} = -0.0498\frac{V}{R}$$

At one time constant, transient term reaches 36.8% of its initial value.

②

$\cdot\ i(5\tau) = \frac{V}{R}e^{-5} = -0.0067\frac{V}{R}$

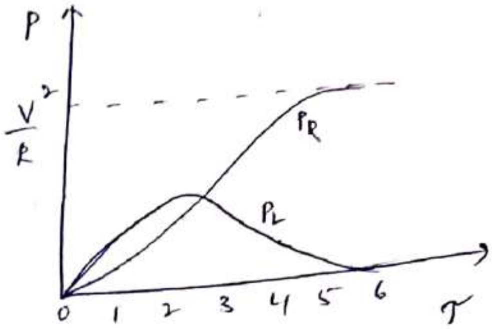


$$V_R = iR = \frac{V}{R} \times R(1-e^{-t/\tau})$$

$$= V(1-e^{-t/\tau})$$

$$V_L = L\frac{di}{dt} = L \times \frac{V}{R} \times -e^{-t/\tau} \times \frac{-R}{L}$$

$$= Ve^{-t/\tau}$$



Power in the resistor $P_R = V_R i$

$$= V(1-e^{-t/\tau}) \times \frac{V}{R}(1-e^{-t/\tau})$$

$$= \frac{V^2}{R}(1-e^{-t/\tau})^2$$

Power in the inductor $P_L = V_L i$

$$= Ve^{-t/\tau} \times \frac{V}{R}(1-e^{-t/\tau})$$

$$= \frac{V^2}{R}(e^{-t/\tau} - e^{-2t/\tau})$$

DC Transient Response of series RC Circuit

Consider a circuit consisting of resistance and capacitance as shown in the fig. The capacitor in the circuit is initially uncharged and is in series with a resistor. When the switch 's' is closed at $t=0$ we can determine the complete solution for the current. Application of the Kirchoff's voltage law to the circuit results in the following differential equation.

$$V = Ri + \frac{1}{C}\int i\,dt$$

$$0 = R\frac{di}{dt} + \frac{1}{C}i$$

$$\frac{di}{dt} + \frac{1}{RC}i = 0$$

$$\frac{d}{dt}\left(ie^{1/RC\,t}\right) = 0 \Rightarrow ie^{1/RC\,t} = c$$

$$i(t) = ce^{-1/RC\,t}$$

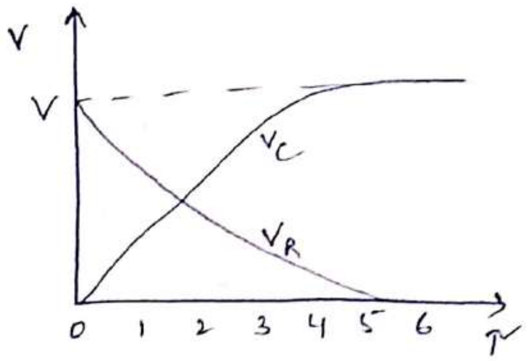
At $t = 0^+$ Capacitor acts as short circuit so $i(0^+) = \frac{V}{R}$.

$\frac{V}{R} = Ce^0 \Rightarrow C = \frac{V}{R}$

$i(t) = \frac{V}{R} e^{-\frac{1}{RC}t}$

$V_R = iR = \frac{V}{R} \times R e^{-1/RC\,t}$

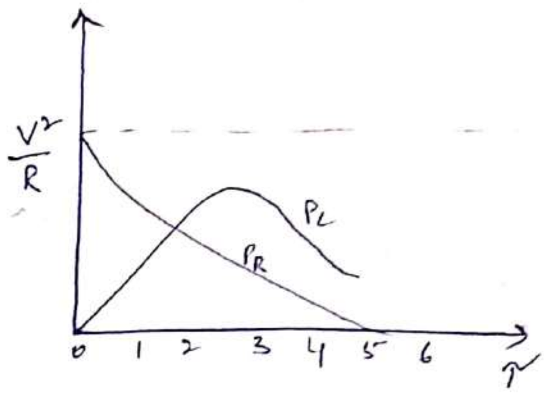
$= Ve^{-1/RC\,t}$



$V_C = \frac{1}{C}\int i\,dt = \frac{1}{C}\int \frac{V}{R} e^{-1/RC\,t}$

$= \frac{1}{\cancel{C}} \times \frac{V}{R} \times e^{-1/RC\,t} \times -RC + c$

$V_C = -Ve^{-1/RC\,t} + c$



At $t = 0^+$ $V_C = 0$

$0 = -V + c \Rightarrow c = V$

$V_C = V - Ve^{-1/RC\,t} = V(1 - e^{-t/\tau})$

$\boxed{\tau = RC}$

$P_R = V_R i = Ve^{-1/RC\,t} \times \frac{V}{R} e^{-1/RC\,t}$

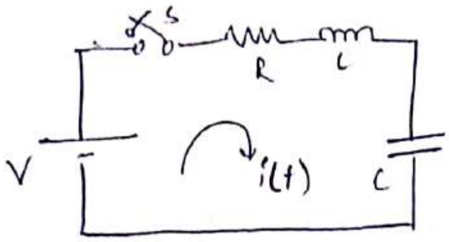
$= \frac{V^2}{R} e^{-2t/\tau}$

$P_C = V_C i = V(1 - e^{-t/\tau}) \times \frac{V}{R} e^{-t/\tau}$

$= \frac{V^2}{R}(e^{-t/\tau} - e^{-2t/\tau})$

DC Transient Response of RLC series circuit :-

Consider a circuit consisting of resistance, inductance & capacitance. The capacitor & inductor are initially uncharged and are in series with a resistor. When switch 'S' is closed at $t=0$, we can determine the complete solution for the current. Application of Kirchoff's voltage law results in the following differential equation.



$$V = Ri + L\frac{di}{dt} + \frac{1}{C}\int i\,dt$$

$$R\frac{di}{dt} + L\frac{d^2i}{dt^2} + \frac{1}{C}i = 0$$

$$\frac{d^2i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{1}{LC}i = 0 \longrightarrow (1)$$

$$s^2 + \frac{R}{L}s + \frac{1}{LC} = 0 \longrightarrow (2)$$

$$s_1, s_2 = \frac{-\frac{R}{L} \pm \sqrt{\left(\frac{R}{L}\right)^2 - 4\times\frac{1}{LC}}}{2} = \frac{-R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$$

$$= -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$$

where $\alpha = \frac{R}{2L}$, $\omega_0 = \frac{1}{\sqrt{LC}}$

'α' is known as neper frequency & units are np/sec, ω_0 is known as resonant frequency or undamped natural frequency of oscillations, units are rad/sec, There are three cases

when ~~E.~~ $\alpha > \omega_0$, s_1, s_2 are unequal & real

$\alpha > \omega_0$, overdamped case:-

$$s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2}, \quad s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2}$$

solution of eq. is

$$i(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$$

when $\alpha = \omega_0$, Critically damped :-

$$s_1 = s_2 = -\alpha$$

$$i(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t} = A_3 e^{-\alpha t}$$

where $A_3 = (A_1 + A_2)$ but there are two storage elements & we should have two initial conditions & two constants so this is wrong. Rewriting eq. ①

$$\left[\because \alpha = \omega_0 = \frac{1}{\sqrt{LC}} \right]$$
in this case

$$\cancel{s^2 + 2\alpha s +} \quad \frac{d^2 i}{dt^2} + 2\alpha \frac{di}{dt} + \alpha^2 i = 0$$

$$\frac{d^2 i}{dt^2} + \alpha \frac{di}{dt} + \alpha \frac{di}{dt} + \alpha^2 i = 0$$

$$\frac{d}{dt}\left[\frac{di}{dt} + \alpha i\right] + \alpha\left[\frac{di}{dt} + \alpha i\right] = 0$$

$$\frac{di}{dt} + \alpha i = f \quad \Rightarrow \cancel{d(ie^{-\alpha t})} =$$

$$\frac{df}{dt} + \alpha f = 0 \quad \Rightarrow d(f e^{-\alpha t}) = 0$$

$$f e^{-\alpha t} = A_1$$

$$f = A_1 e^{+\alpha t}$$

$$\frac{di}{dt} + \alpha i = A_1 e^{-\alpha t} \quad \Rightarrow \cancel{\frac{d}{dt}}(ie^{-\alpha t}) = \cancel{A_1} e^{-\alpha t}$$

$$i\cancel{e^{-\alpha t}} = A_1 \cancel{e^{+\alpha t}} \times \frac{+1}{\alpha}$$

$$\left(\frac{di}{dt} + \alpha i\right) e^{\alpha t} = A_1$$

$$\frac{d}{dt}(ie^{\alpha t}) = A_1 \Rightarrow ie^{\alpha t} = A_1 t + A_2$$

$$i = (A_1 t + A_2) e^{-\alpha t}$$

$$i(t) = (A_1 t + A_2) e^{-\alpha t}$$

when $\alpha < \omega_0$, underdamped

$$i(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$$

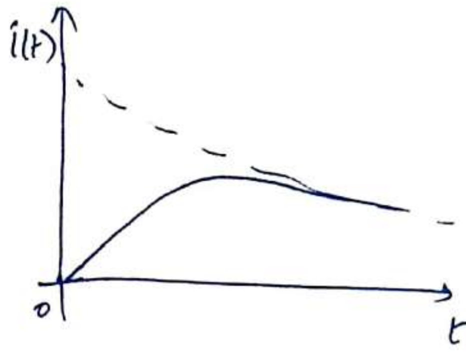
$$= A_1 e^{(-\alpha + j\omega_d)t} + A_2 e^{(-\alpha - j\omega_d)t}$$

$$= A_1 e^{-\alpha t}\left[\cos\omega_d t + j\sin\omega_d t\right] + A_2 e^{-\alpha t}\left[\cos\omega_d t - j\sin\omega_d t\right]$$

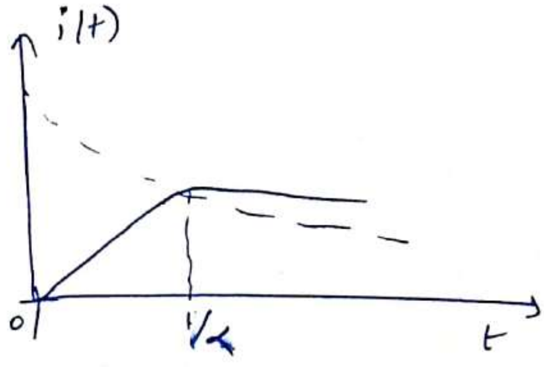
$$= e^{-\alpha t}\left[(A_1 + A_2)\cos\omega_d t + j(A_1 - A_2)\sin\omega_d t\right]$$

$$= e^{-\alpha t}\left[B_1\cos\omega_d t + B_2\sin\omega_d t\right]$$

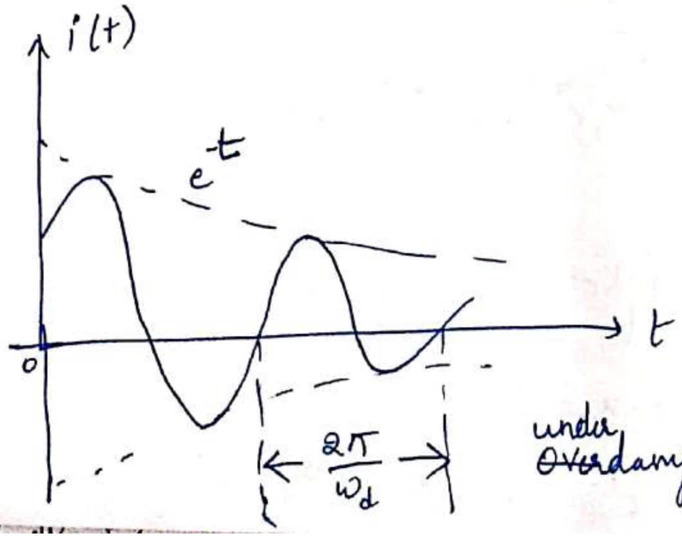
where $B_1 = A_1 + A_2$, $B_2 = j(A_1 - A_2)$



overdamped



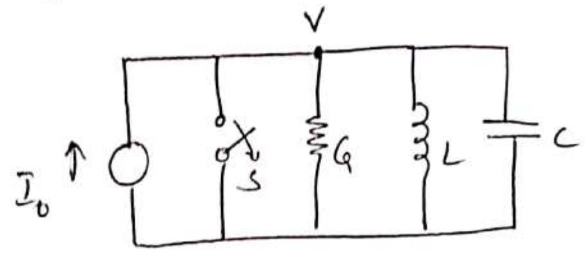
critically damped



under ~~Over~~damped

Transient Response of a Parallel RLC circuit:-

(4)



Let switch 'S' is ~~closed~~ opened at $t=0$

Applying KCL to the circuit

$$I_0 = GV + \frac{1}{L}\int V\,dt + C\frac{dV}{dt} \rightarrow (1)$$

$$C\frac{d^2V}{dt^2} + \frac{1}{L}V + G\frac{dV}{dt} = 0$$

$$\left(D^2 + \frac{G}{C}D + \frac{1}{LC}\right)V = 0$$

$$D^2 + \frac{G}{C}D + \frac{1}{LC} = 0 \qquad x^2 + \frac{G}{C}x + \frac{1}{LC} = 0$$

$$x_1, x_2 = \frac{\frac{-G}{C} \pm \sqrt{\left(\frac{G}{C}\right)^2 - \frac{4}{LC}}}{2} = \frac{-G}{2C} \pm \sqrt{\left(\frac{G}{2C}\right)^2 - \frac{1}{LC}}$$

$\frac{G}{2C} > \frac{1}{LC}$ overdamped :-

At $t = 0^+$, $V(0^+) = 0$, $\frac{1}{L}\int V\,dt = 0$

from (1) $\quad I_0 = 0 + 0 + C\frac{dV}{dt}$

$$C\frac{dV}{dt} = I_0 \Rightarrow \frac{dV}{dt} = \frac{I_0}{C}$$

$$V(t) = \cancel{K_1 e^{-\frac{G}{2C}t} + K_2 e^{-\frac{G}{2C}}}\ K_1 e^{x_1 t} + K_2 e^{x_2 t} \rightarrow (2)$$

At $t = 0^+$, $V(0^+) = 0$

$$0 = K_1 e^0 + K_2 e^0 \Rightarrow K_1 + K_2 = 0$$

From (2) $\quad \frac{dV}{dt} = K_1 x_1 e^{x_1 t} + K_2 x_2 e^{x_2 t}$

$$\frac{dV}{dt}(0^+) = \frac{I_0}{C} \Rightarrow \frac{I_0}{C} = K_1 x_1 e^0 + K_2 x_2 e^0 \Rightarrow K_1 x_1 + K_2 x_2 = \frac{I_0}{C}$$

$K_1 = -K_2 \Rightarrow -K_2 r_1 + K_2 r_2 = \frac{I_0}{C} \Rightarrow K_2(r_2 - r_1) = \frac{I_0}{C} \Rightarrow K_2 = \frac{I_0}{C(r_2 - r_1)}$ (5)

$$K_1 r_1 + \frac{I_0 r_2}{C(r_2 - r_1)} = \frac{I_0}{C} \Rightarrow K_1 r_1 = \frac{I_0}{C}\left[1 - \frac{r_2}{r_2 - r_1}\right] = \frac{I_0}{C}\left[\frac{-r_1}{r_2 - r_1}\right]$$

$$K_1 = \frac{-I_0}{C(r_2 - r_1)}$$

$$v(t) = \frac{-I_0}{C}\left[\frac{1}{r_2 - r_1} e^{r_1 t} - \frac{1}{r_2 - r_1} e^{r_2 t}\right] = \frac{I_0}{C(r_2 - r_1)}\left[e^{r_1 t} - e^{r_2 t}\right]$$

Case (ii) $\frac{G}{2C} = \frac{1}{LC}$

$$v(t) = e^{-G/2C\, t}\left[C_1 t + C_2\right]$$

At $t = 0^+$, $v = 0$

$$0 = e^0\left[0 + C_2\right] \Rightarrow C_2 = 0$$

$$v(t) = e^{-G/2C\, t} C_1 t$$

$$\frac{dv}{dt} = \frac{-G}{2C} e^{-G/2C\, t} C_1 t + e^{-G/2C\, t} C_1$$

$$\frac{I_0}{C} = -G/2C\, e^0 C_1 \times 0 + e^0 C_1$$

$$\Rightarrow C_1 = I_0/C$$

$$v(t) = e^{-G/2C\, t}\left[\frac{I_0}{C} t\right]$$

Case (iii) $\frac{G}{2C} < \frac{1}{LC}$

$$v(t) = e^{-G/2C\, t}\left[C_1 \cos\sqrt{\left(\frac{G}{2C}\right)^2 - \frac{1}{LC}}\; t + C_2 \sin\sqrt{\left(\frac{G}{2C}\right)^2 - \frac{1}{LC}}\; t\right]$$

At $t = 0$, $v(0^+) = 0$

$$0 = e^0\left[C_1 + C_2 \times 0\right] \Rightarrow C_1 = 0.$$

$$v(t) = e^{-G/2C\, t} C_2 \sin\sqrt{\left(\frac{G}{2C}\right)^2 - \frac{1}{LC}}\; t$$

$$\frac{dv}{dt} = C_2 e^{-G/2C\, t}\sqrt{\left(\frac{G}{2C}\right)^2 - \frac{1}{LC}} \cos\sqrt{\left(\frac{G}{2C}\right)^2 - \frac{1}{LC}}\; t - C_2 \times \frac{G}{2C} e^{-G/2C\, t} \sin\sqrt{\left(\frac{G}{2C}\right)^2 - \frac{1}{LC}}\; t$$

At $t = 0^+$ $\frac{dv}{dt} = I_0/C$

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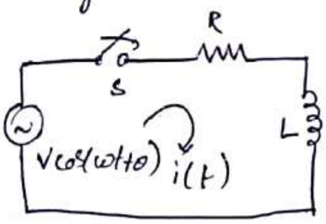
$$\frac{I_0}{C} = C_2 e^0 \cos 0 \times \sqrt{\left(\frac{G}{2C}\right)^2 - \frac{1}{LC}} \Rightarrow C_2 = \frac{I_0}{C}$$

$$v(t) = \frac{I_0 e^{-G/2C\, t}}{C\sqrt{\left(\frac{G}{2C}\right)^2 - \frac{1}{LC}}} \sin \sqrt{\left(\frac{G}{2C}\right)^2 - \frac{1}{LC}}\; t$$

Sinusoidal Response of RL Circuit :-

Consider a circuit consisting of resistance and inductance as shown in Fig.. The switch 'S' is closed at $t=0$, a sinusoidal voltage $V\cos(\omega t+\theta)$ is applied to series RL R-L circuit where V is the amplitude of the wave & θ is the phase angle.



Apply KVL to the circuit

$$V\cos(\omega t+\theta) = Ri + L\frac{di}{dt}$$

$$\frac{di}{dt} + \frac{R}{L}i = \frac{V}{L}\cos(\omega t+\theta) \longrightarrow (1)$$

This eq. has two solutions, Complementary function when (1) is equated to zero & particular integral

$$i(t) = i_c + i_p$$

$$\frac{di}{dt} + \frac{R}{L}i = 0 \Rightarrow i_c = c e^{-R/L\, t}$$

Let $i_p = A\cos(\omega t+\theta) + B\sin(\omega t+\theta)$

$$i_p' = -A\sin(\omega t+\theta)\times\omega + B\omega\cos(\omega t+\theta)$$

Substitute i_p & i_p' in (1).

$$-A\omega\sin(\omega t+\theta) + B\omega\cos(\omega t+\theta) + \frac{R}{L}\left[A\cos(\omega t+\theta) + B\sin(\omega t+\theta)\right]$$
$$= \frac{V}{L}\cos(\omega t+\theta)$$

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$$\left(-A\omega + \frac{BR}{L}\right)\sin(\omega t+\theta) + \left(B\omega + \frac{AR}{L}\right)\cos(\omega t+\theta) = \frac{V}{L}\cos(\omega t+\theta)$$ 6

$$-A\omega + \frac{BR}{L} = 0 \qquad A\omega = \frac{BR}{L} \Rightarrow A = \frac{BR}{\omega L}$$

$$B\omega + \frac{AR}{L} = \frac{V}{L}$$

$$B\omega + \frac{BR^2}{\omega L^2} = \frac{V}{L} \Rightarrow B\left[\omega + \frac{R^2}{\omega L^2}\right] = \frac{V}{L} \Rightarrow B\left[\frac{\omega^2 L^2 + R^2}{\omega L^{\cancel{2}}}\right] = \frac{V}{\cancel{L}}$$

$$B = V\left[\frac{\omega L}{R^2+\omega^2 L^2}\right], \quad A = \frac{V\,\omega \cancel{L}}{R^2+\omega^2 L^2} \times \frac{R}{\omega \cancel{L}} \Rightarrow A = V\left[\frac{R}{R^2+\omega^2 L^2}\right]$$

$$i_p = V\left[\frac{R}{R^2+\omega^2 L^2}\right]\cos(\omega t+\theta) + V\left[\frac{\omega L}{R^2+\omega^2 L^2}\right]\sin(\omega t+\theta)$$

Let $M\cos\phi = \frac{VR}{R^2+\omega^2 L^2}$, $M\sin\phi = \frac{V\omega L}{R^2+\omega^2 L^2}$

$$\frac{\cancel{M}\sin\phi}{\cancel{M}\cos\phi} = \frac{\frac{\omega L}{\cancel{R^2+\omega^2 L^2}}}{\frac{R}{\cancel{R^2+\omega^2 L^2}}} = \frac{\omega L}{R}$$

$$\tan\phi = \frac{\omega L}{R} \Rightarrow \phi = \tan^{-1}\left[\frac{\omega L}{R}\right]$$

$$M^2\cos^2\phi + M^2\sin^2\phi = \frac{V^2R^2}{(R^2+\omega^2L^2)^2} + \frac{V^2\omega^2L^2}{(R^2+\omega^2L^2)^2} = V^2\left[\frac{1}{R^2+\omega^2L^2}\right]$$

$$M = \frac{V}{\sqrt{R^2+\omega^2L^2}}$$

$$i_p = \frac{V}{\sqrt{R^2+\omega^2L^2}}\cos\phi\cos(\omega t+\theta) + \frac{V}{\sqrt{R^2+\omega^2L^2}}\sin\phi\sin(\omega t+\theta)$$

$$= \frac{V}{\sqrt{R^2+\omega^2L^2}}\left[\cos(\omega t+\theta-\phi)\right] = \frac{V}{\sqrt{R^2+\omega^2L^2}}\left[\cos\left(\omega t+\theta-\tan^{-1}\frac{\omega L}{R}\right)\right]$$

$$i(t) = ce^{-R/L\,t} + \frac{V}{\sqrt{R^2+\omega^2L^2}}\cos\left(\omega t+\theta-\tan^{-1}\frac{\omega L}{R}\right)$$

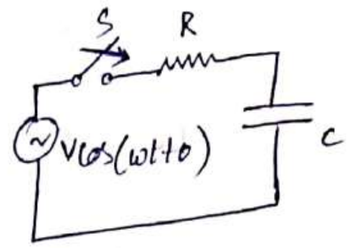
At $t=0$ $i(0^+)=0$

$$0 = ce^0 + \frac{V}{\sqrt{R^2+\omega^2L^2}}\cos\left(\theta-\tan^{-1}\frac{\omega L}{R}\right) \Rightarrow c = \frac{-V}{\sqrt{R^2+\omega^2L^2}}\cos\left(\theta-\tan^{-1}\frac{\omega L}{R}\right)$$

$$i(t) = \frac{-V}{\sqrt{R^2+\omega^2L^2}}\cos\left(\theta - \tan^{-1}\frac{\omega L}{R}\right)e^{-R/L\,t} + \frac{V}{\sqrt{R^2+\omega^2L^2}}\cos\left(\omega t+\theta-\tan^{-1}\frac{\omega L}{R}\right).$$

2

Sinusoidal Response of RC Circuit :-



Consider the circuit consisting of circuit with R & C in series.

Apply KVL to the circuit

$$V\cos(\omega t+\theta) = Ri + \frac{1}{C}\int i\,dt$$

$$-V\omega\sin(\omega t+\theta) = R\frac{di}{dt} + \frac{1}{C}i \longrightarrow (1).$$

$$-\frac{V\omega}{R}\sin(\omega t+\theta) = \frac{di}{dt} + \frac{1}{RC}i \Rightarrow \frac{di}{dt} + \frac{1}{RC}i = 0$$

$$i(t) = i_c + i_p \quad \Rightarrow i_c = ce^{-\frac{1}{RC}t}$$

$$i_p = A\cos(\omega t+\theta) + B\sin(\omega t+\theta)$$

$$i_p' = -A\omega\sin(\omega t+\theta) + B\omega\cos(\omega t+\theta)$$

Put i_p' & i_p in (1).

$$-A\omega\sin(\omega t+\theta) + B\omega\cos(\omega t+\theta) + \frac{1}{RC}\left[A\cos(\omega t+\theta) + B\sin(\omega t+\theta)\right] = -\frac{V\omega\sin(\omega t+\theta)}{R}$$

$$\left(-A\omega + \frac{B}{RC}\right)\sin(\omega t+\theta) + \left(B\omega + \frac{A}{RC}\right)\cos(\omega t+\theta) = \frac{-V\omega}{R}\sin(\omega t+\theta)$$

$$-A\omega + \frac{B}{RC} = \frac{-V\omega}{R}, \quad B\omega + \frac{A}{RC} = 0 \Rightarrow B\omega = -\frac{A}{RC}$$

$$B = \frac{-A}{\omega RC}$$

$$-A\omega - \frac{A}{\omega R^2C^2} = \frac{-V\omega}{R}$$

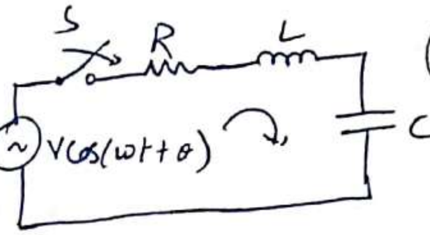
$$-A\left(\frac{\omega^2R^2C^2+1}{\omega R^2C^2}\right) = \frac{-V\omega}{\cancel{R}}$$

$$A = V\frac{\omega^2RC^2}{1+\omega^2R^2C^2}$$

$$B = -V\frac{\omega^{\cancel{2}}\cancel{R}C^{\cancel{2}}}{1+\omega^2R^2C^2} \times \frac{1}{\cancel{\omega}R\cancel{C}}$$

$$= -V\frac{\omega C}{1+\omega^2R^2C^2}$$

Sinusoidal Response of RLC circuit :-



Consider a circuit consisting of resistance, inductance and capacitance in series as shown in fig. Switch 's' is closed at $t=0$. At $t=0$, $V\cos(\omega t+\theta)$ is applied to RLC series circuit.

Apply KVL to the circuit

$$V\cos(\omega t+\theta) = Ri + L\frac{di}{dt} + \frac{1}{C}\int i\,dt$$

Diff.

$$R\frac{di}{dt} + L\frac{d^2 i}{dt^2} + \frac{1}{C} i = -V\omega \sin(\omega t+\theta)$$

$$\left(D^2 + \frac{R}{L}D + \frac{1}{LC}\right) i = \frac{-V\omega}{L}\sin(\omega t+\theta)$$

$$i_p = A\cos(\omega t+\theta) + B\sin(\omega t+\theta)$$

$$i_p' = -A\omega\sin(\omega t+\theta) + B\omega\cos(\omega t+\theta)$$

$$i_p'' = -A\omega^2\cos(\omega t+\theta) - B\omega^2\sin(\omega t+\theta)$$

$$-A\omega^2\cos(\omega t+\theta) - B\omega^2\sin(\omega t+\theta) + \frac{R}{L}\left[-A\omega\sin(\omega t+\theta) + B\omega\cos(\omega t+\theta)\right]$$
$$+ \frac{1}{C}\left[A\cos(\omega t+\theta) + B\sin(\omega t+\theta)\right] = -\frac{V\omega}{L}\sin(\omega t+\theta)$$

$$A\left(\frac{\omega R}{L}\right) + B\left(\omega^2 - \frac{1}{LC}\right) = \frac{V\omega}{L}, \quad -B\omega^2 - A\frac{\omega R}{L} + \frac{B}{LC} = \frac{-V\omega}{L}$$

Cosine terms

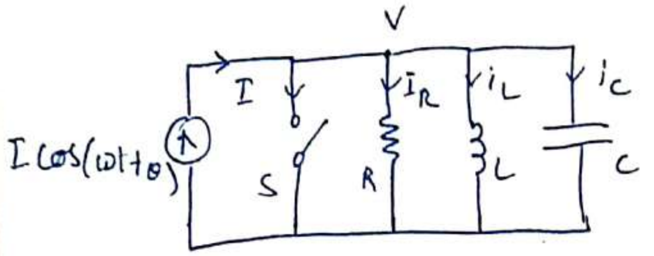
$$-A\omega^2 + \frac{B\omega R}{L} + \frac{A}{LC} = 0$$

$$A\left(\omega^2 - \frac{1}{LC}\right) - B\left(\frac{\omega R}{L}\right) = 0$$

$$A = \frac{V \times \frac{\omega^2 R}{L^2}}{\left(\frac{\omega R}{L}\right)^2 + \left(\omega^2 - \frac{1}{LC}\right)^2} \qquad B = \frac{\left(\omega^2 - \frac{1}{LC}\right)V\omega}{L\left[\left(\frac{\omega R}{L}\right)^2 + \left(\omega^2 - \frac{1}{LC}\right)^2\right]}$$

Sinusoidal Response of Parallel RLC network :- (8)

Consider parallel RLC circuit with switch open at $t=0$



Apply KVL to the circuit

$$I = I_R + I_L + I_C$$

$$I\cos(\omega t+\theta) = \frac{V}{R} + \frac{1}{L}\int V\,dt + C\frac{dV}{dt}$$

$$-I\omega\sin(\omega t+\theta) = \frac{dV/dt}{R} + \frac{1}{L}V + C\frac{d^2V}{dt^2}$$

$$\frac{d^2V}{dt^2} + \frac{1}{RC}\frac{dV}{dt} + \frac{1}{LC}V = \frac{-I\omega}{C}\sin(\omega t+\theta)$$

$$\left(D^2 + \frac{1}{RC}D + \frac{1}{LC}\right)V = \frac{-I\omega}{C}\sin(\omega t+\theta)$$

$$V_p = A\cos(\omega t+\theta) + B\sin(\omega t+\theta)$$

$$V_p' = -A\omega\sin(\omega t+\theta) + B\omega\cos(\omega t+\theta)$$

$$V_p'' = -A\omega^2\cos(\omega t+\theta) - B\omega^2\sin(\omega t+\theta)$$

$$-A\omega^2\cos(\omega t+\theta) - B\omega^2\sin(\omega t+\theta) + \frac{1}{RC}\left[-A\omega\sin(\omega t+\theta) + B\omega\cos(\omega t+\theta)\right]$$

$$+ \frac{1}{LC}\left[A\cos(\omega t+\theta) + B\sin(\omega t+\theta)\right] = \frac{-I\omega}{C}\sin(\omega t+\theta)$$

$$-A\omega^2 + \frac{A}{LC}\cos(\omega t+\theta) + \frac{B\omega}{RC} = 0 \quad , \quad -\frac{A\omega}{RC} - B\omega^2 + \frac{B}{LC} = \frac{-I\omega}{C}$$

$$A = \frac{B\frac{\omega}{RC}}{\omega^2 - \frac{1}{LC}} \quad , \quad B = \frac{\frac{I\omega}{C}\left(\omega^2 - \frac{1}{LC}\right)}{\left(\omega^2 - \frac{1}{LC}\right)^2 + \left(\frac{\omega}{RC}\right)^2}$$

$$V_p = \frac{I\omega^2}{RC^2} \Big/ \left(\omega^2 - \frac{1}{LC}\right)^2 + \left(\frac{\omega}{RC}\right)^2$$

$$V_p = \frac{\frac{I\omega^2}{RC^2}\cos(\omega t+\theta)}{\left[\left(\omega^2 - \frac{1}{LC}\right)^2 + \left(\frac{\omega}{RC}\right)^2\right]} + \frac{\frac{I\omega}{C}\left(\omega^2 - \frac{1}{LC}\right)\sin(\omega t+\theta)}{\left[\left(\omega^2 - \frac{1}{LC}\right)^2 + \left(\frac{\omega}{RC}\right)^2\right]}$$

$$i_p = \frac{\frac{V\omega^2 R}{L^2}}{\left(\frac{\omega R}{L}\right)^2 - \left(\omega^2 - \frac{1}{LC}\right)^2}\cos(\omega t+\theta) + \frac{\left(\omega^2-\frac{1}{LC}\right)V\omega}{L\left[\left(\frac{\omega R}{L}\right)^2 - \left(\omega^2-\frac{1}{LC}\right)^2\right]}\sin(\omega t+\theta)$$

$$m\cos\phi = \frac{\frac{V\omega^2 R}{L^2}}{\left(\frac{\omega R}{L}\right)^2 - \left(\omega^2-\frac{1}{LC}\right)^2}, \quad m\sin\phi = \frac{\left(\omega^2-\frac{1}{LC}\right)V\omega}{L\left[\left(\frac{\omega R}{L}\right)^2-\left(\omega^2-\frac{1}{LC}\right)^2\right]}$$

$$\frac{m\sin\phi}{m\cos\phi} \Rightarrow \tan\phi = \frac{\left(\omega L - \frac{1}{\omega C}\right)}{R}$$

$$m^2\cos^2\phi + m^2\sin^2\phi = \frac{V^2}{R^2+\left(\frac{1}{\omega C}-\omega L\right)^2}$$

$$m = \frac{V}{\sqrt{R^2+\left(\frac{1}{\omega C}-\omega L\right)^2}}$$

$$i_p = \frac{V}{\sqrt{R^2+\left(\frac{1}{\omega C}-\omega L\right)^2}}\cos\left[\omega t+\theta+\tan^{-1}\left(\frac{\frac{1}{\omega C}-\omega L}{R}\right)\right]$$

$$D^2 + \frac{R}{L}D + \frac{1}{LC} = 0$$

$$D_1, D_2 = \frac{-R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$$

$\frac{R}{2L} > \frac{1}{LC}$

$$i_c(t) = c_1 e^{D_1 t} + c_2 e^{D_2 t}$$

$$i = i_p + i_c$$

$\frac{R}{2L} = \frac{1}{LC}$, $i_c(t) = e^{D_1 t}(c_1 t + c_2)$

$$i = i_p + i_c$$

$\frac{R}{2L} < \frac{1}{LC}$, $i_c(t) = c_1 e^{D_1 t} + c_2 e^{D_2 t} \Rightarrow i = i_p + i_c$

$= m\cos\phi \cos(\omega t+\theta) + m\sin\phi \sin(\omega t+\theta)$ (9)

$$m\cos\phi = \frac{\frac{I\omega^2}{RC^2}}{\left(\omega^2-\frac{1}{LC}\right)^2+\left(\frac{\omega}{RC}\right)^2}, \quad m\sin\phi = \frac{\frac{I\omega}{C}\left(\omega^2-\frac{1}{LC}\right)}{\left(\omega^2-\frac{1}{LC}\right)^2+\left(\frac{\omega}{RC}\right)^2}$$

$$\phi = \tan^{-1}\left(R\left(\omega C-\frac{1}{\omega L}\right)\right)$$

$$m^2\cos^2\phi + m^2\sin^2\phi \Rightarrow m = \frac{\frac{I\omega}{C}}{\sqrt{\left(\omega^2-\frac{1}{LC}\right)^2+\left(\frac{\omega}{RC}\right)^2}}$$

$$V_p = \frac{\frac{I\omega}{C}}{\sqrt{\left(\omega^2-\frac{1}{LC}\right)^2+\left(\frac{\omega}{RC}\right)^2}} \cos\left[\omega t+\theta+\tan^{-1}\left(R\left(\omega C-\frac{1}{\omega L}\right)\right)\right]$$

$$\left(D^2+\frac{1}{RC}D+\frac{1}{LC}\right)V = 0$$

$$D_1, D_2 = \frac{-\frac{1}{RC} \pm \sqrt{\left(\frac{1}{RC}\right)^2-\frac{4}{LC}}}{2} = -\frac{1}{2RC} \pm \sqrt{\left(\frac{1}{2RC}\right)^2-\frac{1}{LC}}$$

$\frac{1}{2RC} > \frac{1}{LC}$ $\quad V_c(t) = A_1 e^{D_1 t} + A_2 e^{D_2 t}$

$V(t) = V_p + V_c$

$\frac{1}{2RC} = \frac{1}{LC}$, $\quad V_c(t) = e^{D_1(t)}(A_1 t + A_2)$

$V(t) = V_p + V_c$

$\frac{1}{2RC} < \frac{1}{LC}$ $\quad V_c(t) = A_1 e^{D_1 t} + A_2 e^{D_2 t}$.

$V(t) = V_p + V_c$.

Laplace Transform Method :-

The laplace transform is used to solve differential equations and corresponding initial and final value problems. Laplace transforms are widely used in engineering, particularly when the driving function has discontinuties and appears for a short period only.

In circuit analysis, the input and output functions donot exist forever in time. For casual functions, the function can be defined as $f(t)u(t)$. The integral for the Laplace transform is taken with the lower limit at $t=0$ in order to include the effect of an discontinuity at $t=0$.

Consider a function $f(t)$ which is to be continuous & defined for values of $t \geq 0$. The laplace transform is then

$$L\{f(t)\} = F(s) = \int_0^{\infty} f(t)\underbrace{e^{-st}}_{u(t)} dt = \int_0^{\infty} f(t) e^{-st} dt$$

$s = \sigma + j\omega$

For a function to have laplace transform it must satisfy the condition $\int_0^{\infty} f(t) e^{-st} dt < \infty$.

$$L^{-1}[F(s)] = f(t) = \frac{1}{2\pi j}\int_{-j}^{+j} F(s) e^{st} ds$$

	Function	Laplace transform
1.	1	$1/s$
2.	t	$1/s^2$
3	e^{-at}	$1/s+a$

t^n	$\frac{n!}{s^{n+1}}$
$\sin\omega t$	$\frac{\omega}{s^2+\omega^2}$
$\cos\omega t$	$\frac{s}{s^2+\omega^2}$
$\cosh at$	$\frac{s}{s^2-a^2}$
$\sinh at$	$\frac{a}{s^2-a^2}$
$e^{-at}\cos\omega t$	$\frac{s}{(s+a)^2+\omega^2}$
$e^{-at}\sin\omega t$	$\frac{\omega}{(s+a)^2+\omega^2}$

$L\left\{\frac{df(t)}{dt}\right\} \Rightarrow sF(s)-f(0)$

$L\left\{\frac{d^nf(t)}{dt^n}\right\} \quad s^nF(s)-s^{n-1}f(0)-s^{n-2}f'(0)\ldots$

$L\left\{\int f(t)\right\} \Rightarrow \frac{F(s)}{s}$

$L\left\{t\,f(t)\right\} = -\frac{d}{ds}F(s)$

$L\left\{\int_0^\infty \frac{f(t)}{t}\right\} = \int_0^\infty F(s)\,ds$

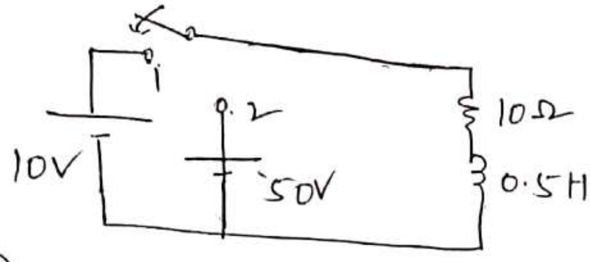
Initial value theorem $\rightarrow \lim_{t\to0} f(t) = \lim_{s\to\infty} sF(s)$

Final value theorem $\rightarrow \lim_{t\to\infty} f(t) = \lim_{s\to0} sF(s)$

First shifting theorem $L\{e^{-at}f(t)\} \rightarrow F(s+a)$

Second shifting theorem $L\{f(t-a)u(t-a)\} \rightarrow e^{-as}F(s)$

→ In the circuit shown, determine the current $i(t)$ when the switch is changed from position 1 to 2. The switch is moved from position 1 to 2 at $t=0$



Sol:

$$i(0) = \frac{10}{10} = 1\,A.$$

When switch is moved from 1 to 2 at $t=0$

$$50 = 10i + 0.5\frac{di}{dt}$$

Apply Laplace method

$$\frac{50}{s} = 10\,I(s) + 0.5\left[sI(s) - i(0)\right]$$

$$= 10I(s) + 0.5sI(s) - 0.5$$

$$\frac{50}{s} + 0.5 = I(s)\left[10 + 0.5s\right]$$

$$\frac{50+0.5s}{s}\ \cancel{\frac{50+.5}{0.5}} = I(s)\left[s + \frac{10}{0.5}\right]\times 0.5$$

$$\cancel{101} = \cancel{I(s)\left[s+20\right]}$$

$$\cancel{I(s) = \frac{101}{s+20}}$$

$$\frac{50}{s(s+20)} = \frac{A}{s} + \frac{B}{s+20}$$

$$50 = A(s+20) + Bs$$

$$A = \frac{5}{2} = 2.5$$

$$B = -5/2 = -2.5$$

$$I(s) = \frac{0.5s+50}{s(s+20)}$$

$$= \frac{0.5}{s+20} + \frac{50}{s(s+20)}$$

$$= \frac{0.5}{s+20} + \frac{A}{s} + \frac{B}{s+20}$$

$$= \frac{0.5}{s+20} + \frac{5/2}{s} - \frac{5/2}{s+20}$$

$$i(t) = 0.5e^{-20t} + \frac{5}{2} - \frac{5}{2}e^{-20t}$$

$$= -2e^{-20t} + 2.5$$

$$= 2.5 - 2e^{-20t}$$

$$\frac{50+0.5s}{s\times0.5} = I(s)(s+20)$$

$$\frac{100+s}{s} = I(s)(s+20)$$

$$I(s) = \frac{s+100}{s(s+20)}$$

$$= \frac{A}{s} + \frac{B}{s+20}$$

$$s+100 = A(s+20) + Bs$$

$$A = \frac{100}{20} = 5$$

$$-20B = 80$$

$$B = -4$$

$$I(s) = \frac{5}{s} - \frac{4}{s+20}$$

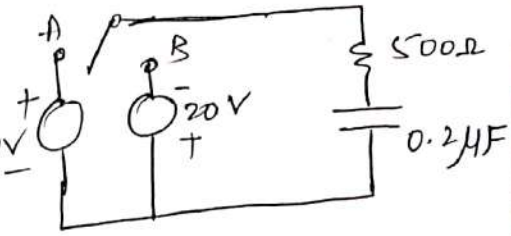
$$i(t) = 5 - 4e^{-20t}$$ ☆

☆ $$\boxed{i(t) = 5 - 4e^{-20t} = i(\infty) + \left[i(0) - i(\infty)\right]e^{-R/L\,t}}$$

$(1-5)$

→ In fig.. the switch K is closed at position A at $t=0$. After a lapse of one time constant the switch is moved to position B. Determine complete current.

11

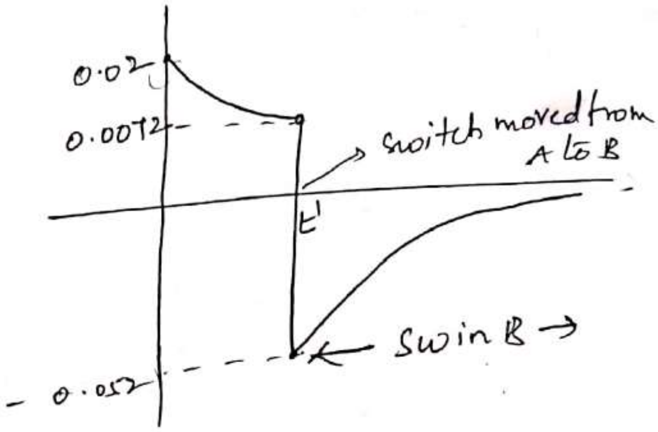


Sol.

At $t=0$, switch is closed.

So with 10V, Capacitor C is charged to 10V

for $0<t<\tau$



$$V = Ri + \frac{1}{C}\int i\,dt$$

$$10 = 500i + \frac{1}{0.2\times10^{-6}}\int i\,dt$$

$$0 = 500\frac{di}{dt} + \frac{i}{0.2\times10^{-6}}$$

$$\frac{di}{dt} + 10000i = 0$$

$$i = Ce^{-10000t}$$

At $t=0$, Capacitor acts as S.C, so $i(0) = \frac{10}{500} = 0.02$

$$0.02 = C$$

$$i = 0.02e^{-10000t}\ A.$$

$$\tau = RC = 500\times0.2\times10^{-6} = 1\times10^{-4}\ sec.$$

At one time constant $i = 0.02\times e^{-10000\times10^{-4}} = 0.02\,e^{-1} = 7.35\times10^{-3}\ A$

" $V_C = V(1-e^{-t/\tau})$

$= 10(1-e^{-\tau/\tau}) = 6.32\ V.$

Now switch is moved to B.

$$-20 = 500i' + \boxed{\frac{1}{C}\int i\,dt}\ 6.37$$

$$-20 = 500i' + \frac{1}{C}\int i\,dt\ \ 6.37$$

$$\cancel{-20 = 500\frac{di'}{dt} + \frac{1}{0.2\times10^{-6}}i'}$$

$$i' = \cancel{Ce^{-1000t}}$$

At $t=\tau$, $i = \cancel{7.35\times10^{-3}\ A}$

$$\cancel{7.35\times10^{-3} = Ce^{1000\times10^{-4}}}$$

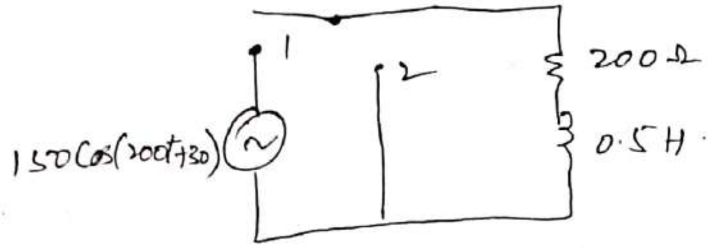
$$\cancel{C = \frac{7.35\times10^{-3}}{e^{-1}} = 0.0199}$$

$$-20 - 6.37 = 500i'$$

$$i' = -0.052$$

$$i = -0.052e^{-1000(t-t')}\ A$$

→ For the circuit shown in figure, determine the transient current when the switch is moved from position 1 to 2 at $t=0$. The circuit is in steady state with switch in position 1.



Sol:

when switch is in position ①

$$i(0) = \frac{150\cos(200\times0+30)}{200} = \frac{15\cos 30}{200} = \frac{150\times\frac{\sqrt3}{2}}{200} = 0.649.$$

when switch is in position ②

$$200i + 0.5\frac{di}{dt} = 0.$$

$$200I(s) + 0.5\left[sI(s) - i(0)\right] = 0.$$

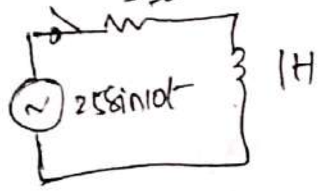
$$200I(s) + 0.5\left[sI(s) - 0.649\right] = 0.$$

$$I(s)\left[200 + 0.5s\right] = 0.3245$$

$$I(s) = \frac{0.3245}{0.5\left(s + \frac{200}{0.5}\right)} = \frac{0.649}{s+400}$$

$$i(t) = 0.649e^{-400t}.$$

→ A sinusoidal voltage $25\sin 10t$ is applied at $t=0$ to series RL circuit having $R=5\Omega$, $L=1H$. Find $i(t)$.



$s = \pm j10$

$$25\sin 10t = 5i + \frac{di}{dt}$$

$$25\times\frac{10}{s^2+100} = 5I(s) + sI(s) - i(0)$$

$$= I(s)(s+5)$$

$$I(s) = \frac{250}{(s^2+100)(s+5)} = \frac{A}{s+5} + \cancel{\frac{Bs+C}{s^2}} \frac{B}{(s+j10)} + \frac{C}{(s-j10)}$$

$$= \frac{A(s^2+100) + B(s+5)(s-j10) + C(s+5)(}{(s+5)(s^2+100)}$$

~~i(t) =~~

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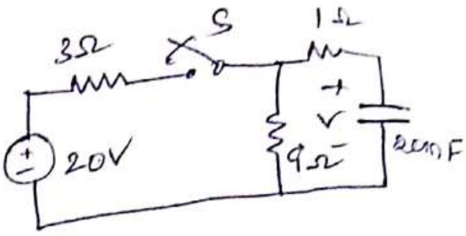
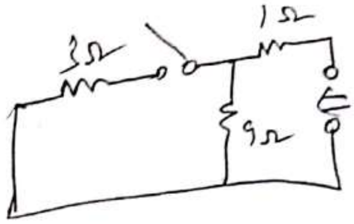
→ The switch in the circuit has been closed for a long time and it is opened at $t=0$, Find $v(t)$ for $t \geq 0$. Calculate the initial energy stored in the capacitor.



Sol:..

when switch has been closed for a long time



At $t \leq 0$

$V_c(0^-) =$ voltage across 9Ω resistor

$$= \frac{9}{9+3} \times (20) = 15V.$$

when switch is opened at $t=0$

$V_c(0^+) = 15V$

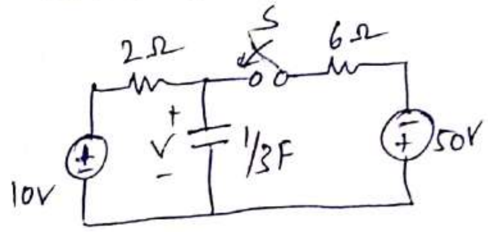
$$\tau = R_{th} C = (9+1) \times 20 \times 10^{-3}$$

$$= 10 \times 20 \times 10^{-3} = 0.2$$

$$V_c(t) = V_c(0) e^{-t/\tau} = 15 e^{-t/0.2} = 15 e^{-5t} V$$

$$W_c(0) = \frac{1}{2} C V_c^2(0) = \frac{1}{2} \times 20 \times 10^{-3} \times 15^2 = 2.25J.$$

→ Find $v(t)$ for $t>0$ in the circuit in fig. Assume the switch has been open for a long time and is closed at $t=0$. Calculate $v(t)$ at $t=0.5$ sec.



Sol:.. when switch is in open for long time, $v_c(0^-) = 10V$

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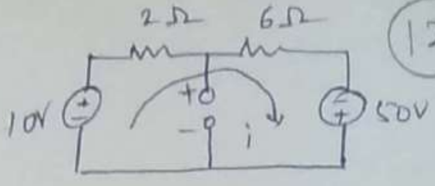
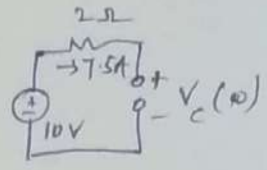
(12)

When switch is closed at $t=0$



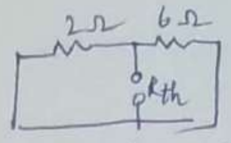
$10-2i-6i+50=0$

$8i=60 \Rightarrow i=60/8=15/2=7.5A$



$V_c(\infty) = 10-2\times7.5 = 10-15 = -5V$

$V_c(t) = V_c(\infty) + [V_c(0) - V_c(\infty)]e^{-t/\tau}$



$R_{th} = \frac{2\times6}{8} = 3/2$

$\tau = R_{th}\times C = \frac{3}{2}\times\frac{1}{3} = \frac{1}{2}$

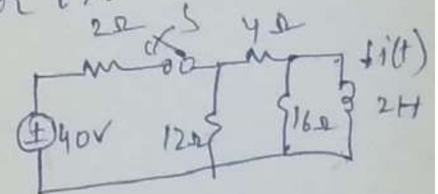
$V_c(t) = -5+[10-(-5)]e^{-t/\frac{1}{2}}$

$= (-5+15e^{-2t})\,V$

At $t=0.5$ sec

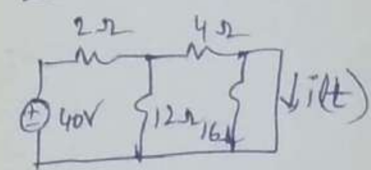
$V_c(t) = -5+15e^{-2\times0.5} = -5+15e^{-1} = 0.518V$

→ The switch in the circuit has been closed for a long time. At $t=0$, the switch is opened. Calculate $i(t)$ for $t>0$



Sol:

switch is closed for a long time, so inductor acts as short circuit.



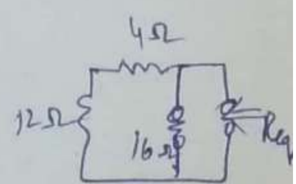
12 & 4 are parallel

So current flowing through 4Ω is $i_0(0^-)$

$\frac{12\times4}{16}+2 = 5\Omega$

$i = \frac{40}{5} = 8A$

$i(0^-) = 8\times\frac{12}{12+4} = 8\times\frac{12}{16} = 6A$



$R_{eq} = (12+4)//16$

$= 16//16 = 8\Omega$

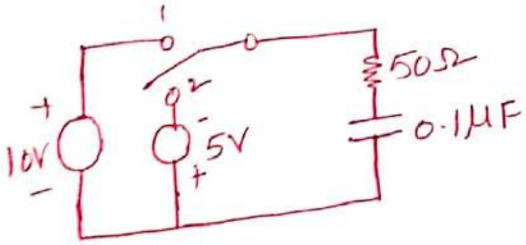
When switch is open at $t=0$

$\tau = \frac{L}{R_{eq}} = \frac{2}{8} = 1/4$ sec

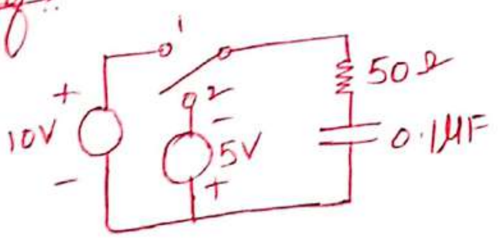
$i(t) = i(0)e^{-t/\tau} = 6e^{-t/1/4} = 6e^{-4t}$ Amps

Problem :-

Steady state is reached in the circuit shown in Fig. When the switch is at 1 at $t=0$, the switch is moved to 2. Find the drop across the capacitor as well as at the resistor. Find the drop across the capacitor after $t = 0.1$ msec. Also find the energy stored across the capacitor.



Sol:



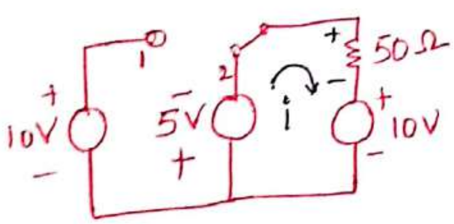
Given that

When switch is at position 1, steady state is reached. Therefore capacitor is open circuited and charged to 10V

$V_c(0^-) = 10V$, $i(0^-) = 0A$

When switch is moved to position 2 , $V_c(0^+) = 10V$, but

$i(0^+) = ?$

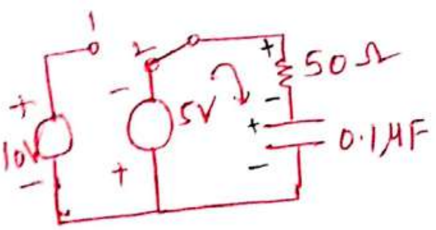


Apply KVL to the loop

$$-5 - 50i - 10 = 0$$

$$-15 = 50i \Rightarrow i = \frac{-15}{50} = -0.3$$

$$\therefore \; i(0^+) = -0.3A$$



Apply KVL to the loop

$$-5 - 50i - \frac{1}{0.1\times10^{-6}}\int i\,dt = 0$$

$$-5 = 50i + \frac{1}{0.1\times10^{-6}}\int i\,dt$$

Differentiate on both sides

$$50\frac{di}{dt} + \frac{1}{0.1\times10^{-6}}i = 0 \Rightarrow \frac{di}{dt} + \frac{1}{50\times0.1\times10^{-6}}i = 0$$

$$\frac{di}{dt} + 0.2\times10^{6} i = 0$$

$$i(t) = K_1 e^{-0.2\times10^{6}t}$$

At $t = 0^+$ $\quad i(0^+) = -0.3$

$$-0.3 = K_1 e^{0}$$

$$K_1 = -0.3$$

$$\therefore i(t) = -0.3 e^{-0.2\times10^{6}t} \rightarrow (1)$$

Voltage across Capacitor

$$V_c(t) = \frac{1}{C}\int i\,dt = \frac{1}{0.1\times10^{-6}}\int -0.3 e^{-0.2\times10^{6}t}\,dt$$

$$= \frac{1}{0.1\times10^{-6}} \times \cancel{-}0.3 \times \frac{e^{-0.2\times10^{6}t}}{\cancel{-}0.2\times10^{6}} + K_2$$

$$V_c(t) = 15 e^{-0.2\times10^{6}t} + K_2 \longrightarrow (2)$$

At $t = 0^+$, $V_c(0^+) = 10V$

$$10 = 15e^{0} + K_2 \Rightarrow K_2 = 10 - 15 = -5$$

Substitute K_2 in (2)

$$V_c(t) = 15e^{-0.2\times10^{6}t} - 5 = V_c(\text{transient}) - V_c(\text{steady state})$$ at position 2

** *

$$\therefore \boxed{V_c(t) = V_c(\infty) + \left(V_c(0) - V_c(\infty)\right)e^{-t/\tau}}$$

$$= -5 + \left(10 - (-5)\right) e^{-t/0.1\times10^{-6}\times50}$$

$$= -5 + 15 e^{-0.2\times10^{6}t}$$

Voltage across Resistor $V_R(t) = -5 - \left(-5 + 15e^{-0.2\times10^{6}t}\right)$

$$= -5 + 5 - 15e^{-0.2\times10^{6}t}$$

$$= -15e^{-0.2\times10^{6}t}$$

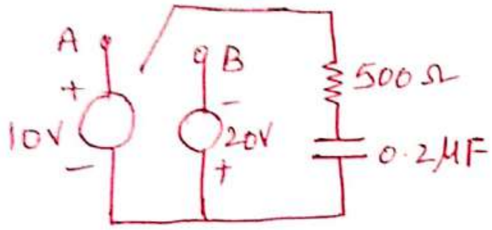
Energy stored in Capacitor at $t = 0.1$msec, $V_c(0.1\times10^{-3}) = 15e^{-0.2\times10^{6}\times0.1\times10^{-3}} - 5$

$= -5V$

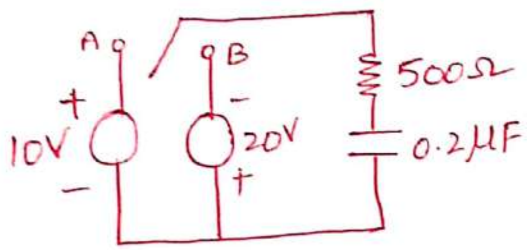
$= \frac{1}{2}CV_c^2 = \frac{1}{2}\times0.1\times10^{-6}\times(5)^2 = 1.25\mu J$

Problem :-

In fig the switch K is closed at position A at $t=0$. After the lapse of time equivalent to one time constant, the switch is moved to position B. Determine the complete current



Sol:-



Switch is closed at position A at $t=0$

then $i(t) = i(0^+)e^{-t/\tau}$

$= \frac{10}{500} e^{-t/500\times0.2\times10^{-6}}$

$= 0.02e^{-t/100\times10^{-6}}$

$= 0.02e^{-10^4 t}$

$\tau = RC = 500\times0.2\times10^{-6}$

$= 10^{-4}$

At $t=\tau$ i.e., one time constant

$i(\tau) = 0.02e^{-10^4\times10^{-4}}$

$= 0.02e^{-1}$

$= 0.0073$ A.

$V_c(t) = V(1-e^{-t/\tau})$

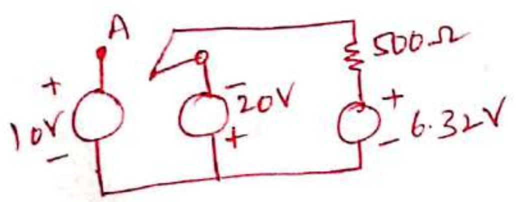
$= 10(1-e^{-t/(500\times0.2\times10^{-6})})$

$= 10(1-e^{-10^4 t})$

At $t=\tau$ (one time constant)

$V_c(\tau) = 10(1-e^{-10^4\times10^{-4}}) = 10(1-e^{-1}) = 6.32V$

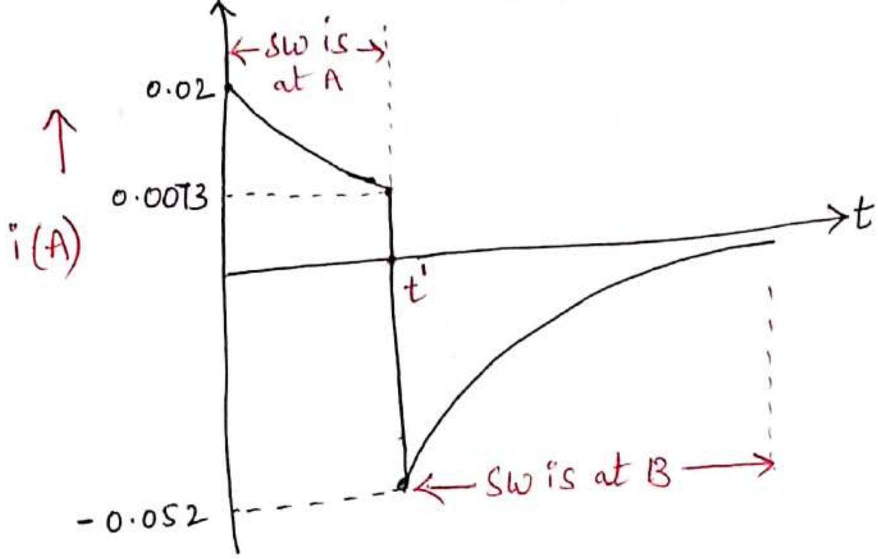
That means when capacitor is charged to 6.32V when switch is at position A. Before it is charged to 10V, switch is moved to position B.



$i(0^+) = \frac{-20-6.32}{500} = -0.052$ A

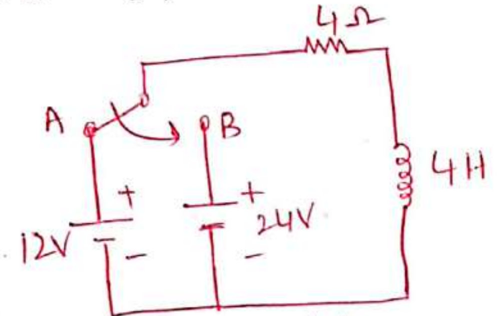
$i(t) = i(0^+)e^{(t-t')/\tau} = -0.052e^{(t-t')/500\times0.2\times10^{-6}}$

$= -0.052e^{(t-t')/10^{-4}} = -0.052e^{10^4(t-t')}$

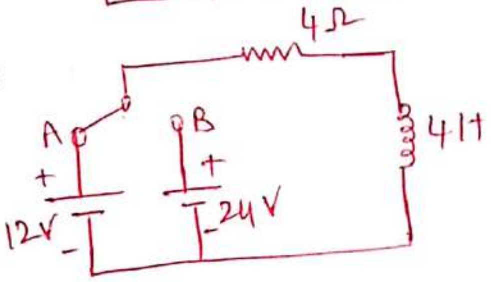


Problem :-

find $i(t)$ when switch K is moved from A to B at $t=0$.



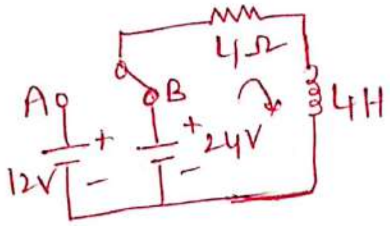
Sol:



When switch is position A steady state is reached, so inductor is s.c

so $i(0^-) = \frac{12}{4} = 3A$

When switch is moved from A to B at $t=0$, $i(0^+) = 3A$



Apply KVL to the loop

$$24 = 4i + 4\frac{di}{dt} \Rightarrow \frac{di}{dt} + i = 6$$

$$i(t) = e^{-t}\int 6e^{t}dt + C_1 e^{-t}$$

$$\Rightarrow i(t) = \cancel{e^{-t}} \times \cancel{e^{t}} \times 6 + C_1 e^{-t} \Rightarrow i(t) = 6 + C_1 e^{-t}, \quad \text{At } t=0^+, i(0^+) = 3$$

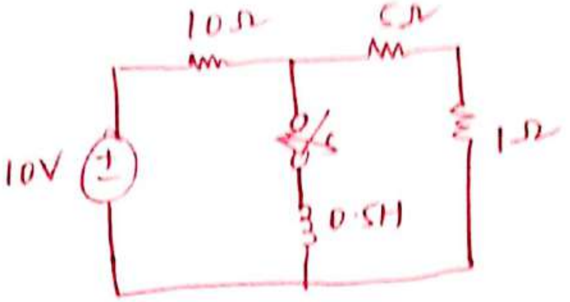
$$3 = 6 + C_1 e^{0} \Rightarrow C_1 = 3 - 6 = -3 \Rightarrow i(t) = (6 - 3e^{-t})A$$

$$i(t) = i(\infty) + [i(0) - i(\infty)]e^{-t/\tau} = \frac{24}{4} + \left[3 - \frac{24}{4}\right]e^{-t/L/R}$$

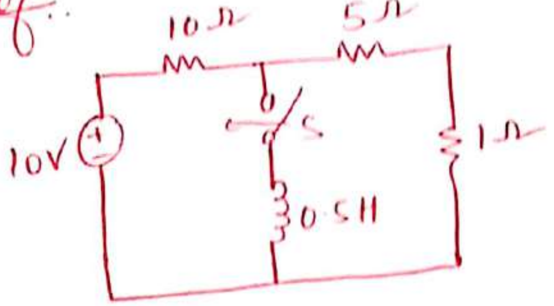
$$= 6 + [3-6]e^{-t/4/4} = 6 - 3e^{-t}$$

Problem:-

Find the current through inductor at $t=0^+$ as well as the voltage after closing the switch 's' at $t=0$.

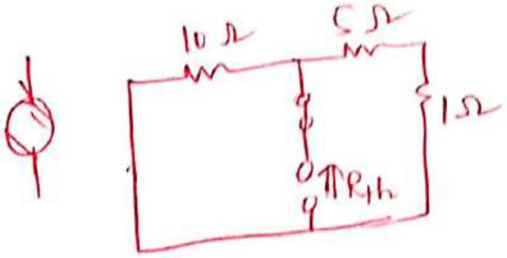


Sol:-



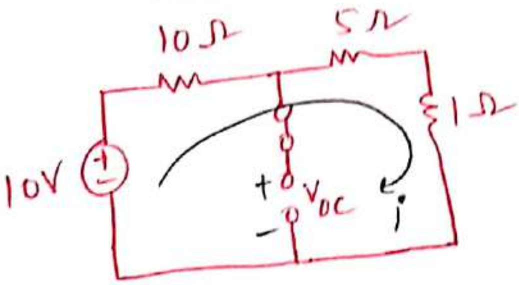
Time Constant $\tau = \frac{L}{R_{th}}$

To find R_{th}, short circuit voltage source and observe from inductor terminals

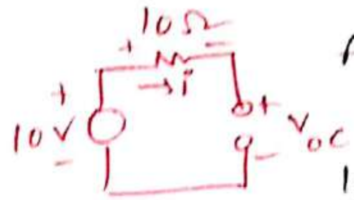


$R_{th} = 6\Omega // 10\Omega = \frac{60}{16} = 3.75\Omega$

$\tau = \frac{0.5}{3.75} = 0.133$



$i = \frac{10}{16} = 0.625$



Apply KVL

$10 - 10i - V_{oc} = 0$

$V_{oc} = 10 - 10 \times 0.625$

$= 3.75V$

$\therefore i_L(t) = \frac{V_{oc}}{R_{th}} \left(1 - e^{-t/\tau}\right)$

$= \frac{3.75}{3.75}\left(1 - e^{-t/0.133}\right) = 1 - e^{-7.51t}$

$V_L(t) = V_{oc}\, e^{-t/\tau} = 3.75\, e^{-t/0.133} = 3.75 e^{-7.51t}$

Network Topology

I. Basic definitions:

Network Topology:

- Is another method of solving electric circuits
- Is generalized approach

Network:

A combination of two or more network elements is called a network.

Topology:

Topology is a branch of geometry which is concerned with the properties of a geometrical figure, which are not changed when the figure is physically distorted, provided that, no parts of the figure are cut open or joined together.

The geometrical properties of a network are independent of the types of elements and their values.

Every element of the network is represented by a line segment with dots at the ends irrespective of its nature and value.

Circuit:

If the network has at least one closed path it is a circuit.

Note that every circuit is a network but every network is not a circuit.

Branch:

Representation of each element (component) of a electric network by a line segment is a branch.

Node:

A point at which two or more elements are joined is a node. End points of the branches are called nodes.

Graph:

It is collection of branches and nodes in which each branch connects two nodes.

Graph of a Network:

The diagram that gives network geometry and uses lines with dots at the ends to represent network element is usually called a graph of a given network. For example,

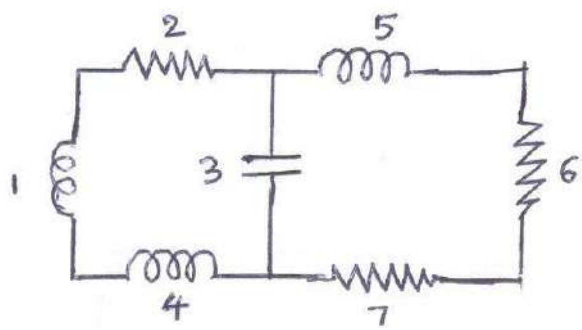


Fig.1 Network

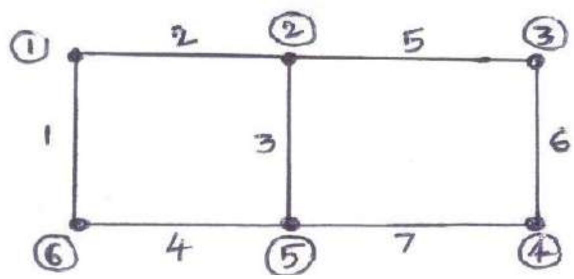


Fig.2 Graph

Sub-graph:

A sub-graph is a subset of branches and nodes of a graph for example branches 1, 2, 3 & 4 forms a sub -graph. The sub-graph may be connected or unconnected. The sub- graph of graph shown in figure 2 is shown in figure 3.

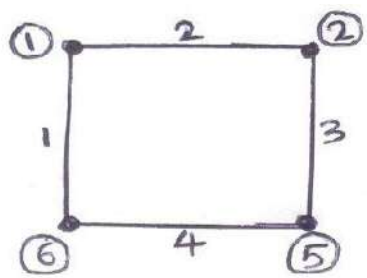


Fig.3 Sub-graph

Connected Graph:

If there exists at least one path from each node to every other node, then graph is said to be connected. Example,

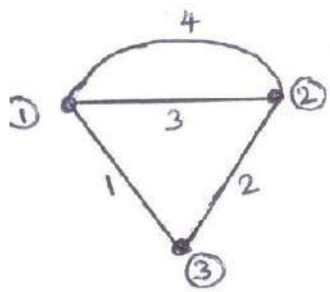


Fig.4 Connected Graph

Un-connected Graph:

If there exists no path from each node to every other node, the graph is said to be un-connected graph. For example, the network containing a transformer (inductively coupled parts) its graph could be un-connected.

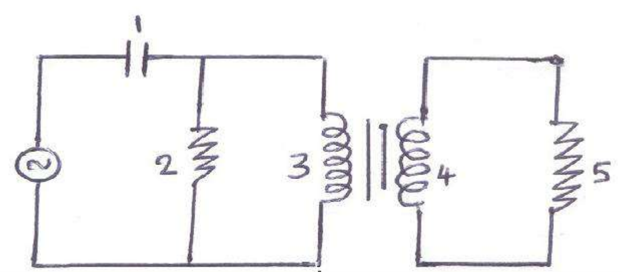


Fig.5 Network

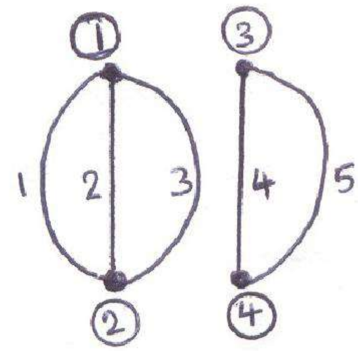


Fig.6 Un-connected Graph

Path (Walk):

A sequence of branches going from one node to other is called path. The node once considered should not be again considered the same node.

Loop (Closed Path):

Loop may be defined as a connected sub -graph of a graph, which has exactly two branches of the sub-graph connected to each of its node.

For example, the branches1, 2 & 3 in figure 7 constitute a loop.

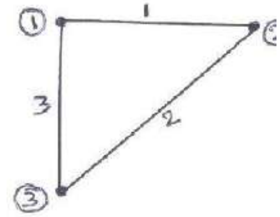


Fig.7 Loop

Planar and Non-planar Graphs:

A planar graph is one where the branches do not cross each other while drawn on a plain sheet of paper. If they cross, they are non-planar.

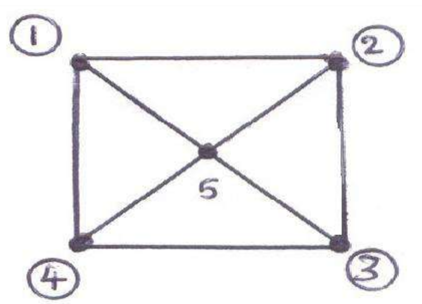


Fig.8 Planar Graph

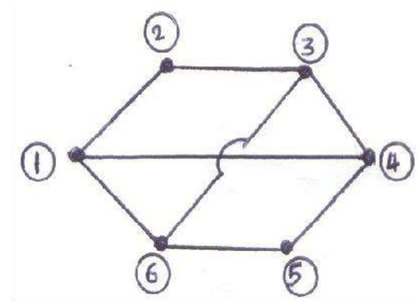


Fig.9 Non-planar graph

Oriented Graph:

The graph whose branches carry an orientation is called an oriented graph

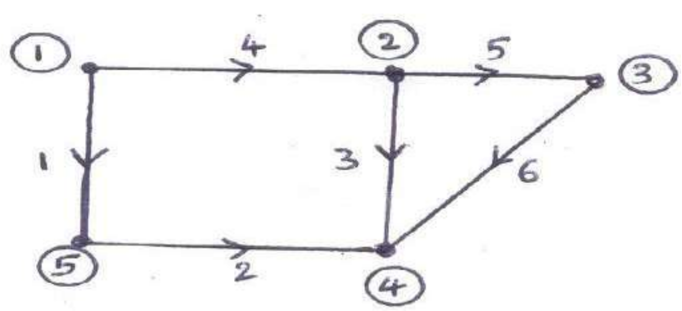
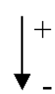


Fig.10 Oriented Graph

The current and voltage references for a given branches are selected with a +ve sign at tail side and –ve sign at head



Tree:

Tree of a connected graph is defined as any set of branches, which together connect all the nodes of the graph without forming any loops. The branches of a tree are called **Twigs.**

Co-tree:

Remaining branches of a graph, which are not in the tree form a co-tree. The branches of a co-tree are called **links** or **chords.**

The tree and co-tree for a given oriented graph shown in figure11 is shown in figure12 and figure13.

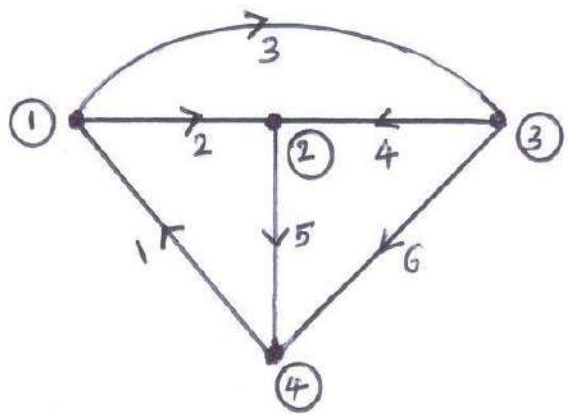


Fig. 11 Oriented Graph

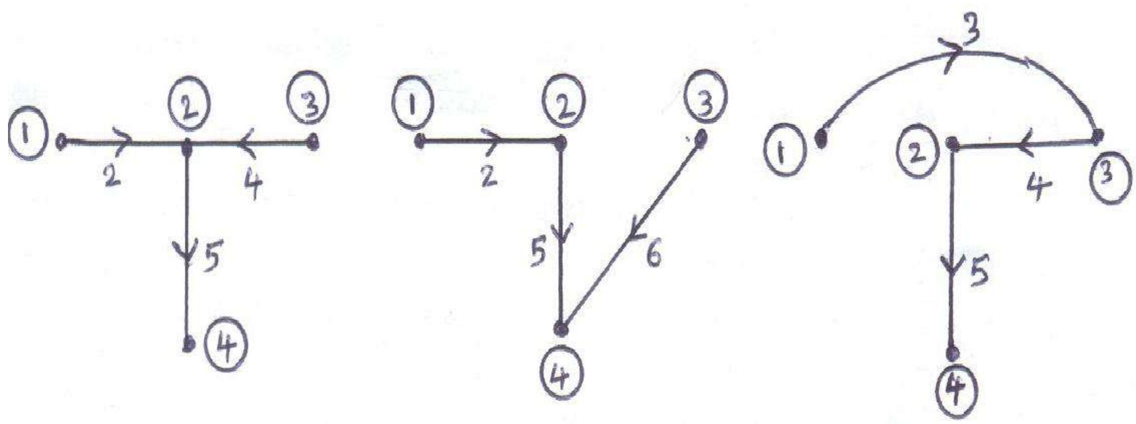


Fig.12 Trees

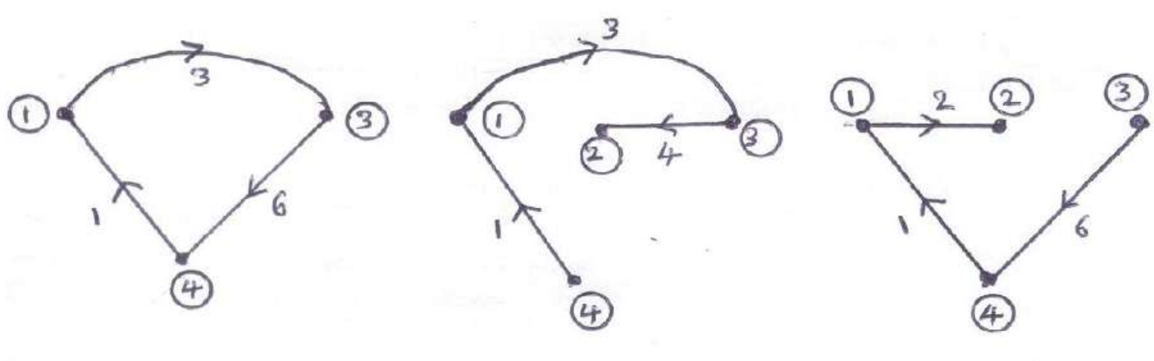


Fig.13 Co-trees

Tree	Twigs	Links (Chords)
1	2,4&5	1,3&6
2	3,4&5	1,2&6
3	2,5&6	1,3&4

Properties of Tree:

i) It contains all the nodes of the graph.

ii) It contains (n_t-1) branches. Where 'n_t' is total number of nodes in the given graph.

iii) There are no closed paths.

Total number of tree branches, n = (n_t-1)

Where n_t = Total number of nodes

Total number of links, l = (b-n)

Where b = Total number of branches in the graph.

Degree of Node:

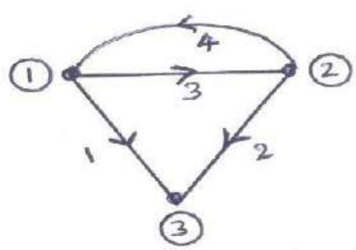
The number of branches attached to the node is degree of node.

II. Complete Incidence Matrix (A_a):

Incidence matrix gives us the information about the branches, which are joined to the nodes and the orientation of the branch, which may be towards a node or away from it.

Nodes of the graph form the rows and branches form the columns. If the branch is not connected to node, corresponding element in the matrix is given the value **'0'**. If a branch is joined, it has two possible orientations. If the orientation is away from the node, the corresponding matrix element is written as **'+1'**. If it is towards the node, the corresponding matrix element is written as **'-1'**.

Example: 1) Obtain complete incidence matrix for the graph shown



Solution:

A_a =

Nodes	**Branches**			
	1	2	3	4
1	1	0	1	-1
2	0	1	-1	1
3	-1	-1	0	0

$$A_a = \begin{array}{cccc} 1 & 0 & 1 & -1 \\ 0 & 1 & -1 & 1 \\ -1 & -1 & 0 & 0 \end{array}$$

Properties of Incidence Matrix:

i) Each column has only two non-zero elements and all other elements are zero.

ii) If all the rows of '$\mathbf{A_a}$' are added, the sum will be a row whose elements equal zero.

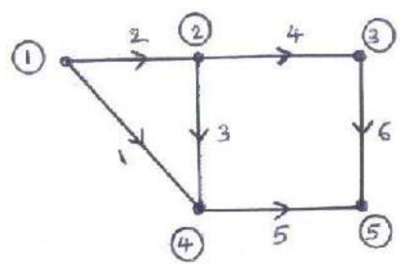
If the graph has '**b**' branches and '$\mathbf{n_t}$' nodes, the complete incidence matrix is of the order (n_t x b).

III. Reduced Incidence Matrix (A):

When one row is eliminated from the complete incidence matrix, the remaining matrix is called **reduced incidence matrix**

If the graph has '**b**' branches and '$\mathbf{n_t}$' nodes, the reduced incidence matrix is of the order (n_t-1) x b.

Example: 2) Write the complete and reduced incidence matrix for the given graph shown



Solution:

A_a =

Nodes	Branches					
	1	2	3	4	5	6
1	1	1	0	0	0	0
2	0	-1	1	1	0	0
3	0	0	0	-1	0	1
4	-1	0	-1	0	1	0
5	0	0	0	0	-1	-1

Complete Incidence Matrix, $\mathbf{A_a} =$

$$\begin{array}{cccccc} 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 1 \\ -1 & 0 & -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & -1 & -1 \end{array}$$

Reduced Incidence Matrix, $\mathbf{A} =$

$$\begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 1 \\ -1 & & 0\text{-}10 & 1 & 0 & \end{bmatrix}$$

Example: 3) Draw the oriented graph of incidence matrix shown below

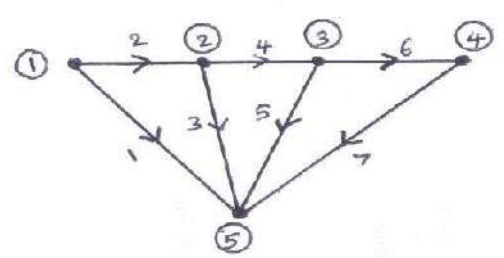
$$\mathbf{A} = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 1 \end{bmatrix}$$

Solution: The given matrix is a reduced incidence matrix. Obtain the complete incidence matrix in order to draw the oriented graph.

$$\mathbf{A_a} = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & 1 & 0 & 0 & 0 \\ 0 & & 0 & 0\text{-}1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0\text{–}1 & & 1 \\ -1 & 0 & & -1 & 0\text{-}1 & 0 & -1 \end{bmatrix}$$

Total number of nodes = $n_t = 5$

Total number of branches = b = 7



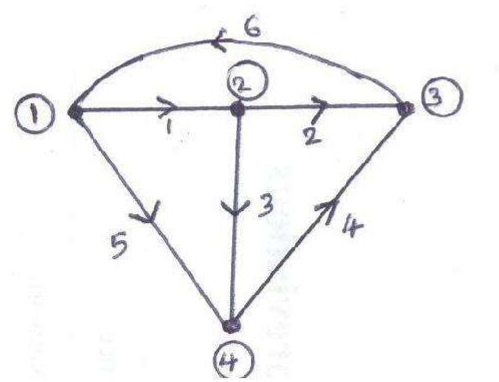
Oriented Graph

Example: 4) Draw the oriented graph of incidence matrix shown below

$$A_a = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & -1 \\ -1 & 1 & 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & -1 & 0 & 1 \\ 0 & 0 & -1 & 1 & -1 & 0 \end{bmatrix}$$

Solution:

Total number of nodes = $n_t = 4$
Total number of branches = b = 6



Oriented Graph

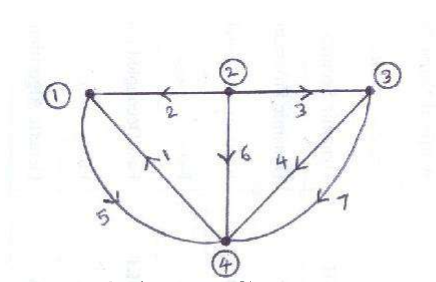
Example: 5) Draw the oriented graph of incidence matrix shown below

$$A = \begin{bmatrix} -1 & -1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & 1 & 0 & 0 & 1 \end{bmatrix}$$

Solution: The given matrix is a reduced incidence matrix. Obtain the complete incidence matrix in order to draw the oriented graph.

$$A_a = \begin{bmatrix} -1 & -1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & -1 & -1 & -1 & -1 \end{bmatrix}$$

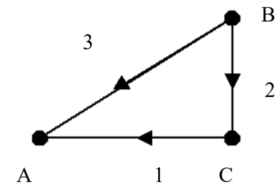
Total number of nodes = $n_t = 4$
Total number of branches = b = 7



Oriented Graph

Example: 6) Show that determinant of the incidence matrix of a closed loop is zero.

Proof: let us consider a closed path ABC



Total number of nodes = $n_t = 3$
Total number of branches = b = 3

The complete incidence matrix is

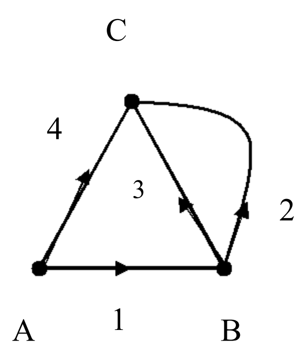
$$\mathbf{A_a} = \begin{pmatrix} -1 & 0 & -1 \\ 0 & 1 & 1 \\ 1 & -1 & 0 \end{pmatrix}$$

The determinant of complete incidence matrix of the closed loop is

$$\begin{vmatrix} -1 & 0 & -1 \\ 0 & 1 & 1 \\ 1 & -1 & 0 \end{vmatrix} = -1(0+1)-0-1(0-1)=-1+1=0$$

IV. Number of Possible Trees of a Graph:

For the given network graph, it is possible to write several trees. The number of possible trees is equal to determinant of $[A]\ [A]^t$. Where [A] is the reduced incidence matrix obtained by removing any one row from complete incidence matrix and $[A]^t$ is the transpose of [A].



A_a =

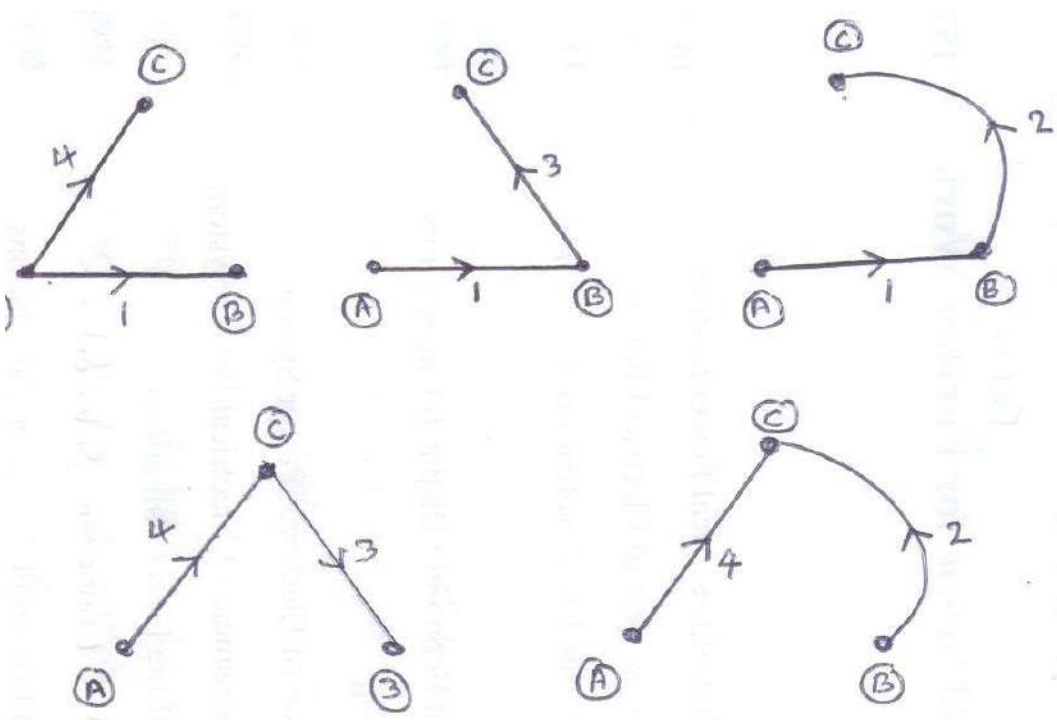
Node No.	Branches 1	2	3	4
A	1	0	0	1
B	-1	1	1	0
c	0	-1	-1	-1

$$[A_a] = \begin{matrix} 1 & 0 & 0 & 1 \\ -1 & 1 & 1 & 0 \\ 0 & -1 & -1 & -1 \end{matrix}$$

$$[A] = \begin{pmatrix} 1 & 0 & 0 & 1 \\ -1 & 1 & 1 & 0 \end{pmatrix}$$

$$[A]\,[A]^t = \begin{pmatrix} 1 & 0 & 0 & 1 \\ -1 & 1 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & -1 \\ 0 & 1 \\ 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 1+1 & -1 \\ -1 & 1+1+1 \end{pmatrix} = \begin{pmatrix} 2 & -1 \\ -1 & 3 \end{pmatrix}$$

$$\left|\,[A]\,[A]^t\,\right| = \begin{vmatrix} 2 & -1 \\ -1 & 3 \end{vmatrix} = (6-1) = 5$$



Node Pair Voltages:

The voltage between any two nodes of a network is known as the node-pair voltages. In general all branch voltages are node-pair voltages.

Network Variables:

In Loop analysis, the loop currents are unknown parameters. Once, they are evaluated, all branch currents can be determined in terms of these loop currents.

Similarly, in nodal analysis, the node-pair voltages are the unknown parameters. Once, they are evaluated, the voltages across any two nodes of the network can be found. Hence, the **node-pair voltages** and **loop currents** are called network variables.

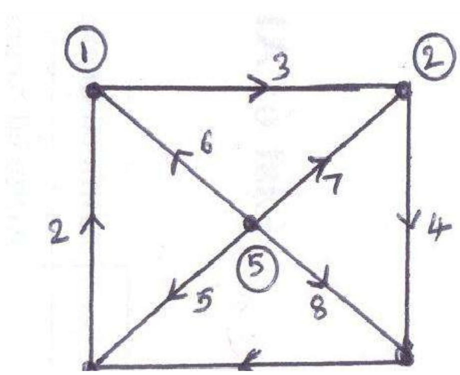
The network variables are independent variables and all other quantities depend on these values.

V. Tie-set:

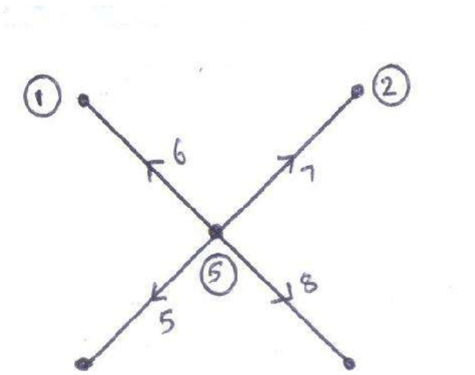
A tie-set is a set of branches contained in a loop such that each loop contains one link or chord and remainder are tree branches.

Or

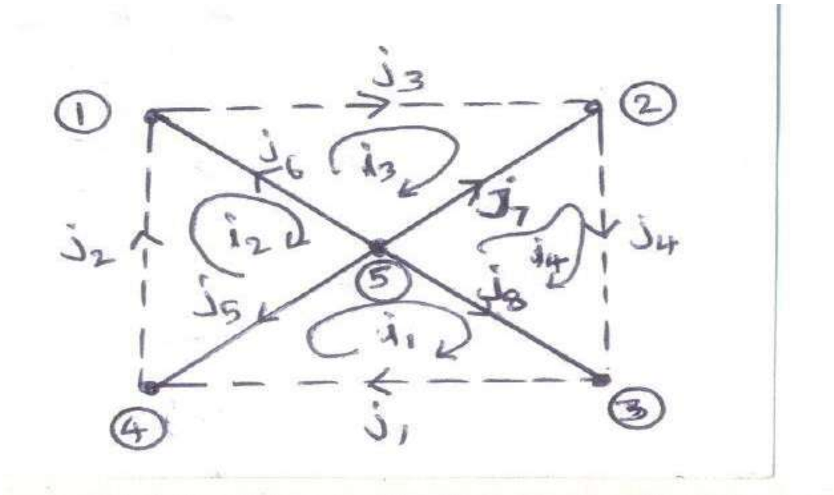
The set of branches forming the closed loop in which link or loop current circulate is called a **Tie-set.**



Oriented Graph



Tree



Let the branch currents in the network graph denoted by the symbol **'j'** and various loop currents by symbol **'i'.**

The orientation of a closed loop will be chosen to be the same as that of its connecting link.

For the given network graph,
Number of branches, b = 8
Number of nodes, n_t= 5
Number of closed loops = [b–(n_t - 1)]
Where (n_t-1) = Number of tree branches.

VI. Tie-set Schedule:

For a given network tree, a systematic way of indicating the links is through use of a schedule called **Tie-set Schedule**
The tie-set schedule for the given network oriented graph is shown below

Link Current or Number	**Branches**							
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**
1	**1**	**0**	**0**	**0**	**-1**	**0**	**0**	**1**
2	**0**	**1**	**0**	**0**	**1**	**-1**	**0**	**0**
3	**0**	**0**	**1**	**0**	**0**	**1**	**-1**	**0**
4	**0**	**0**	**0**	**1**	**0**	**0**	**1**	**-1**

The tie-set schedule can be written in matrix form is known as **Tie-set matrix (B).**

$$\mathbf{B} = \begin{pmatrix} 1 & 0 & 0 & 0 & -1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & -1 \end{pmatrix}$$

After writing the schedule, the columns of a schedule or matrix gives branch currents in terms of link currents. Thus column 1 gives equation for j_1 in terms of link currents

$$\left.\begin{array}{ll} \text{i.e. } j_1 = i_1 & j_5 = i_2 - i_1 \\ \text{Similarly, } j_2 = i_2 & j_6 = i_3 - i_2 \\ j_3 = i_3 & j_7 = i_4 - i_3 \\ j_4 = i_4 & j_8 = i_1 - i_4 \end{array}\right\} \qquad (1)$$

The rows of the schedule give KVL equations in terms of coefficients of the schedule or matrix. i. e. $e_1 - e_5 + e_8 = 0$

$$\left.\begin{array}{l} e_2 + e_5 - e_6 = 0 \\ e_3 + e_6 - e_7 = 0 \\ e_4 + e_7 - e_8 = 0 \end{array}\right\} \qquad (2)$$

The set of equation (1) can be expressed in the matrix form as

$$\begin{bmatrix} j_1 \\ j_2 \\ j_3 \\ j_4 \\ j_5 \\ j_6 \\ j_7 \\ j_8 \end{bmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 1 \\ 1 & 0 & 0 & -1 \end{pmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \end{bmatrix} \qquad (3)$$

In compact form

$$[\mathbf{I_b}] = [\mathbf{B}]^{\mathbf{T}} [\mathbf{I_l}] \qquad (4)$$

Where

$[\mathbf{I_b}]$ = is a column matrix of branch currents of the order (bx1).

$[\mathbf{B}]^{\mathbf{T}}$ = is the transpose of the fundamental tie-set matrix B.

$[\mathbf{I_l}]$ = is a column matrix of loop currents or link currents of the order (lx1).

Where 'l' is the number of independent loops.

From set of equation (2),

$$\begin{pmatrix} 1 & 0 & 0 & 0 & & -1001 \\ 0 & 1 & 0 & 0 & 1 & -100 \\ 0 & 0 & 1 & 0 & 0 & 1\text{-}10 \\ 0 & 0 & 0 & 1 & 0 & 01\text{-}1 \end{pmatrix} \begin{bmatrix} e_1 \\ e_2 \\ e_3 \\ e_4 \\ e_5 \\ e_6 \\ e_7 \\ e_8 \end{bmatrix} = 0 \qquad (5)$$

In compact form

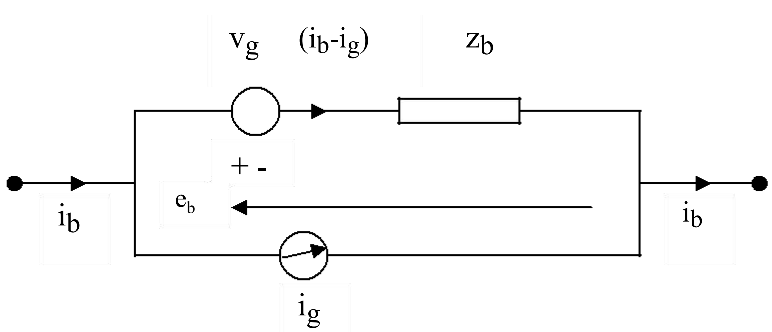
$$[\mathbf{B}]\,[\mathbf{E_b}] = \mathbf{0} \qquad (6)$$

$[\mathbf{E_b}]$ = is a column matrix of branch voltages of the order (bx1).

Equations (4) & (6) are the basic equations required to deriving equilibrium equations on **loop current basis**

VII. Equilibrium Equations with Loop Currents as Variables:

Consider a general branch of a network as shown in figure



Where v_g = Total series voltage in the branch

i_g = Total current source connected across the

branch z_b = Total impedance of the branch i_b =

Branch current

The voltage current relation for the branch can be written as

$$e_b = v_g + z_b\,(i_b - i_g) \qquad \text{(i)}$$

and $$i_b = i_g + y_b\,(i_b - i_g) \qquad \text{(ii)}$$

Where y_b = Total admittance of the branch.

For a network with more number of branches, equation (i) & (ii) may be written as

$$[E_b] = [V_g] + [Z_b] ([I_b] - [I_g]) \quad \text{(iii)}$$

$$[I_b] = [I_g] + [Y_b] ([E_b] - [V_g]) \quad \text{(iv)}$$

Where

$[E_b]$, $[V_g]$, $[I_b]$, & $[I_g]$ are (bx1) matrices of branch voltages, source voltages in the branches, branch currents and source currents in the branches respectively.

$[Z_b]$ & $[Y_b]$ are branch impedance and branch admittance matrices of the order (bxb).

We have

$$[E_b] = [V_g] + [Z_b] ([I_b] - [I_g]) \quad (1)$$

$$[I_b] = [B]^T [I_l] \quad (2)$$

$$[B] [E_b] = 0 \quad (3)$$

Substituting (1) in (3), we get

$$[B] [V_g] + [B][Z_b] ([I_b] - [I_g]) = 0$$

$$[B][Z_b] [I_b] = [B][Z_b] [I_g] - [B] [V_g] \quad (4)$$

Substituting (2) in (4)

$$[B][Z_b] [B]^T [I_l] = [B] [Z_b] [I_g] - [B] [V_g] \quad (5)$$

Let $[B] [Z_b] [I_g] - [B] [V_g] = [V_l]$

and $[B] [Z_b] [B]^T = [Z_l]$ = Loop Impedance matrix

Then equation (5) may be written as

$[Z_l] [I_l] = [V_l]$ or

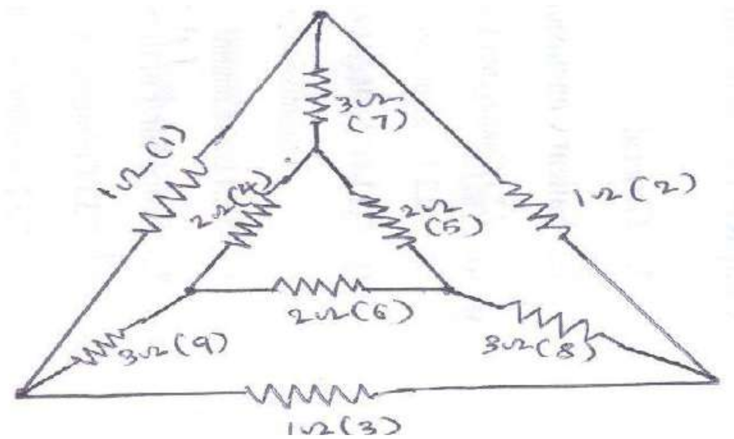
$$[I_l] = [Z_l]^{-1} [V_l] \quad (6)$$

Equations (5) and (6) represent a set of **Equilibrium Equations** with loop currents as independent variables. On solving these equations, loop currents are obtained. Once the loop currents are known, all the branch currents can be found. If the elements of the branches are known, then all branch voltages also can be found.

Example: 1)

(a) For the given network shown. Draw the graph, select a tree with branches 9, 4, 7, 5, & 8 and write the tie-set matrix. The number inside the brackets indicates branch numbers.

(b) Using the above tie-set matrix formulate equilibrium equations.

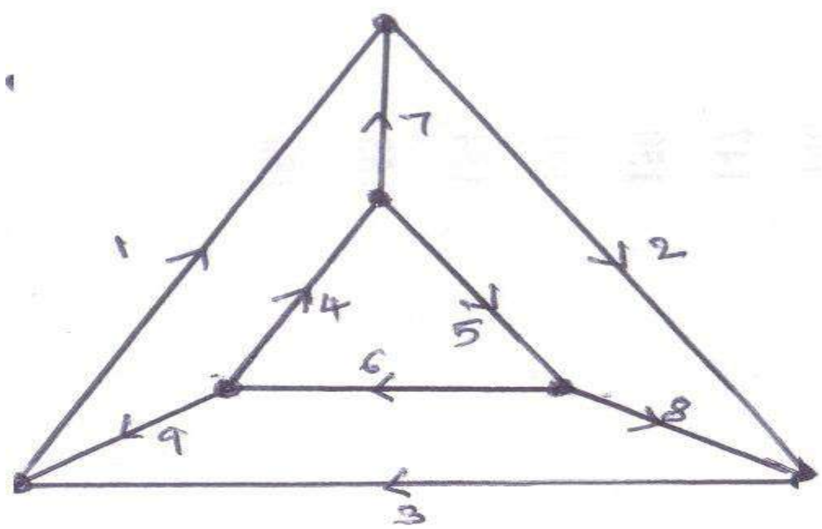


Solution: Total number of branches, b = 9;

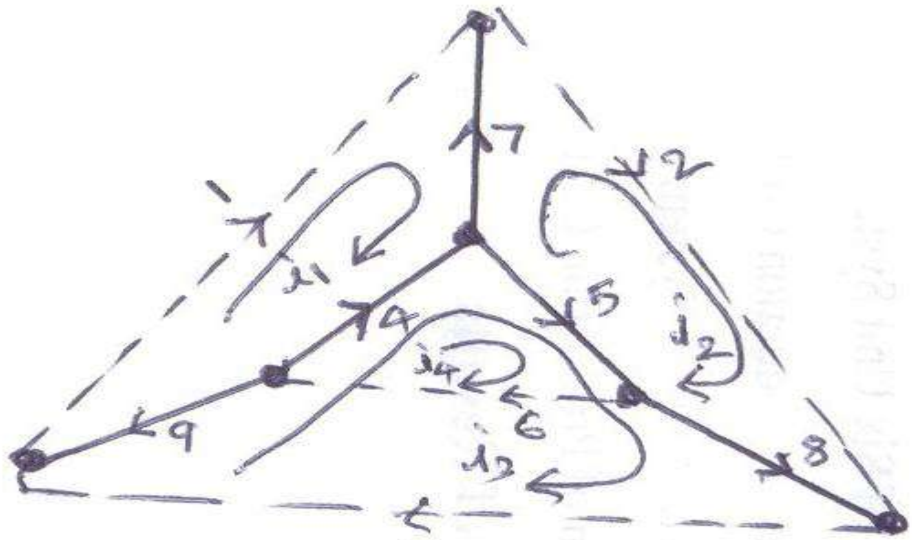
Total number of nodes n_t =6;

Total number of tree branches, n = (n_t-1) = (6-1) = 5;

Total number of links, l = (b– n) = (9 – 5) = 4



Oriented Graph



i_1, i_2, i_3 and i_4 are the loop currents

$$\mathbf{B} = \begin{pmatrix} 1 & 0 & 0 & -1 & 0 & 0 & -1 & 0 & 1 \\ 0 & 1 & 0 & 0 & -1 & 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \end{pmatrix}$$

Rows of above matrix give KVL Equations

$$\left.\begin{array}{l} e_1 - e_4 - e_7 + e_9 = 0 \\ e_2 - e_5 + e_7 - e_8 = 0 \\ e_3 + e_4 + e_5 + e_8 - e_9 = 0 \\ e_4 + e_5 + e_6 = 0 \end{array}\right\} \qquad (1)$$

Columns of above matrix give Branch Currents

$$\left.\begin{array}{l} j_1 = i_1 \\ j_2 = i_2 \\ j_3 = i_3 \\ j_4 = -i_1 + i_3 + i_4 \\ j_5 = -i_2+i_3+i_4 \\ j_6 = i_4 \\ j_7 = -i_1+i_2 \\ j_8 = -i_2+i_3 \\ j_9 = i_1 - i_3 \end{array}\right\} \qquad (2)$$

We have, branch voltage is given by $e_k = r_k\, j_k$.   Where k=1, 2, 3 …9.

$$
\begin{aligned}
e_1 &= r_1\, j_1\\
e_2 &= r_2\, j_2\\
e_3 &= r_3\, j_3\\
e_4 &= r_4\, j_4\\
e_5 &= r_5\, j_5\\
e_6 &= r_6\, j_6\\
e_7 &= r_7\, j_7\\
e_8 &= r_8\, j_8\\
e_9 &= r_9\, j_9
\end{aligned}
\tag{3}
$$

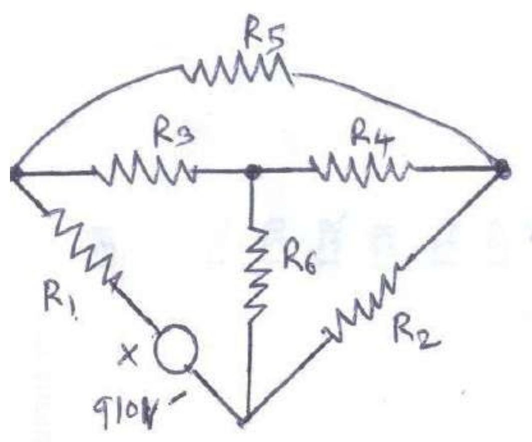
Substituting equation (2) in (3) and resulting equations are substituting in equation (1)
We get,

$$
\begin{aligned}
9i_1-3i_2-5i_3-2i_4 &= 0\\
-3i_1+9i_2-5i_3-2i_4 &= 0\\
-5i_1-5i_2+11i_3+4i_4 &= 0\\
-2i_1-2i_2+4i_3+6i_4 &= 0
\end{aligned}
\tag{4}
$$

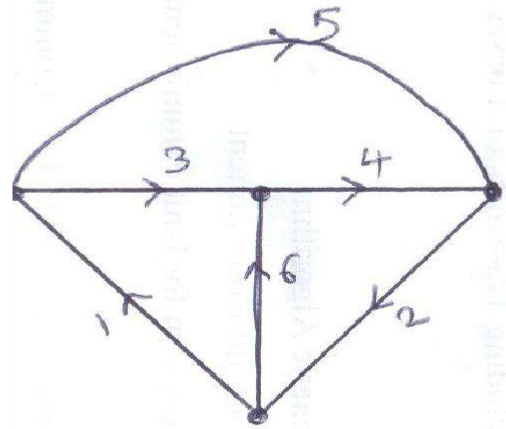
Set of equations (4) are called **Equilibrium Equations.** Solutions of these equations will give the **Link currents or Loop currents**. Then using these link currents, we can find out the branch currents

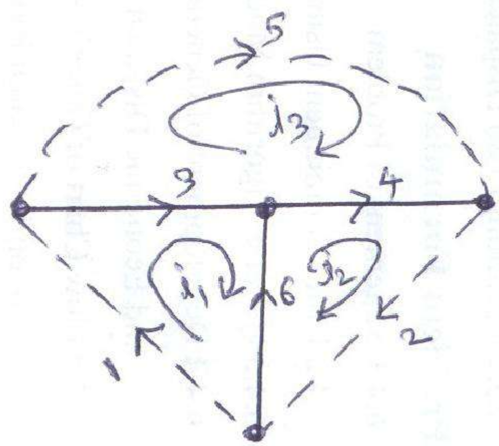
Example: 2)

For the given resistive network, write a tie-set schedule and equilibrium equations on the current basis. Obtain values of branch current and branch voltages. Given that $R_1=5$; $R_2=5$; $R_3=R_4=R_6= 10$ and $R_5= 2$.



Solution:





Total number of branches, b = 6;
Total number of nodes n_t = 4;
Total number of tree branches, n = (n_t-1) = (4-1) = 3;
Total number of links, l = (b– n) = (6 – 3) = 3

Tie-set Schedule:

Link Currents	**Branches**					
	1	**2**	**3**	**4**	**5**	**6**
$\mathbf{i_1}$	**1**	**0**	**1**	**0**	**0**	**-1**
$\mathbf{i_2}$	**0**	**1**	**0**	**1**	**0**	**1**
$\mathbf{i_3}$	**0**	**0**	**-1**	**-1**	**1**	**0**

The columns of the above schedule give branch currents in terms of link currents

$$\left.\begin{aligned} j_1 &= i_1 \\ j_2 &= i_2 \\ j_3 &= (i_1 - i_3) \\ j_4 &= (i_2 - i_3) \\ j_5 &= i_3 \\ j_6 &= (-i_1 + i_2) \end{aligned}\right\} \qquad (1)$$

The rows of above schedule, give the KVL Equations

$$\left.\begin{aligned} e_1+e_3-e_6 &= 0\\ e_2+e_4+e_6 &= 0\\ -e_3-e_4+e_5 &= 0 \end{aligned}\right\} \qquad (2)$$

We have, branch voltage is given by $e_k = r_k\, j_k$. Where k=1, 2, 3 …6.

$$\left.\begin{aligned} e_1 &= r_1\, j_1 - 910 = 5i_1 - 910\\ e_2 &= r_2\, j_2 = 5i_2\\ e_3 &= r_3\, j_3 = 10i_1 - 10i_3\\ e_4 &= r_4\, j_4 = 10i_2 - 10i_3\\ e_5 &= r_5\, j_5 = 2i_3\\ e_6 &= r_6\, j_6 = -10i_1 + 10i_2 \end{aligned}\right\} \qquad (3)$$

Substituting equation (3) in (2) we get,

$$\left.\begin{aligned} 25i_1 - 10i_2 - 10i_3 &= 910\\ -10i_1 + 25i_2 - 10i_3 &= 0\\ -10i_1 - 10i_2 + 22i_3 &= 0 \end{aligned}\right\} \qquad (4)$$

The set of equations (4) are called **Equilibrium Equations.**

$$\begin{pmatrix} 25 & -10 & -10\\ -10 & 25 & -10\\ -10 & -10 & 22 \end{pmatrix} \begin{bmatrix} i_1\\ i_2\\ i_3 \end{bmatrix} = \begin{pmatrix} 910\\ 0\\ 0 \end{pmatrix}$$

$i_1 = \Delta_1/\Delta = 40950/4550 = 90A$

Where

$$= \begin{vmatrix} 25 & -10 & -10\\ -10 & 25 & -10\\ -10 & -10 & 22 \end{vmatrix} \quad \text{and} \quad \Delta_1 = \begin{vmatrix} 910 & -10 & -10\\ 0 & 25 & -10\\ 0 & -10 & 22 \end{vmatrix}$$

$$i_2 = \Delta_2/\Delta = 64A; \text{ where } \Delta_2 = \begin{vmatrix} 25 & 910 & -10\\ -10 & 0 & -10\\ -10 & 0 & 22 \end{vmatrix}$$

$$i_3 = \Delta_3/\Delta = 70A; \text{ where } \Delta_3 = \begin{vmatrix} 25 & -10 & 910\\ -10 & 20 & 0\\ -10 & -10 & 0 \end{vmatrix}$$

From equation (1), Branch currents in terms of link currents are given by

$j_1 = i_1 = 90A$
$j_2 = i_2 = 64A$
$j_3 = (i_1 - i_3) = (90-70) = 20A$
$j_4 = (i_2 - i_3) = (64-70) = -6A$
$j_5 = i_3 = 70A$
$j_6 = (-i_1 + i_2) = (-90+64) = -26A$

From equation (3), Branch Voltages in terms of branch currents or link currents are given by

$e_1 = r_1 j_1 - 910 = 5i_1 - 910 = 460V$
$e_2 = r_2 j_2 = 5i_2 = 320V$
$e_3 = r_3 j_3 = 10i_1 - 10i_3 = 200V$
$e_4 = r_4 j_4 = 10i_2 - 10i_3 = -60V$
$e_5 = r_5 j_5 = 2i_3 = 140V$
$e_6 = r_6 j_6 = -10i_1 + 10i_2 = -260V$

VIII. Cut-set:

Tree branches connect all the nodes in the network graph. Hence, it is possible to trace the path from one node to any other node by traveling along the tree branch only. Therefore, potential difference between any two nodes called node-pair voltage can be expressed in terms of tree branch voltages.

The cut set is a minimal set of branches of the graph, removal of which cuts the graph into two parts. It separates the nodes of the graph into two groups. The cut-set consists of only one tree branch and remainders are links. Each branch of the cut-set has one of its terminal incident at a node in one group and its other end at a node in the other group and its other end at a node in the other group. The orientation of the cut-set is same as orientation of tree branch.

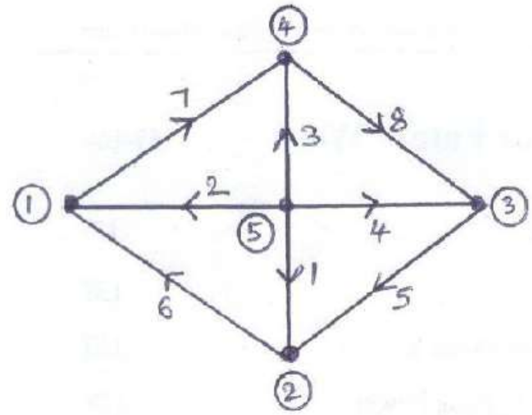
The number of cut -sets is equal to number of tree branches [i.e. (nt-1) = n where n_t is total number of nodes in the network graph].

IX. Cut-set schedule:

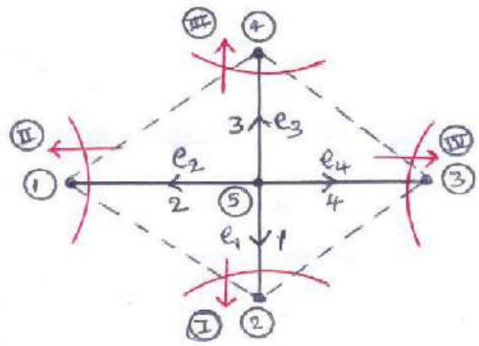
For a given network tree, a systematic way of indicating the tree branch voltage through use of a schedule called **cut-set schedule**

To write the cut-set schedule for network graph,

(i) Consider an oriented network graph
(ii) Write any one possible tree of the network graph
(iii) Assume tree branch voltages as (e1, e2…en) independent variables.
(iv) Assume the independent voltage variable is same direction as that of a tree branch voltage
(v) Mark the cut-sets (recognize) in the network graph.



Oriented Graph



Tree and Cut-sets

The tree branch voltages e_1, e_2, e_3, & e_4 entered in the first column of the schedule correspond to 4 branches 1, 2, 3 & 4. In order to fill the first row corresponding to the tree branch voltages e_1 , by looking into the direction of currents in the branches connected to the cut-set under consideration. If the direction of current in the cut-set branch is towards the cut-set node, write **'+1'** in the branch column of concerned cut-set branch. If the direction of current in the cut-set branch is away from the cut-set node, write **'-1'** in that particular cut-set branch column. Write **'0'** in the branch columns, which are not in that particular, cut-set.

Cut-set schedule:

Tree Branch Voltages	Branches							
	1	2	3	4	5	6	7	8
e_1	1	0	0	0	1	-1	0	0
e_2	0	1	0	0	0	1	-1	0
e_3	0	0	1	0	0	0	1	-1
e_4	0	0	0	1	-1	0	0	1

The columns of the cut-set schedule give branch voltage equations;

i.e.

$$\left.\begin{aligned} v_1&=e_1\\ v_2&=e_2\\ v_3&=e_3\\ v_4&=e_4\\ v_5&=(e_1-e_4)\\ v_6&=(-e_1+e_2)\\ v_7&=(-e_2+e_3)\\ v_8&=(-e_3+e_4)\end{aligned}\right\} \qquad (1)$$

The rows of cut-set schedule give KCL equations;

$$\left.\begin{aligned} j_1+j_5-j_6&=0\\ j_2+j_6-j_7&=0\\ j_3+j_7-j_8&=0\\ j_4-j_5+j_8&=0\end{aligned}\right\} \qquad (2)$$

The cut-set schedule can be written in matrix form is known as **cut-set matrix**. This matrix can be represented by **Q.**

$$Q = \begin{pmatrix} 1 & 0 & 0 & 0 & 1 & -1 & 0 & 0\\ 0 & 1 & 0 & 0 & 0 & 1 & -1 & 0\\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & -1\\ 0 & 0 & 0 & 1 & -1 & 0 & 0 & 1 \end{pmatrix}$$

From set of equations (1)

$$\begin{bmatrix} v_1\\ v_2\\ v_3\\ v_4\\ v_5\\ v_6\\ v_7\\ v_8 \end{bmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\\ 1 & 0 & 0 & -1\\ -1 & 1 & 0 & 0\\ 0 & -1 & 1 & 0\\ 0 & 0 & -1 & 1 \end{pmatrix} \begin{bmatrix} e_1\\ e_2\\ e_3\\ e_4 \end{bmatrix}$$

In compact form;

$$[\mathbf{V_b}] = [\mathbf{Q}]^{\mathbf{T}}[\mathbf{V_T}] \qquad (3)$$

Where
$[V_b]$ = Branch Voltage Column Matrix of order **bx1**
$[V_T]$ = Tree Branch Voltage Column Matrix of order **nx1**
[Q] = Cut-set Matrix of order **nxb**

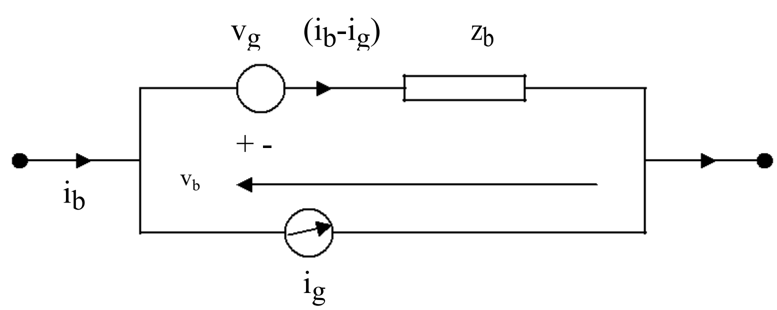
$$\begin{bmatrix} 10001\text{-}100 \\ 010001\text{-}10 \\ 0010001\text{-}1 \\ 0001\text{-}1001 \end{bmatrix} \begin{bmatrix} j_1 \\ j_2 \\ j_3 \\ j_4 \\ j_5 \\ j_6 \\ j_7 \\ j_8 \end{bmatrix} = 0$$

In compact form;

$[Q]\,[I_b] = 0$ **(4)**

Where
$[I_b]$ = Branch Current Column Matrix of order
bx1 [Q] = Cut-set Matrix of order **nxb**

Consider a general branch of a network as shown in figure



Where v_g = Total series voltage in the branch

i_g = Total current source connected across the branch

z_b = Total impedance of the branch i_b = Branch current

The voltage current relation for the branch can be written as

$$v_b = v_g + z_b\,(i_b - i_g) \qquad (1)$$

and $$i_b = i_g + y_b\,(v_b - v_g) \qquad (2)$$

Where y_b = Total admittance of the branch.
For a network with more number of branches, equation (i) & (ii) may be written as

$$[V_b] = [V_g] + [Z_b]\,([I_b] - [I_g]) \qquad (3)$$

$$[I_b] = [I_g] + [Y_b]\,([V_b] - [V_g]) \qquad (4)$$

Where
$[V_b]$, $[V_g]$, $[I_b]$, & $[I_g]$ are (bx1) matrices of branch voltages, source voltages in the branches, branch currents and source currents in the branches respectively.
$[Z_b]$ & $[Y_b]$ are branch impedance and branch admittance matrices of the order (bxb).

We have

$$[V_b] = [V_g] + [Z_b]\,([I_b] - [I_g])$$

$$[V_b] = [Q]^T\,[V_T] \qquad (5)$$

$$[Q]\,[I_b] = 0 \qquad (6)$$

Substituting (4) in (6), we get
$$[Q]\,[I_g] + [Q]\,[Y_b]\,\{[V_b] - [V_g]\} = 0$$

$$[Q]\,[I_g] + [Q]\,[Y_b]\,[V_b] - [Q]\,[Y_b][V_g] = 0 \qquad (7)$$

Substituting (5) in (7), we get

$$[Q]\,[I_g] + [Q]\,[Y_b]\,[Q]^T\,[V_T] - [Q]\,[Y_b][V_g] = 0 \qquad (8)$$

Let,

$[Q]\,[Y_b]\,[Q]^T = [Y_c]$ = Cut-set admittance matrix

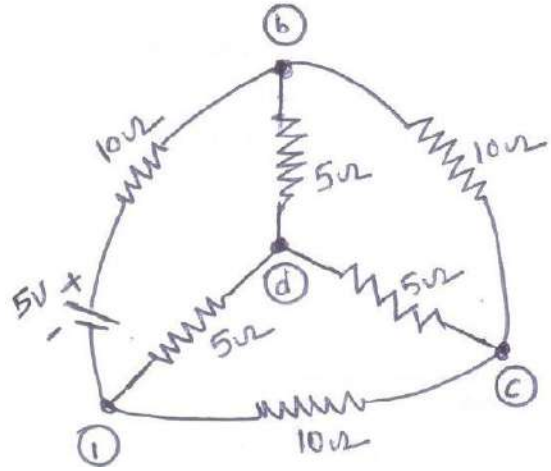
Equation (8) becomes,
$$[Y_c]\,[V_T] = [Q]\,[Y_b][V_g] - [Q]\,[I_g]$$
Or
$$[V_T] = [Y_c]^{-1}\,\{[Q]\,[Y_b][V_g] - [Q]\,[I_g]\} \qquad (9)$$

This equation (9) is known as **Equilibrium Equation**.

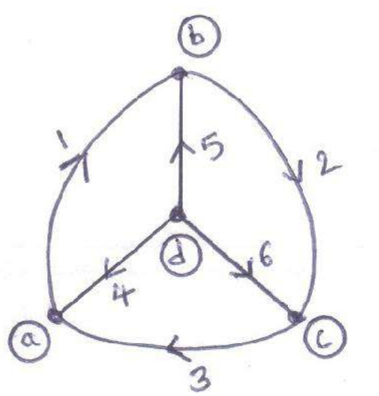
The solutions of equation (9) give the tree branch voltages and branch currents can be find using equation (5) and (4).

Example: 1)

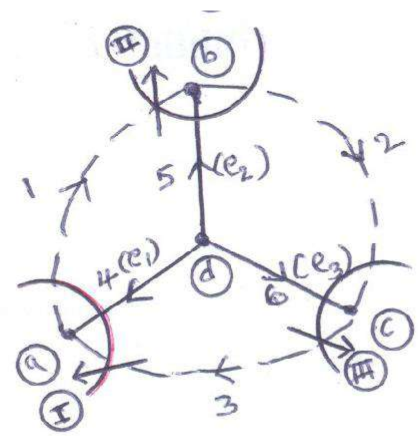
For the given resistive network, write a cut-set schedule and obtain equilibrium equations on the voltage basis. Solve these equations and hence calculate values of branch voltages and branch currents.



Solution:



Oriented Graph



Tree and Cut-sets

Cut-set Schedule:

Cut-sets or Tree branch Voltages	**Branches**					
	1	**2**	**3**	**4**	**5**	**6**
1 or (e_1)	-1	0	1	1	0	0
2 or (e_2)	1	-1	0	0	1	0
3 or (e_3)	0	1	-1	0	0	1

Columns of the above schedule give branch voltages in terms of tree branch voltages

$$\left.\begin{aligned} v_1 &= -e_1 + e_2 \\ v_2 &= -e_2+e_3 \\ v_3 &= e_1-e_3 \\ v_4 &= e_1 \\ v_5 &= e_2 \\ v_6 &= e_3 \end{aligned}\right\} \quad (1)$$

Rows of the above schedule give KCL equations

$$\left.\begin{aligned} -j_1+j_3+j_4 &= 0 \\ j_2-j_3+j_6 &= 0 \end{aligned}\right\} \qquad j_1-j_2+j_5 = 0 \quad (2)$$

We have from given network, $(v_k \pm v_g) = j_k \times r_k$ Where k=1, 2, 3 …6.

Branch currents $j_k = (v_k \pm v_g)/r_k$. Hence,

$j_1= (v_1 + 5)/10 = (-e_1 + e_2 + 5)/10 = -0.1e_1 + 0.1e_2 +$

0.5 $j_2= v_2/10 = (-e_2 + e_3)/10 = -0.1e_2 + 0.1e_3$

$j_3= v_3/10 = (e_1- e_3)/10 = 0.1e_1 - 0.1e_3$ (3)

$j_4= v_4/5 = e_1/5 = 0.2e_1$

$j_5= v_5/5 = e_2/5 = 0.2e_2$

$j_6= v_6/5 = e_3/5 = 0.2e_3$

Substituting set of equations (3) in (2), we get

$$\begin{aligned} 0.4e_1 - 0.1e_2 - 0.1e_3 &= 0.5 \\ -0.1e_1 + 0.4e_2 - 0.1e_3 &= -0.5 \\ -0.1e_1 - 0.1e_2 + 0.4e_3 &= 0 \end{aligned} \quad (4)$$

The set of equations (4) are called **Equilibrium Equations**

$$\begin{pmatrix} \mathbf{0.4} & \mathbf{-0.1} & \mathbf{-0.1} \\ \mathbf{-0.1} & \mathbf{0.4} & \mathbf{-0.1} \\ \mathbf{-0.1} & \mathbf{-0.1} & \mathbf{0.4} \end{pmatrix} \begin{pmatrix} \mathbf{e_1} \\ \mathbf{e_2} \\ \mathbf{e_3} \end{pmatrix} = \begin{pmatrix} \mathbf{0.5} \\ \mathbf{-0.5} \\ \mathbf{0} \end{pmatrix}$$

$$= \begin{vmatrix} \mathbf{0.4} & \mathbf{-0.1} & \mathbf{-0.1} \\ \mathbf{-0.1} & \mathbf{0.4} & \mathbf{-0.1} \\ \mathbf{-0.1} & \mathbf{-0.1} & \mathbf{0.4} \end{vmatrix} = \mathbf{50} \qquad = \begin{vmatrix} \mathbf{0.5} & \mathbf{-0.1} & \mathbf{-0.1} \\ \mathbf{-0.5} & \mathbf{0.4} & \mathbf{-0.1} \\ \mathbf{0} & \mathbf{-0.1} & \mathbf{0.4} \end{vmatrix} = \mathbf{50}$$

$$= \begin{vmatrix} \mathbf{0.4} & \mathbf{0.5} & \mathbf{-0.1} \\ \mathbf{-0.1} & \mathbf{-0.5} & \mathbf{-0.1} \\ \mathbf{-0.1} & \mathbf{0} & \mathbf{0.4} \end{vmatrix} = \mathbf{-50}$$

$$= \begin{vmatrix} \mathbf{.4} & \mathbf{-0.1} & \mathbf{0.5} \\ \mathbf{-0.1} & \mathbf{0.4} & \mathbf{-0.5} \\ \mathbf{-0.1} & \mathbf{-0.1} & \mathbf{0} \end{vmatrix} = \mathbf{0}$$

$e_1 = v_4 = {}_1/ \quad = 1V$

$e_2 = v_5 = {}_2/ \quad = -1V$

$e_3 = v_6 = {}_3/ \quad = 0V$

$v_1 = -e_1+e_2 = -1-1 = -2V$

$v_2 = -e_2+e_3 = 1+0 = 1V$

$v_3 = e_1-e_3 = 1-0 = 1V$

From equation (3),

$j_1 = (v_1 + 5)/10 = (-2+5)/10 = 0.3A$

$j_2 = v_2/10 = 1/10 = 0.1\ A$

$j_3 = v_3/10 = 1/10 = 0.1\ A$

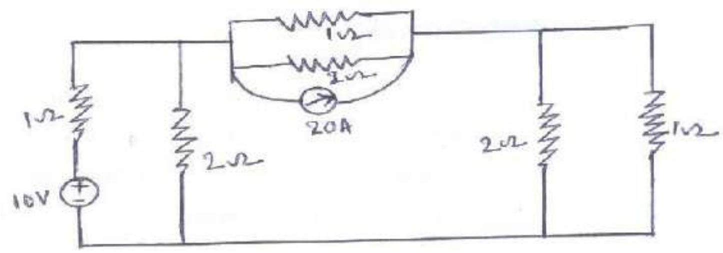
$j_4 = v_4/5 = 1/5 = 0.2\ A$

$j_5 = v_5/5 = 1/5 = 0.2\ A$

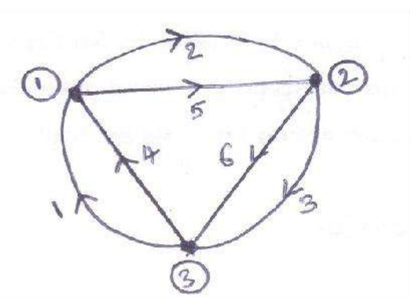
$j_6 = v_6/5 = 0/5 = 0\ A$

Example: 2)

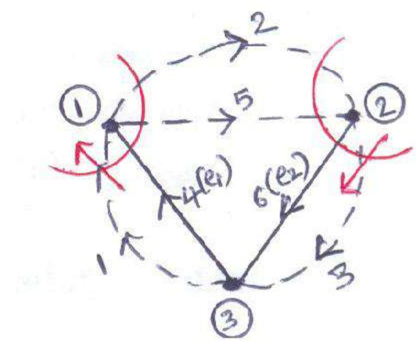
Draw the graph for the network shown in figure below. Write the cut-set schedule & obtain equilibrium equations and hence calculate values of branch voltages and branch currents



Solution:



Oriented Graph



Tree and Cut-sets

Cut-set Schedule:

Cut-sets or Tree branch Voltages	Branches					
	1	2	3	4	5	6
1 or (e_1)	1	-1	0	1	-1	0
2 or (e_2)	0	-1	1	0	-1	1

Columns of the above schedule give branch voltages in terms of tree branch voltages

$$\left.\begin{aligned} v_1 &= e_1 \\ v_2 &= -e_1 - e_2 \\ v_3 &= e_2 \\ v_4 &= e_1 \\ v_5 &= -e_1 - e_2 \\ v_6 &= e_2 \end{aligned}\right\} \qquad (1)$$

Rows of the above schedule give KCL equations

$$\left.\begin{array}{l} j_1 - j_2 + j_4 - j_5 = 0 \\ -j_2 + j_3 - j_5 + j_6 = 0 \end{array}\right\} \qquad (2)$$

We have from given network, $(v_k \pm v_g) = j_k \times r_k$

Branch currents $j_k = (v_k \pm v_g) / r_k$. Hence,

$$\left.\begin{array}{l} j_1 = (v_1 + 10)/1 = (e_1 + 10)/1 = e_1 + 10 \\ j_2 = v_2/1 = (-e_1 - e_2)/1 = -e_1 - e_2 \\ j_3 = v_3/1 = e_2/1 = e_2 \\ j_4 = v_4/2 = e_1/2 = 0.5e_1 \\ j_5 = (v_5 + 40)/2 = [(-e_1 - e_2) + 40]/2 = -0.5e_1 - 0.5e_2 + 20 \\ j_6 = v_6/2 = e_2/2 = 0.5e_2 \end{array}\right. \qquad (3)$$

Substituting set of equations (3) in (2), we get

$$\left.\begin{array}{l} 3e_1 + 1.5e_2 = 10 \\ 1.5e_1 + 3e_2 = 20 \end{array}\right\} \qquad (4)$$

The set of equations (4) are called **Equilibrium Equations**

$$\begin{pmatrix} 3 & 1.5 \\ 1.5 & 3 \end{pmatrix} \begin{pmatrix} e_1 \\ e_2 \end{pmatrix} = \begin{pmatrix} 10 \\ 20 \end{pmatrix}$$

$$e_1 = \Delta_1 / \Delta \quad \text{where} \quad \Delta = \begin{vmatrix} 3 & 1.5 \\ 1.5 & 3 \end{vmatrix} = 6.75 \text{ and } \Delta_1 = \begin{vmatrix} 10 & 1.5 \\ 20 & 3 \end{vmatrix} = 0$$

$e_1 = 0/6.75 = 0$ V.

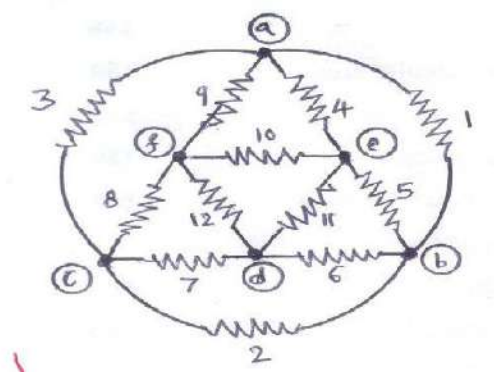
$$e_2 = \Delta_2 / \Delta \quad \text{where } \Delta = \begin{vmatrix} 3 & 1.5 \\ 1.5 & 3 \end{vmatrix} = 6.75 \text{ and } \Delta_2 = \begin{vmatrix} 10 & 1.5 \\ 20 & 3 \end{vmatrix} = 45$$

$e_2 = 45/6.75 = 6.667$ V.

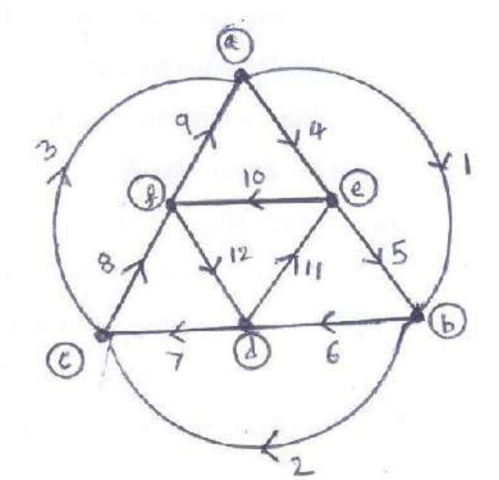
The branch voltages are	The branch currents are
$v_1 = e_1 =$ **0 V**	$j_1 = (v_1 + 10)/1 =$ **10 A**
$v_2 = -e_1 - e_2 = 0 - 6.667 =$ **-6.667 V**	$j_2 = v_2/1 =$ **6.667 A**
$v_3 = e_2 =$ **6.667 V**	$j_3 = v_3/1 =$ **6.667 A**
$v_4 = e_1 =$ **0 V**	$j_4 = v_4/2 =$ **0 A**
$v_5 = -e_1 - e_2 = -0 - 6.667 =$ **-6.667V**	$j_5 = (v_5 + 40)/2 = (-6.667 + 40)/2 =$ **16.66 A**
$v_6 = e_2 =$ **6.667 V**	$j_6 = v_6/2 =$ **3.33 A**

Example: 3)

For the given network, draw the oriented graph and a tree. Select suitable tree branch voltages and write the cut-set schedule and also write the equations for the branch voltages in terms of the tree branch voltages.



Solution: **Oriented Graph**



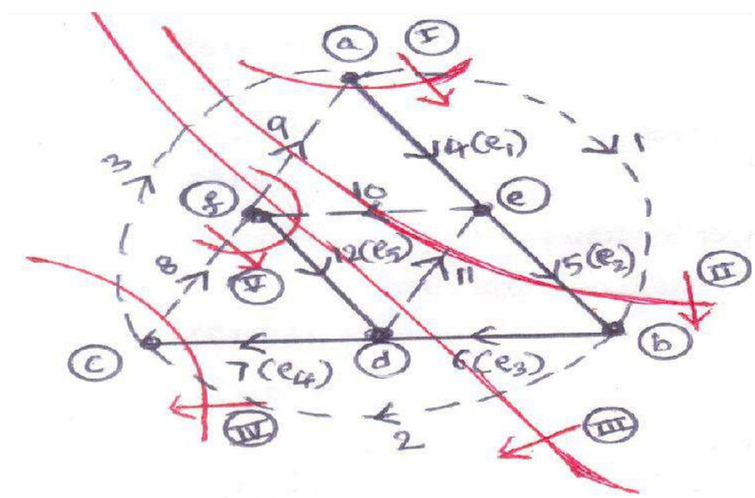
For the given network graph,

Number of branches, b = 12

Number of nodes, n_t= 6

Number of Cut-sets = [n_t - 1]

= [6-1] =5



Tree and Cut-sets

Cut-set Schedule:

Tree Branch Voltages or Cut-set	Branches											
	1	2	3	4	5	6	7	8	9	10	11	12
e_1	1	0	-1	1	0	0	0	0	-1	0	0	0
e_2	1	0	-1	0	1	0	0	0	-1	1	-1	0
e_3	0	1	-1	0	0	1	0	0	-1	1	-1	0
e_4	0	1	-1	0	0	0	1	-1	0	0	0	0
e_5	0	0	0	0	0	0	0	-1	1	-1	0	1

Columns of the above schedule will give branch voltages in terms of tree branch voltages

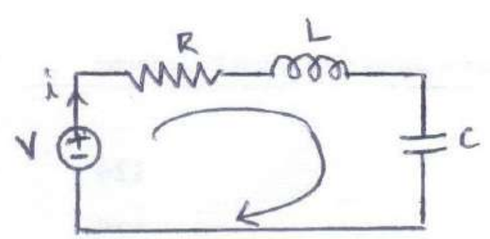
$v_1 = e_1+e_2$
$v_2 = e_3+e_4$
$v_3 = -e_1- e_2 - e_3 -e_4$
$v_4= e_1$
$v_5= e_2$
$v_6= e_3$
$v_7 = e_4$
$v_8 = -e_4 - e_5$
$v_9 = -e_1-e_2 - e_3 - e_5$
$v_{10} = e_2 +e_3-e_5$
$v_{11} = -e_1- e_3$
$v_{12} = e_5$

XI. Principle of Duality:

Duality: is the mutual relationship.

We come across a number of similarities in analyzing the network on current (Loop) basis and voltage (Node) basis. The principal quantities (and concepts) involved in the two methods form pairs. Each of the quantity in such a pair thus plays a dual role. These quantities (or concepts) forming pair are called **dual quantities.**

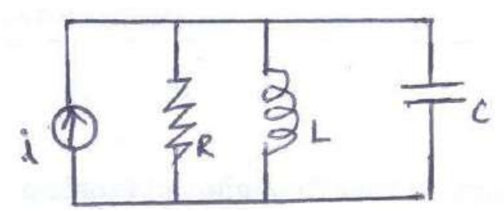
Consider a network containing R, L & C elements connected in series and excited by a voltage source as shown.



The integro- differential equations for the circuit is

$$R\, i + L\, (di/dt) + (1/C)\ i\, dt = v \qquad (1)$$

Consider a network containing R, L & C elements connected in parallel and driven by a current source as shown.



The integro- differential equations for the circuit is

$$(1/R)\ v + C\ (dv/dt) + (1/L)\ \ v\ dt = i \qquad (2)$$

OR

$$G\ v + C\ (dv/dt) + (1/L)\ \ v\ dt = i \qquad (2)$$

If we observe both the equations, the solutions of these equations are the same.

Therefore, these two networks are called **duals**

XII. Construction of a Dual of a Network:

Only planar networks without mutual inductances have **duals.**

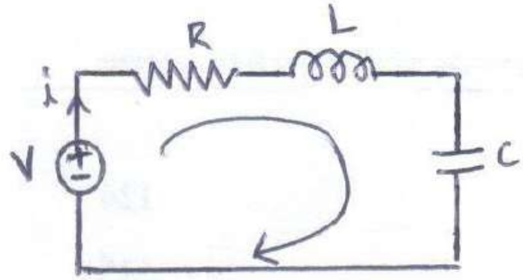
S.N.	Quantity or Concept	Dual Quantity or Concept
1	Current	Voltage
2	Resistance	Conductance
3	Inductance	Capacitance
4	Branch Current	Branch Voltage
5	Mesh	Node
6	Loop	Node-Pair
7	Number of Loops	Number of Node-Pairs
8	Loop Current	Node-Pair Voltage
9	Mesh Current	Node Voltage or Node Potential
10	Link	Tree Branch
11	Tie-set	Cut-set
12	Short Circuit	Open Circuit
13	Parallel Path	Series Path
14	Charge (Q)	Flux Linkages ( )

Procedure to draw a Dual Network:

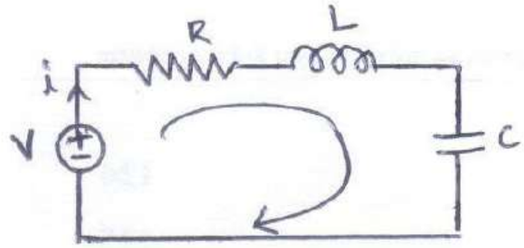
Step 1: In each Loop of a given network place a node and place an extra node called reference node outside the network.

Step 2: Draw the lines connecting adjacent nodes passing through each element and also to the reference node by placing the dual of each element in the line passing through original elements.

Example: 1) Draw the dual of a network for given network shown in figure.



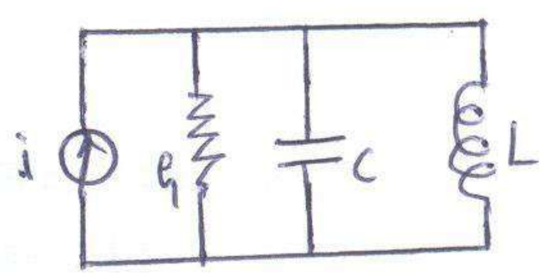
Solution:



The integro- differential equations for the circuit is

$$R\,i + L\,(di/dt) + (1/C)\ idt = v \qquad (1)$$

Dual Network

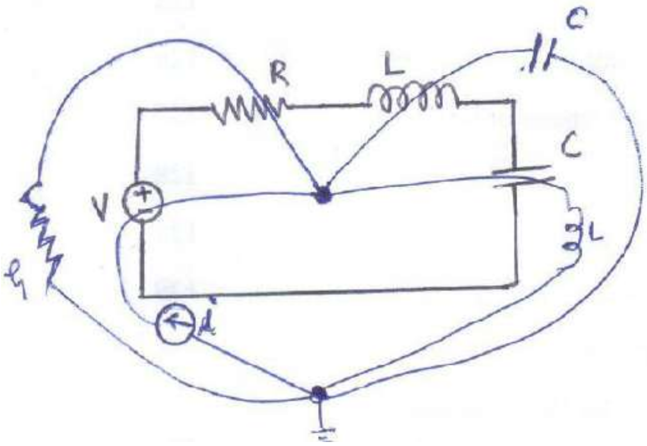


The integro- differential equations for the circuit is

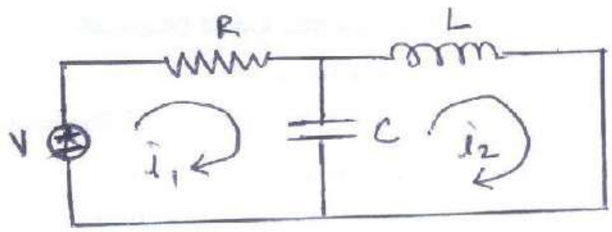
$(1/R)\ v + C\ (dv/dt) + (1/L)\ \ v\ dt = i$ (2)

OR

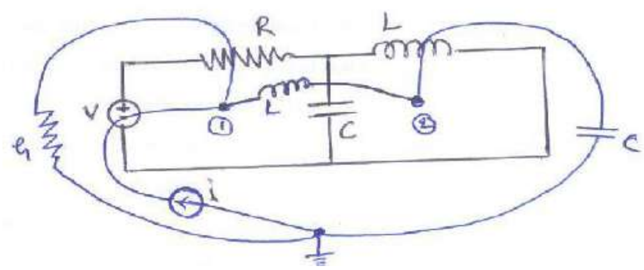
$G\ v + C\ (dv/dt) + (1/L)\ \ v\ dt = i$



Example: 2) Draw the dual of the network shown in figure.



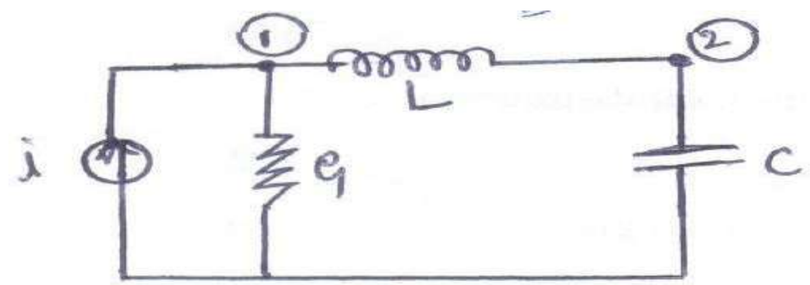
Solution:



The integro- differential equations for the network is

$R\ i_1 + (1/C)\ \ i_1\text{-}i_2)\ dt = v\ (t)$ (1)

$L\ (di_2/dt) + (1/C)\ \ i_2\text{-}i_1)\ dt = 0$ (2)

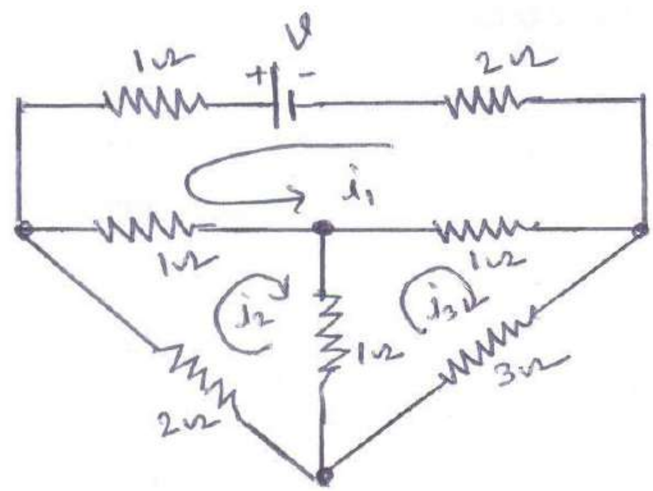


The integro- differential equations for the network is

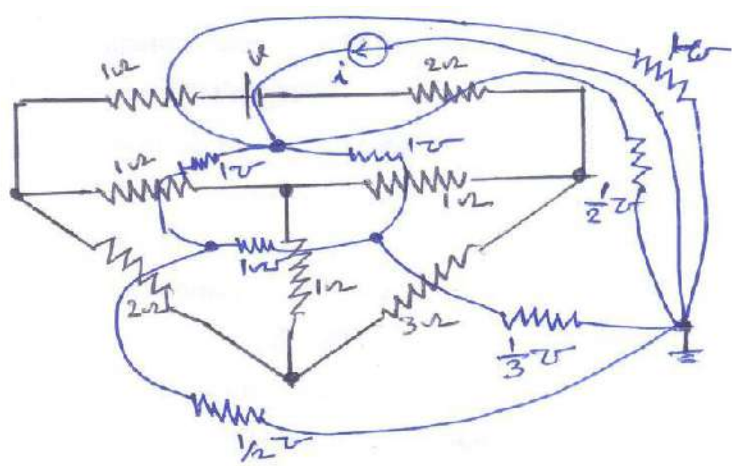
$G\, v_1 + (1/L)\quad v_1 - v_2)\, dt = i$ (1)

$C\, (dv_2/dt) + (1/L)\quad v_2 - v_1)\, dt = 0$ (2)

Example: 3) Draw the dual of the network shown in figure.

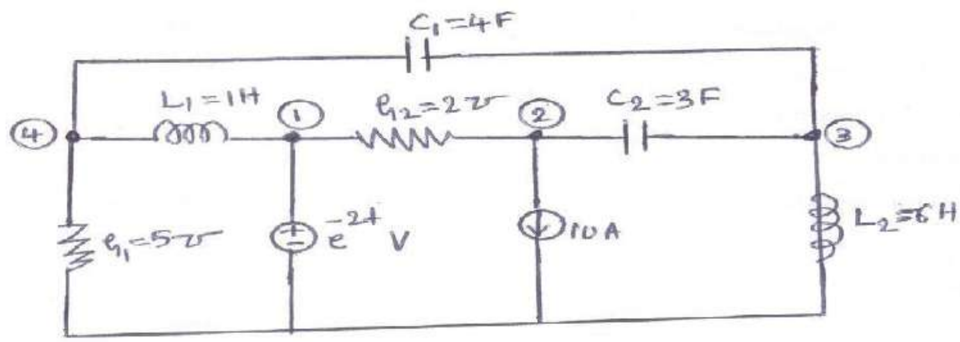


Solution:



Example: 4)

For the network shown in figure below, write the node equations. Draw the dual of this network and write mesh equations for the dual network. Verify whether these two sets of equations are dual equations



Node Equations:

At node (1);

$$(I/L_1)\ (e^{-2t} - v_4)\ dt + G_2\ (e^{-2t} - v_2) = 0 \qquad (1)$$

At node (2);

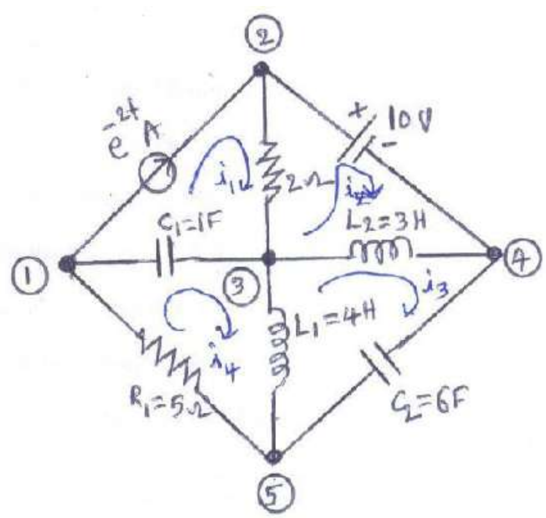
$$(G_2)\ (v_2 - e^{-2t}) + C_2\ [d\ (v_2 - v_3)\ /dt] + 10 = 0 \qquad (2)$$

At node (3);

$$(I/L_2)\ v_3\ dt + C_2\ [d\ (v_3 - v_2)\ /dt] + C_1\ [d\ (v_3 - v_4)\ /dt] = 0 \qquad (3)$$

At node (4);

$$G_1\ v_4 + (1/L_1)\ (v_4 - e^{-2t})\ dt + C_1\ [d\ (v_4 - v_3)\ /dt] = 0 \qquad (4)$$



Loop Equations for dual network are

$$(I/C_1)\ (e^{-2t} - i_4)\ dt + R_2\ (e^{-2t} - i_2) = 0 \quad (5)$$

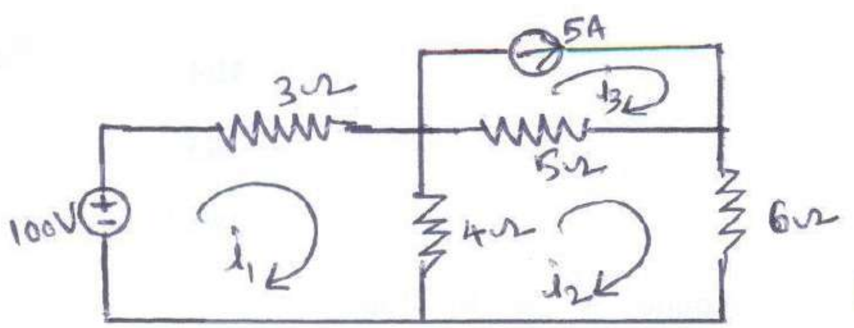
$$R_2\ \ (i_2 - e^{-2t}) + L_2\ [d\ (i_2 - i_3)\ /dt]\ +10 = 0 \quad (6)$$

$$(I/C_2)\ \ i_3\ dt + L_2[d\ (i_3 - i_2)\ /dt]+ L_1\ \ [d\ (i_3 - i_4)\ /dt] = 0 \quad (7)$$

$$R_1\ i_4 + (1/C_1)\ \ (i_4 - e^{-2t})\ dt + L_1\ [d\ (i_4 - i_3)\ /dt] = 0 \quad (8)$$

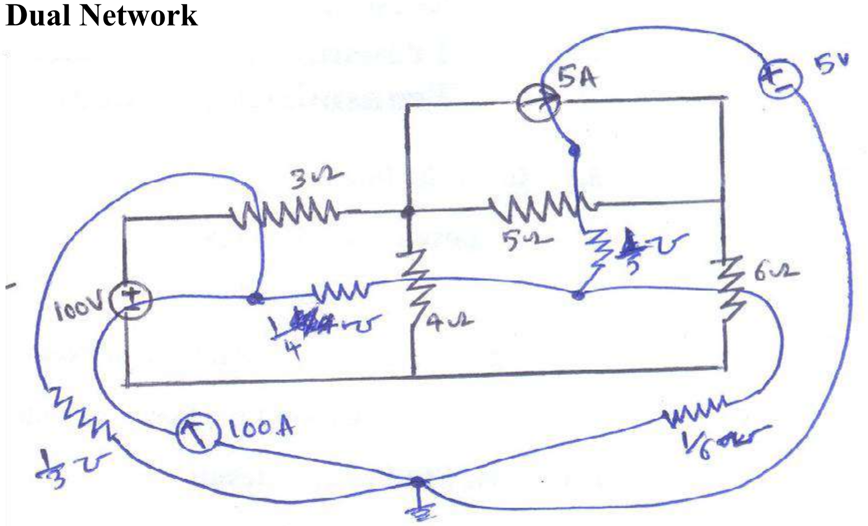
By looking into two sets of equations, we can say that two sets of equations are dual equations.

Example: 5) Draw the dual of the network shown below



Solution:

Dual Network



KVL Equations;

$-100 + 3i_1 + 4(i_1 - i_2) = 0$

$7i_1 - 4i_2 = 100$ (1)

$4(i_2 - i_1) + 5(i_2 - i_3) + 6i_2 = 0$

$-4i_1 + 15i_2 - 5i_3 = 0$ (2)

$-5i_2 + 5i_3 = 0$

$-5i_2 + 25 = 0$ (3)

Since $i_3 = 5A$

KCL Equations;

$100 = (1/3)v_1 + (1/4)(v_1 - v_2)$

$(7/12)v_1 - (1/4)v_2 = 100$ (1)

$(1/4)(v_2 - v_1) + (1/5)(v_2 - v_3) + (1/6)v_2 = 0$

$(-1/4)v_1 + (9/20)v_2 - (1/5)v_3 = 0$ (2)

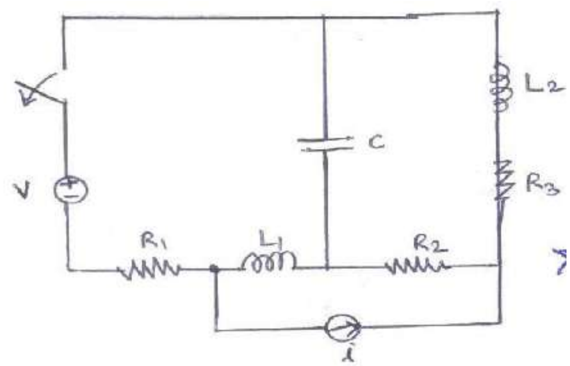
$(-1/5)v_2 + (1/5)v_3 = 0$

$(-1/5)v_2 + 1 = 0$ (3)

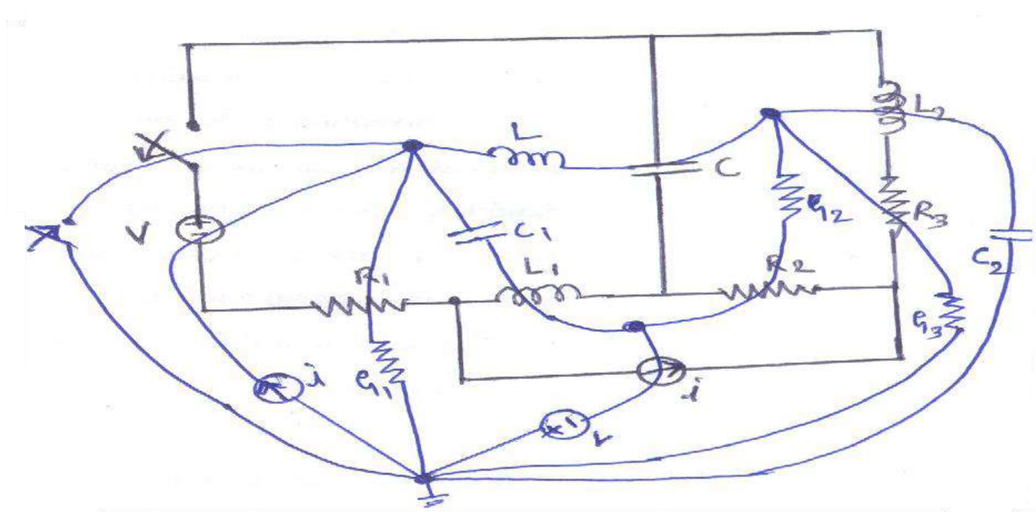
Since $v_3 = 5v$

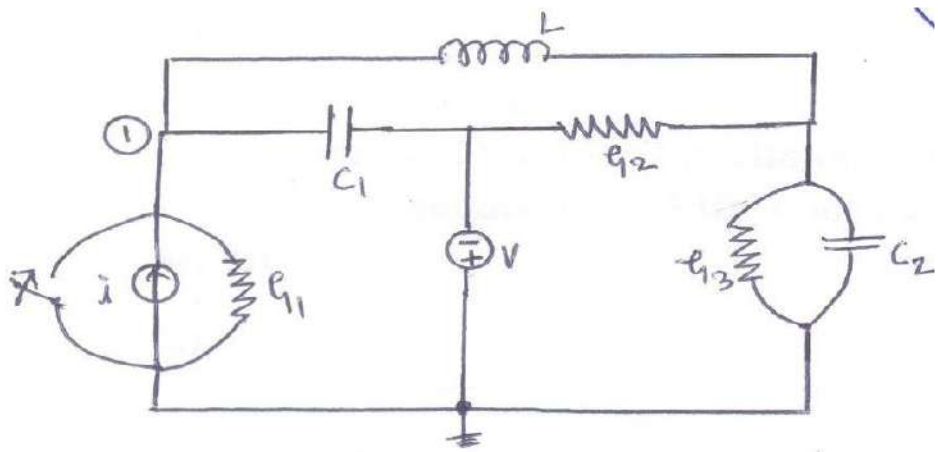
By looking into two sets of equations, we can say that two sets of equations dual equations

Example: 6) Draw the dual of the network shown below

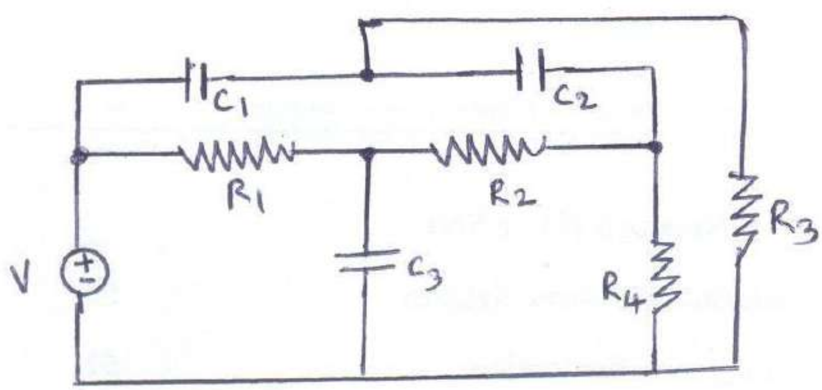


Solution:

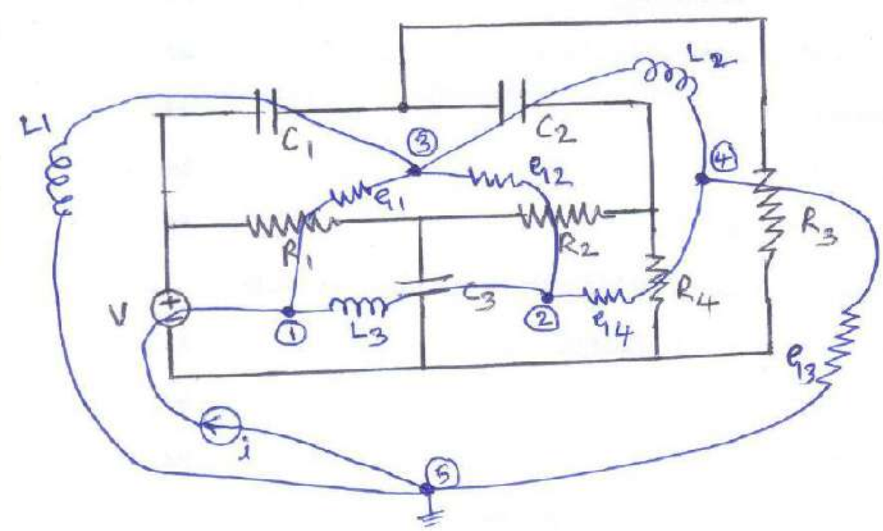




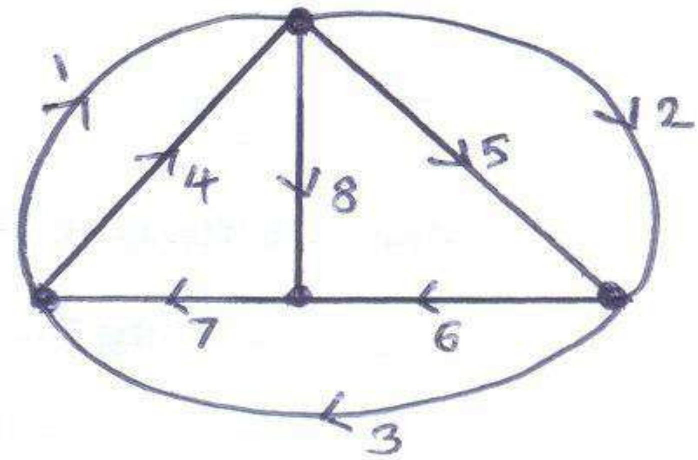
Example: 7) Draw the dual of the network shown below



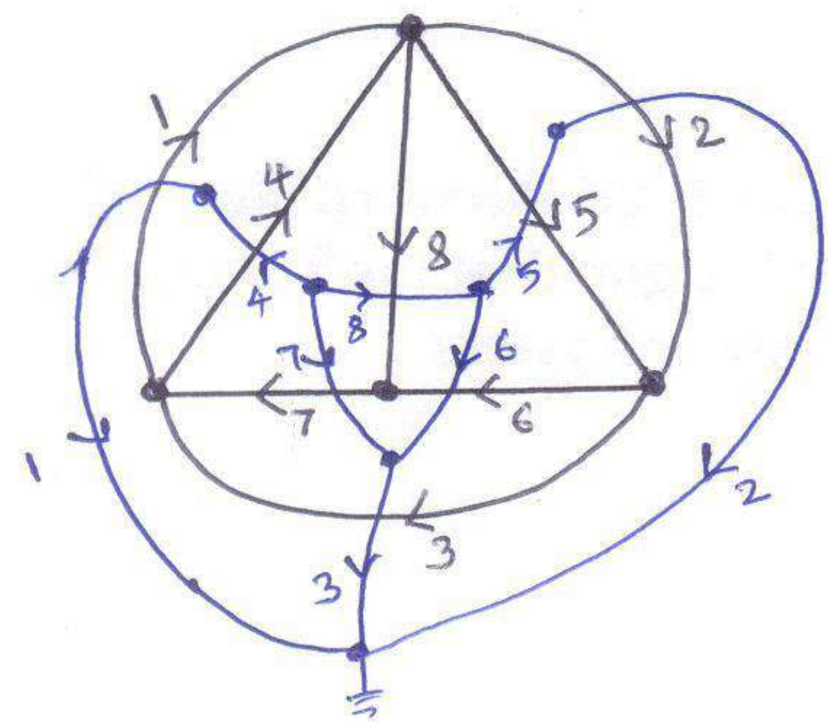
Solution:



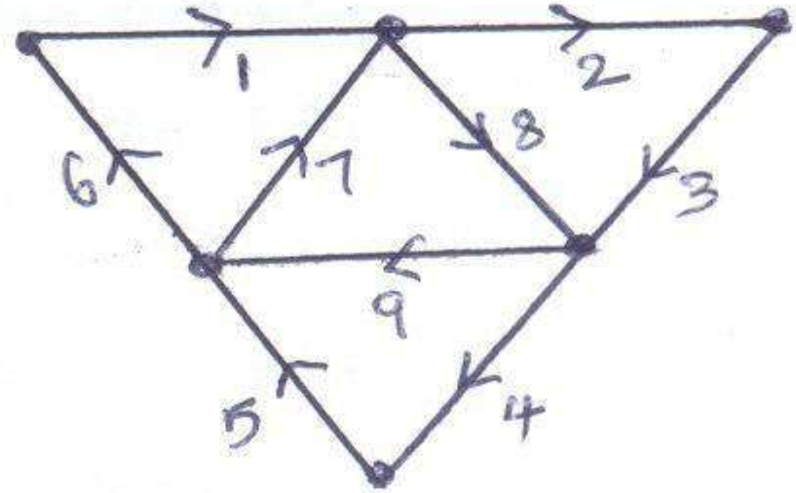
Example: 8) Draw the dual of the given oriented graph



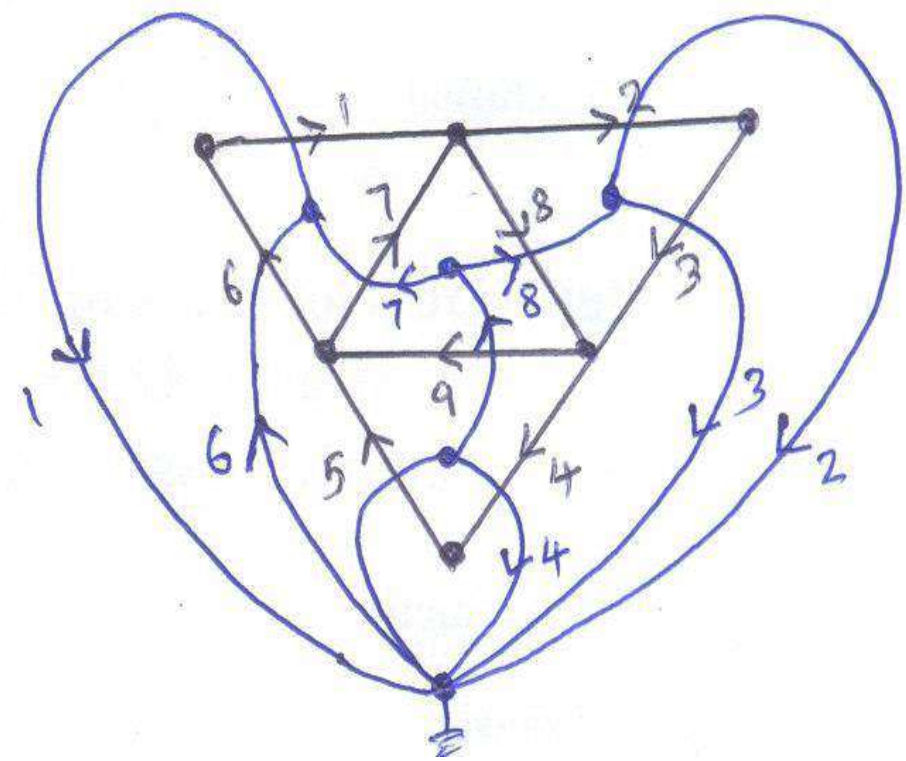
Solution:



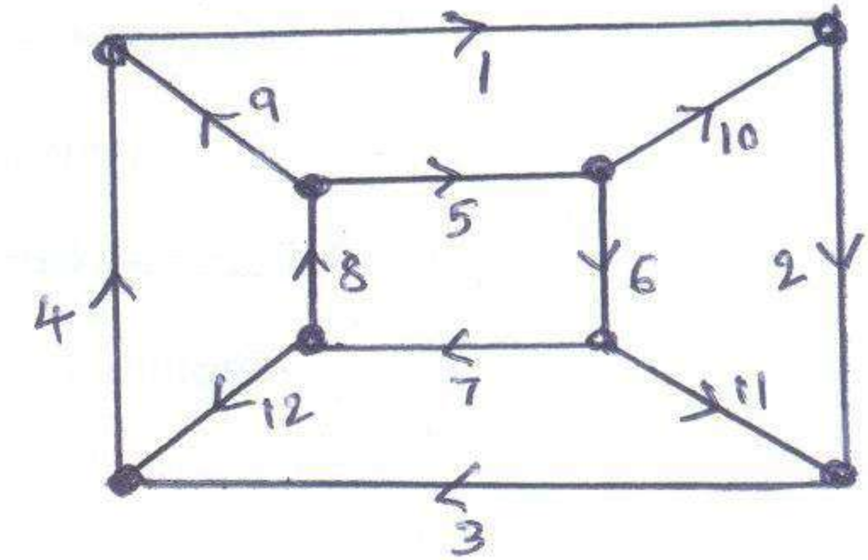
Example: 9) Draw the dual of the given oriented graph



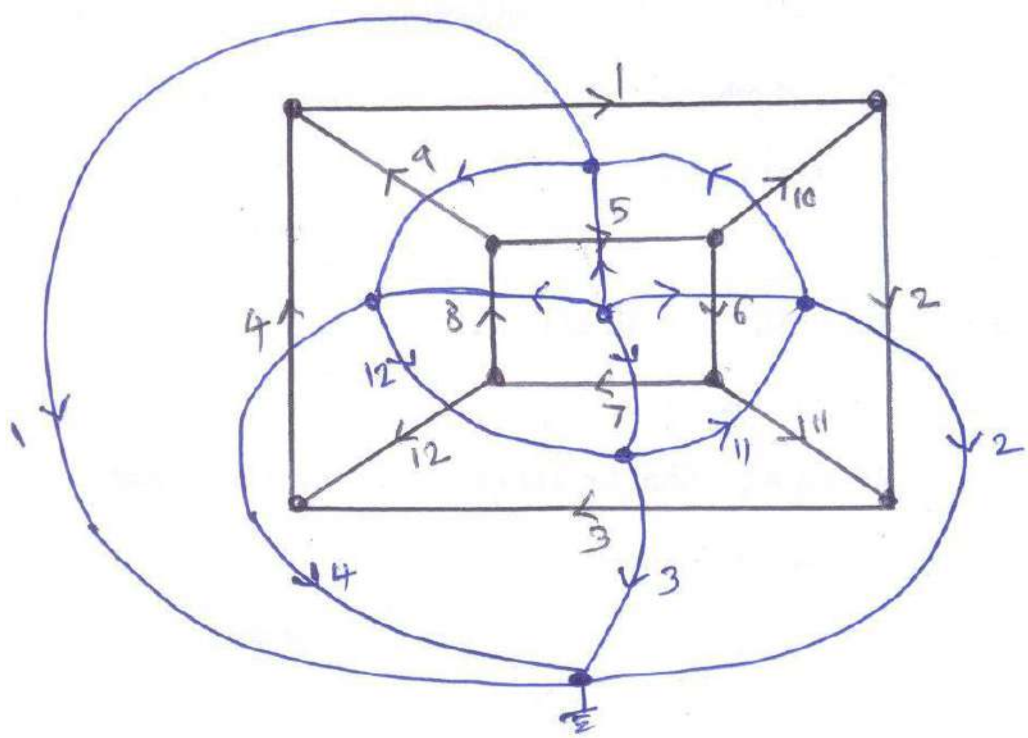
Solution:



Example: 10) Draw the dual of the given oriented graph



Solution:



UNIT - IV

Two port Networks

Introduction:

A two-port network has two pairs of terminals, one pair at the input known as input port and one pair at the output known as output port as shown in figure: 6.1. There are four variables V_1, V_2, I_1 and I_2 associated with a two port network. Two of these variables can be expressed in terms of the other two variables. Thus, there will be two dependent variables and two independent variables. The number of possible combinations generated by four variables taken two at a time is ${}^4C_2$, i.e., six. There are six possible sets of equations describing a two-port network.

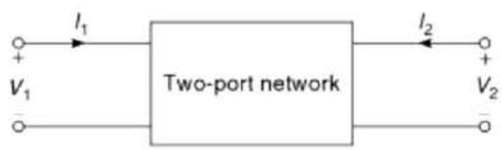


Figure: 6.1 Two-port network

Two-Port Parameters:

Parameter	Variables		Equation
	Express	In terms of	
Open-Circuit Impedance	V_1, V_2	I_1, I_2	$V_1 = Z_{11}I_1 + Z_{12}I_2$ $V_2 = Z_{21}I_1 + Z_{22}I_2$
Short-Circuit Admittance	I_1, I_2	V_1, V_2	$I_1 = Y_{11}V_1 + Y_{12}V_2$ $I_2 = Y_{21}V_1 + Y_{22}V_2$
Transmission	V_1, I_1	V_2, I_2	$V_1 = A\,V_2 - B\,I_2$ $I_1 = C\,V_2 - D\,I_2$
Inverse Transmission	V_2, I_2	V_1, I_1	$V_2 = A'V_1 - B'I_1$ $I_2 = C'V_1 - D'I_1$

Hybrid	V_1,I_2	I_1,V_2	$V_1 = h_{11}I_1 + h_{12}V_2$ $I_2 = h_{21}I_1 + h_{22}V_2$
Inverse Hybrid	I_1,V_2	V_1,I_2	$I_1 = g_{11}V_1 + g_{12}I_2$ $V_2 = g_{21}V_1 + g_{22}I_2$

Open-Circuit Impedance Parameters (Z Parameters)

The Z parameters of a two-port network may be defined by expressing two-port voltages V_1 and V_2in terms of two-port currents I_1 andI_2.

$(V_1, V_2)= f(I_1, I_2)$

$$V_1 = Z_{11}I_1 + Z_{12}I_2$$

$$V_2 = Z_{21}I_1 + Z_{22}I_2$$

In matrix form, we can write $\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$

$$[V] = [Z][I]$$

The individual Z parameters for a given network can be defined by setting each of the port currents equal to zero.

Case 1: When the output port is open-circuited, i.e., $I_2 = 0$

$$Z_{11} = \frac{V_1}{I_1}/I_2 = 0$$

Where Z_{11} is the driving-point impedance with the output port open-circuited. It is also called open-circuit input impedance.

Similarly, $Z_{21} = \frac{V_2}{I_1}/I_2 = 0$

Where Z_{21} is the transfer impedance with the output port open-circuited. It is also called open-circuit forward transfer impedance.

Case 2: When the input port is open-circuited, i.e., $I_1 = 0$

$$Z_{12} = \frac{V_1}{I_2}/I_1 = 0$$

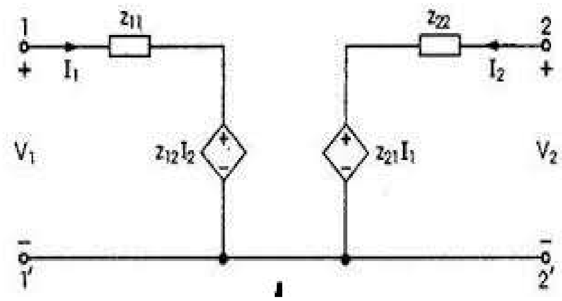
Where Z_{12} is the transfer impedance with the input port open-circuited. It is also called open-circuit reverse transfer impedance.

Similarly, $$Z_{22} = \frac{V_2}{I_2}/I_1 = 0$$

Where Z_{22} is the open-circuit driving-point impedance with the input port open-circuited. It is also called open-circuit output impedance.

As these impedance parameters are measured with either the input or output port open-circuited, these are called open-circuit impedance parameters.

The equivalent circuit of the two-port network in terms of Z parameters is shown in figure: 6.3



Equivalent circuit of the two-port network in terms of Z parameter.

Condition for Reciprocity:

If Z_{12}= Z_{21} ,the network is said to be reciprocal network.

Condition for Symmetry:

If Z_{11}=Z_{22}, the network is said to be symmetrical network.

Short-Circuit Admittance Parameters (Y Parameters)

The Y parameters of a two-port network may be defined by expressing two-port currents I_1 and I_2 in terms of the two-port voltages V_1 and V_2.

$$(I_1, I_2) = f(V_1, V_2)$$
$$I_1 = Y_{11}V_1 + Y_{12}V_2$$
$$I_2 = Y_{21}V_1 + Y_{22}V_2$$

In matrix form, we can write $$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$$[I] = [Y][V]$$

The individual Y parameters for a given network can be defined by setting each of the port voltages equal to zero.

Case 1: When the output port is short-circuited, i.e., $V_2 = 0$

$$Y_{11} = \frac{I_1}{V_1} / V_2 = 0$$

Where Y_{11} is the driving-point admittance with the output port short-circuited. It is also called short-circuit input admittance.

Similarly, $Y_{21} = \frac{I_2}{V_1} / V_2 = 0$

Where Y_{21} is the transfer admittance with the output port short-circuited. It is also called short-circuit forward transfer admittance.

Case 2: When the input port is short-circuited, i.e., $V_1 = 0$

$$Y_{12} = \frac{I_1}{V_2} / V_1 = 0$$

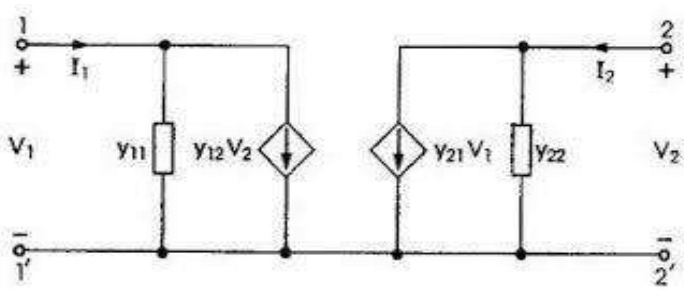
Where Y_{12} is the transfer admittance with the input port short-circuited. It is also called short-circuit reverse transfer admittance.

Similarly, $Y_{22} = \frac{I_2}{V_2} / V_1 = 0$

Where Y_{22} is the short-circuit driving-point admittance with the input port short-circuited. It is also called short-circuit output admittance.

As these admittance parameters are measured with either input or output port short-circuited, these are called short-circuit admittance parameters.

The equivalent circuit of the two-port network in terms of Y parameters is shown in figure: 6.4



Equivalent circuit of the two-port network in terms of Y-parameters

Condition for Reciprocity:

If $Y_{12} = Y_{21}$,the network is said to be reciprocal network.

Condition for Symmetry:

If $Y_{11} = Y_{22}$, the network is said to be symmetrical network.

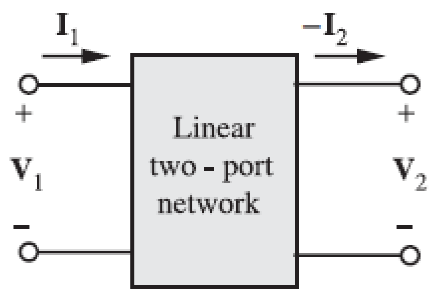
Transmission Parameters (ABCD Parameters)

The transmission parameters or chain parameters or ABCD parameters serve to relate the voltage and current at the input port to voltage and current at the output port.

In equation form,

$$(V_1, I_1) = f(V_2, -I_2)$$
$$V_1 = A\,V_2 - B\,I_2$$
$$I_1 = C\,V_2 - D\,I_2$$

Here, the negative sign is used with I_2 and not for parameters B and D. The reason the current I_2 carries a negative sign is that in transmission field, the output current is assumed to be coming out of the output port instead of going into the port.



Terminal variables used to define ABCD parameters

In matrix form, we can write $\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$

Where matrix $\begin{bmatrix} A & B \\ C & D \end{bmatrix}$ is called transmission matrix.

For a given network, these parameters are determined as follows:

Case 1 When the output port is open-circuited, i.e., $I_2=0$

$$A = \frac{V_1}{V_2} / \, I_2 = 0$$

Where A is the reverse voltage gain with the output port open-circuited.

Similarly, $C = \frac{I_1}{V_2} / \, I_2 = 0$

Where C is the transfer admittance with the output port open-circuited.

Case 2 When the output port is short-circuited, i.e., V_2=0

$$B = -\frac{V_1}{I_2} / \, V_2 = 0$$

Where B is the transfer impedance with the output port short-circuited.

Similarly, $D = -\frac{I_1}{I_2} / \, V_2 = 0$

Where D is the reverse current gain with the output port short-circuited.

Condition for Reciprocity:
If AD-BC=1, the network is said to be reciprocal network.
Condition for Symmetry:
If A=D, the network is said to be symmetrical network.

Hybrid Parameters (h Parameters)

The hybrid parameters of a two-port network may be defined by expressing the voltage of input port V_1 and current of output port I_2 in terms of current of input port I_1 and voltage of output port V_2.

$$(V_1, I_2) = f\,(I_1, V_2)$$

$$V_1 = h_{11}I_1 + h_{12}V_2$$
$$I_2 = h_{21}I_1 + h_{22}V_2$$

In matrix form, we can write $\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$

These parameters are particularly important in transistor circuit analysis.

Case 1 When the output port is short-circuited, i.e., V_2=0

$$h_{11} = \left.\frac{V_1}{I_1}\right/ V_2 = 0$$

Where h_{11} is called as short-circuit input impedance.

Similarly, $h_{21} = \left.\frac{I_2}{I_1}\right/ V_2 = 0$

Where h_{21} is called as short-circuit forward current gain.

Case 2 When the input port is open-circuited, i.e.,I_1=0

$$h_{12} = \left.\frac{V_1}{V_2}\right/ I_1 = 0$$

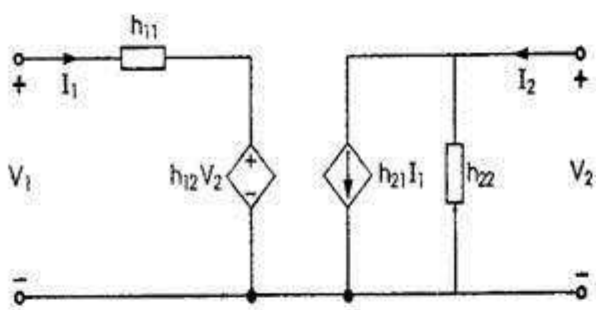
Where h_{12} is called as open circuit reverse voltage gain.

Similarly, $h_{22} = \left.\frac{I_2}{V_2}\right/ I_1 = 0$

Where h_{22} is called as open-circuit output admittance.

Since h parameters represent dimensionally impedance, admittance, voltage gain and current gain, these are called hybrid parameters.

The equivalent circuit of the two-port network in terms of hybrid parameters is shown in figure: 6.6



Equivalent circuit of the two-port network in terms of h-parameters

Condition for Reciprocity:

If $h_{21} = -h_{12}$, the network is said to be reciprocal network.

Condition for Symmetry:

If $h_{11}h_{22} - h_{12}h_{21} = 1$ $(\Delta h = 1)$, the network is said to be symmetrical network.

Inter-Relationships between the Parameters:

When it is required to find out two or more parameters of a particular network then finding each parameter will be tedious. But if we find a particular parameter then the other parameters can be found if the inter-relationship between them is known.

1. *Z*-parameters in terms of other parameters:

a) Z-parameters in terms of Y-parameters:

We known that

$$I_1 = Y_{11}V_1 + Y_{12}V_2$$
$$I_2 = Y_{21}V_1 + Y_{22}V_2$$

By Cramer's rule, $V_1 = \frac{\begin{vmatrix} I_1 & Y_{12} \\ I_2 & Y_{22} \end{vmatrix}}{\begin{vmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{vmatrix}} = \frac{Y_{22}I_1 - Y_{12}I_2}{Y_{11}Y_{22} - Y_{12}Y_{21}} = \frac{Y_{22}}{\Delta Y}I_1 - \frac{Y_{12}}{\Delta Y}I_2$

Where $\Delta Y = Y_{11}Y_{22} - Y_{12}Y_{21}$

Comparing with $V_1 = Z_{11}I_1 + Z_{12}I_2$

$$Z_{11} = \frac{Y_{22}}{\Delta Y}$$

$$Z_{12} = -\frac{Y_{12}}{\Delta Y}$$

Also, $V_2 = \frac{\begin{vmatrix} Y_{11} & I_1 \\ Y_{21} & I_2 \end{vmatrix}}{\Delta Y}$

$$= \frac{Y_{11}}{\Delta Y}I_2 - \frac{Y_{21}}{\Delta Y}I_1$$

Comparing with $V_2 = Z_{21}I_1 + Z_{22}I_2$

$$Z_{22} = \frac{Y_{11}}{\Delta Y}$$

$$Z_{21} = -\frac{Y_{21}}{\Delta Y}$$

b) Z-parameters in terms of ABCD parameters:

We know that $V_1 = A\,V_2 - B\,I_2$

$I_1 = C\,V_2 - D\,I_2$

Rewriting the second equation,

$$V_2 = \frac{1}{C} I_1 + \frac{D}{C} I_2$$

Comparing with $$V_2 = Z_{21} I_1 + Z_{22} I_2$$

$$Z_{21} = \frac{1}{C}$$

$$Z_{22} = \frac{D}{C}$$

Also, $V_1 = A\left[\frac{1}{C} I_1 + \frac{D}{C} I_2\right] - BI_2$

$$= \frac{A}{C} I_1 + \left[\frac{AD}{C} - B\right] I_2$$

$$= \frac{A}{C} I_1 + \left[\frac{AD - BC}{C}\right] I_2$$

Comparing with $$V_1 = Z_{11} I_1 + Z_{12} I_2$$

$$Z_{11} = \frac{A}{C}$$

$$Z_{12} = \frac{AD - BC}{C}$$

c) *Z-parameters in terms of A'B'C'D' parameters:*

We know that $V_2 = A'V_1 - B'I_1$
$I_2 = C'V_1 - D'I_1$

Rewriting the second equation,

$$V_1 = \frac{D'}{C'} I_1 + \frac{1}{C'} I_2$$

Comparing with $$V_1 = Z_{11} I_1 + Z_{12} I_2$$

$$Z_{11} = \frac{D'}{C'}$$

$$Z_{12} = \frac{1}{C'}$$

Also, $V_2 = A'\left[\frac{D'}{C'}I_1 + \frac{1}{C'}I_2\right] - B'I_1 = \left[\frac{A'D'-B'C'}{C'}\right]I_1 + \frac{A'}{C'}I_2$

Comparing with $$V_2 = Z_{21}I_1 + Z_{22}I_2$$

$$Z_{21} = \left[\frac{A'D' - B'C'}{C'}\right]$$

$$Z_{22} = \frac{A'}{C'}$$

d) *Z*-parameters in terms of Hybrid parameters:

We know that

$$V_1 = h_{11}I_1 + h_{12}V_2$$
$$I_2 = h_{21}I_1 + h_{22}V_2$$

Rewriting second equation,

$$V_2 = -\frac{h_{21}}{h_{22}}I_1 + \frac{1}{h_{22}}I_2$$

Comparing with $$V_2 = Z_{21}I_1 + Z_{22}I_2$$

$$Z_{21} = -\frac{h_{21}}{h_{22}}$$
$$Z_{22} = \frac{1}{h_{22}}$$

Also, $$V_1 = h_{11}I_1 + h_{12}\left[-\frac{h_{21}}{h_{22}}I_1 + \frac{1}{h_{22}}I_2\right]$$

$$= h_{11}I_1 + \frac{h_{12}}{h_{22}}I_2 - \frac{h_{12}h_{21}}{h_{22}}I_1$$

$$= \left[\frac{h_{11}h_{22} - h_{12}h_{21}}{h_{22}}\right]I_1 + \frac{h_{12}}{h_{22}}I_2$$

Comparing with $$V_1 = Z_{11}I_1 + Z_{12}I_2$$

$$Z_{11} = \frac{h_{11}h_{22} - h_{12}h_{21}}{h_{22}} = \frac{\Delta h}{h_{22}}$$

$$Z_{12} = \frac{h_{12}}{h_{22}}$$

2. Y-parameters in terms of other parameters:

a) Y-parameters in terms of Z-parameters:

We known that

$$V_1 = Z_{11}I_1 + Z_{12}I_2$$

$$V_2 = Z_{21}I_1 + Z_{22}I_2$$

By Cramer's rule,

$$I_1 = \frac{\begin{vmatrix} V_1 & Z_{12} \\ V_2 & Z_{22} \end{vmatrix}}{\begin{vmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{vmatrix}}$$

$$= \frac{Z_{22}V_1 - Z_{12}V_2}{Z_{11}Z_{22} - Z_{12}Z_{21}}$$

$$= \frac{Z_{22}}{\Delta Z}V_1 - \frac{Z_{12}}{\Delta Z}V_2$$

Where $\Delta Z = Z_{11}Z_{22} - Z_{12}Z_{21}$

Comparing with $I_1 = Y_{11}V_1 + Y_{12}V_2$

$$Y_{11} = \frac{Z_{22}}{\Delta Z}$$

$$Y_{12} = -\frac{Z_{12}}{\Delta Z}$$

Also, $I_2 = \dfrac{\begin{vmatrix} Z_{11} & V_1 \\ Z_{21} & V_2 \end{vmatrix}}{\Delta Z}$

$$= \frac{Z_{11}V_2 - Z_{12}V_1}{\Delta Z}$$

$$= -\frac{Z_{21}}{\Delta Z}V_1 + \frac{Z_{11}}{\Delta Z}V_2$$

Comparing with $I_2 = Y_{21}V_1 + Y_{22}V_2$

$$Y_{21} = -\frac{Z_{21}}{\Delta Z}$$

$$Y_{22} = \frac{Z_{11}}{\Delta Z}$$

b) Y-parameters in terms of ABCD parameters:

We know that $$V_1 = AV_2 - BI_2$$

$$I_1 = CV_2 - DI_2$$

Rewriting the first equation, $I_2 = -\frac{1}{B}V_1 + \frac{A}{B}V_2$

Comparing with $$I_2 = Y_{21}V_1 + Y_{22}V_2$$

$$Y_{21} = -\frac{1}{B}$$

$$Y_{22} = \frac{A}{B}$$

Also, $I_1 = CV_2 - D\left[-\frac{1}{B}V_1 + \frac{A}{B}V_2\right]$

$$= \frac{D}{B}V_1 + \left[\frac{BC - AD}{B}\right]V_2$$

Comparing with $$I_1 = Y_{11}V_1 + Y_{12}V_2$$

$$Y_{11} = \frac{D}{B}$$

$$Y_{12} = \frac{BC - AD}{B}$$

c) Y-parameters in terms of A'B'C'D' parameters:

We know that $V_2 = A'\ V_1 - B'\ I_1$
$I_2 = C'\ V_1 - D'\ I_1$

Rewriting the first equation, $I_1 = \frac{A'}{B'}V_1 - \frac{1}{B'}V_2$

Comparing with $$I_1 = Y_{11}V_1 + Y_{12}V_2$$

$$Y_{11} = \frac{A'}{B'}$$

$$Y_{12} = -\frac{1}{B'}$$

Also, $I_2 = C'V_1 - D'\left[\frac{A'}{B'}V_1 - \frac{1}{B'}V_2\right]$

$$= -\left[\frac{A'D' - B'C'}{B'}\right]V_1 + \frac{D'}{B'}V_2$$

Comparing with $$I_2 = Y_{21}V_1 + Y_{22}V_2$$

$$Y_{21} = -\frac{A'D' - B'C'}{B'}$$
$$Y_{22} = \frac{D'}{B'}$$

d) Y-parameters in terms of Hybrid parameters:

We know that,

$$V_1 = h_{11}I_1 + h_{12}V_2$$
$$I_2 = h_{21}I_1 + h_{22}V_2$$

Rewriting the first equation,

$$I_1 = \frac{1}{h_{11}}V_1 - \frac{h_{12}}{h_{11}}V_2$$

Comparing with $$I_1 = Y_{11}V_1 + Y_{12}V_2$$
$$Y_{11} = \frac{1}{h_{11}}$$
$$Y_{12} = -\frac{h_{12}}{h_{11}}$$

Also, $I_2 = h_{21}\left[\frac{1}{h_{11}}V_1 - \frac{h_{12}}{h_{11}}V_2\right] + h_{22}V_2$

$$= \frac{h_{21}}{h_{11}}V_1 + \left[\frac{h_{11}h_{22} - h_{12}h_{21}}{h_{11}}\right]V_2$$

Comparing with $$I_2 = Y_{21}V_1 + Y_{22}V_2$$

$$Y_{21} = \frac{h_{21}}{h_{11}}$$

$$Y_{22} = \frac{h_{11}h_{22} - h_{12}h_{21}}{h_{11}}$$

3. ABCD parameters in terms of other parameters:

a) ABCD parameters in terms of Z-parameters:

We know that

$$V_1 = Z_{11}I_1 + Z_{12}I_2$$

$$V_2 = Z_{21}I_1 + Z_{22}I_2$$

Rewriting the second equation,

$$I_1 = \frac{1}{Z_{21}}V_2 - \frac{Z_{22}}{Z_{21}}I_2$$

Comparing with, $I_1 = CV_2 - DI_2$

$$C = \frac{1}{Z_{21}}$$

$$D = \frac{Z_{22}}{Z_{21}}$$

Also, $V_1 = Z_{11}\left[\frac{1}{Z_{21}}V_2 - \frac{Z_{22}}{Z_{21}}I_2\right] + Z_{12}I_2$

$$= \frac{Z_{11}}{Z_{21}}V_2 - \frac{Z_{22}Z_{11}}{Z_{21}}I_2 + Z_{12}I_2$$

$$= \frac{Z_{11}}{Z_{21}}V_2 - \left[\frac{Z_{11}Z_{22} - Z_{12}Z_{21}}{Z_{21}}\right]I_2$$

Comparing with, $V_1 = AV_2 - BI_2$

$$A = \frac{Z_{11}}{Z_{21}}$$

$$B = \frac{Z_{11}Z_{22} - Z_{12}Z_{21}}{Z_{21}}$$

b) ABCD parameters in terms of Y-parameters:

We know that

$$I_1 = Y_{11}V_1 + Y_{12}V_2$$
$$I_2 = Y_{21}V_1 + Y_{22}V_2$$

Rewriting the second equation,

$$V_1 = -\frac{Y_{22}}{Y_{21}}V_2 + \frac{1}{Y_{21}}I_2$$

Comparing with $\quad V_1 = AV_2 - BI_2$

$$A = -\frac{Y_{22}}{Y_{21}}$$

$$B = -\frac{1}{Y_{21}}$$

Also, $I_1 = Y_{11}\left[-\frac{Y_{22}}{Y_{21}}V_2 + \frac{1}{Y_{21}}I_2\right] + Y_{12}V_2$

$$= \left[\frac{Y_{12}Y_{21} - Y_{11}Y_{22}}{Y_{21}}\right]V_2 + \frac{Y_{11}}{Y_{21}}I_2$$

Comparing with, $I_1 = CV_2 - DI_2$

$$C = \left[\frac{Y_{12}Y_{21} - Y_{11}Y_{22}}{Y_{21}}\right] = -\frac{\Delta Y}{Y_{21}}$$

$$D = -\frac{Y_{11}}{Y_{21}}$$

c) ABCD parameters in terms of hybrid parameters:

We know that,
$$V_1 = h_{11}I_1 + h_{12}V_2$$
$$I_2 = h_{21}I_1 + h_{22}V_2$$

Rewriting the second equation, $I_1 = -\frac{h_{22}}{h_{21}}V_2 + \frac{1}{h_{21}}I_2$

Comparing with, $I_1 = CV_2 - DI_2$

$$C = -\frac{h_{22}}{h_{21}}$$

$$D = -\frac{1}{h_{21}}$$

Also, $V_1 = h_{11}\left[\frac{1}{h_{21}}I_2 - \frac{h_{22}}{h_{21}}V_2\right] + h_{12}V_2$

$$= \left[\frac{h_{12}h_{21} - h_{11}h_{22}}{h_{21}}\right]V_2 + \frac{h_{11}}{h_{21}}I_2$$

Comparing with $V_1 = AV_2 - BI_2$

$$A = \frac{h_{12}h_{21} - h_{11}h_{22}}{h_{21}}$$

$$B = -\frac{h_{11}}{h_{21}}$$

4. Hybrid parameters in terms of other parameters:

a) Hybrid parameters in terms of Z-parameters:

We know that

$$V_1 = Z_{11}I_1 + Z_{12}I_2$$
$$V_2 = Z_{21}I_1 + Z_{22}I_2$$

Rewriting the second equation, $I_2 = -\frac{Z_{21}}{Z_{22}}I_1 + \frac{1}{Z_{22}}V_2$

Comparing with

$$I_2 = h_{21}I_1 + h_{22}V_2$$
$$h_{21} = -\frac{Z_{21}}{Z_{22}}$$
$$h_{22} = \frac{1}{Z_{22}}$$

Also, $V_1 = Z_{11}I_1 + Z_{12}\left[-\frac{Z_{21}}{Z_{22}}I_1 + \frac{1}{Z_{22}}V_2\right]$

$$= \left[\frac{Z_{11}Z_{22} - Z_{12}Z_{21}}{Z_{22}}\right]I_1 + \frac{Z_{12}}{Z_{22}}V_2$$

Comparing with

$$V_1 = h_{11}I_1 + h_{12}V_2$$
$$h_{11} = \frac{Z_{11}Z_{22} - Z_{12}Z_{21}}{Z_{22}} = \frac{\Delta Z}{Z_{22}}$$
$$h_{12} = \frac{Z_{12}}{Z_{22}}$$

b) Hybrid parameters in terms of Y-parameters:

We know that

$$I_1 = Y_{11}V_1 + Y_{12}V_2$$
$$I_2 = Y_{21}V_1 + Y_{22}V_2$$

Rewriting the first equation, $V_1 = \frac{1}{Y_{11}}I_1 - \frac{Y_{12}}{Y_{11}}V_2$

Comparing with

$$V_1 = h_{11}I_1 + h_{12}V_2$$
$$h_{11} = \frac{1}{Y_{11}}$$
$$h_{12} = -\frac{Y_{12}}{Y_{11}}$$

Also, $I_2 = Y_{21}\left[\frac{1}{Y_{11}}I_1 - \frac{Y_{12}}{Y_{11}}V_2\right] + Y_{22}V_2$

$$= \left[\frac{Y_{11}Y_{22} - Y_{12}Y_{21}}{Y_{11}}\right]V_2 + \frac{Y_{21}}{Y_{11}}I_1$$

Comparing with $\quad I_2 = h_{21}I_1 + h_{22}V_2$

$$h_{22} = \frac{Y_{11}Y_{22} - Y_{12}Y_{21}}{Y_{11}} = \frac{\Delta Y}{Y_{11}}$$

$$h_{21} = \frac{Y_{21}}{Y_{11}}$$

c) Hybrid parameters in terms of ABCD parameters:

We know that

$$V_1 = AV_2 - BI_2$$
$$I_1 = CV_2 - DI_2$$

Rewriting the second equation, $\quad I_2 = -\frac{1}{D}I_1 + \frac{C}{D}V_2$

Comparing with $\quad I_2 = h_{21}I_1 + h_{22}V_2$

$$h_{21} = -\frac{1}{D}$$

$$h_{22} = \frac{C}{D}$$

Also, $V_1 = AV_2 - B\left[-\frac{1}{D}I_1 + \frac{C}{D}V_2\right]$

$$= \frac{B}{D}I_1 + \left[\frac{AD - BC}{D}\right]V_2$$

Comparing with $\quad V_1 = h_{11}I_1 + h_{12}V_2$

$$h_{11} = \frac{B}{D}$$

$$h_{12} = \frac{AD-BC}{D} = \frac{\Delta T}{D}$$

Inter-relationship between parameters:

INTER-RELATIONSHIP BETWEEN PARAMETERS

	z	y	T	h
z	$\begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix}$	$\begin{bmatrix} \frac{y_{22}}{\Delta y} & \frac{-y_{12}}{\Delta y} \\ \frac{-y_{21}}{\Delta y} & \frac{y_{11}}{\Delta y} \end{bmatrix}$	$\begin{bmatrix} \frac{A}{C} & \frac{\Delta T}{C} \\ \frac{1}{C} & \frac{D}{C} \end{bmatrix}$	$\begin{bmatrix} \frac{\Delta h}{h_{22}} & \frac{h_{12}}{h_{22}} \\ \frac{-h_{21}}{h_{22}} & \frac{1}{h_{22}} \end{bmatrix}$
y	$\begin{bmatrix} \frac{z_{22}}{\Delta z} & \frac{-z_{12}}{\Delta z} \\ \frac{-z_{21}}{\Delta z} & \frac{z_{11}}{\Delta z} \end{bmatrix}$	$\begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix}$	$\begin{bmatrix} \frac{D}{B} & \frac{-\Delta T}{B} \\ \frac{-1}{B} & \frac{A}{B} \end{bmatrix}$	$\begin{bmatrix} \frac{1}{h_{11}} & \frac{-h_{22}}{h_{11}} \\ \frac{h_{21}}{h_{11}} & \frac{\Delta h}{h_{11}} \end{bmatrix}$
T	$\begin{bmatrix} \frac{z_{11}}{z_{21}} & \frac{\Delta z}{z_{21}} \\ \frac{1}{z_{21}} & \frac{z_{22}}{z_{21}} \end{bmatrix}$	$\begin{bmatrix} \frac{-y_{22}}{y_{21}} & \frac{-1}{y_{21}} \\ \frac{-\Delta y}{y_{21}} & \frac{-y_{11}}{y_{21}} \end{bmatrix}$	$\begin{bmatrix} A & B \\ C & D \end{bmatrix}$	$\begin{bmatrix} \frac{-\Delta h}{h_{21}} & \frac{-h_{11}}{h_{21}} \\ \frac{-h_{22}}{h_{21}} & \frac{-1}{h_{21}} \end{bmatrix}$
h	$\begin{bmatrix} \frac{\Delta z}{z_{22}} & \frac{z_{12}}{z_{22}} \\ \frac{-z_{21}}{z_{22}} & \frac{1}{z_{22}} \end{bmatrix}$	$\begin{bmatrix} \frac{1}{y_{11}} & \frac{-y_{12}}{y_{11}} \\ \frac{y_{21}}{y_{11}} & \frac{\Delta y}{y_{11}} \end{bmatrix}$	$\begin{bmatrix} \frac{B}{D} & \frac{\Delta T}{D} \\ -\frac{1}{D} & \frac{C}{D} \end{bmatrix}$	$\begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix}$

$\Delta z = z_{11}z_{22} - z_{12}z_{21},\ \Delta y = y_{11}y_{22} - y_{12}y_{21},\ \Delta h = h_{11}h_{22} - h_{12}h_{21},\ \Delta T = AD - BC$

Interconnection of two-port networks:

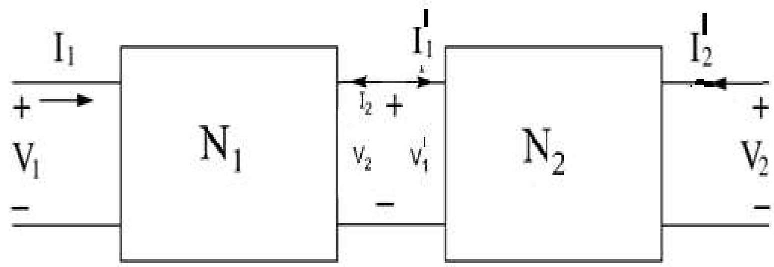
Interconnection of two-port networks, namely, cascade, parallel, series-parallel and parallel-series are discussed below and the relation between the input and output quantities of the combined two-port networks is derived.

Cascade Connection:

Transmission Parameter Representation:

Figure: 6.8.1 shows two-port networks connected in cascade. In the cascade connection, the output port of the first network becomes the input port of the second network. Since it is assumed that input and output currents are positive when they enter the network, we have

$$I_1' = -I_2$$



Cascade Connection

Let A_1, B_1, C_1, D_1 be the transmission parameters of the network N_1 and A_2, B_2, C_2, D_2 be the transmission parameters of the network N_2.

For the network N_1,

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A_1 B_1 \\ C_1 D_1 \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix} \text{\_\_\_\_\_\_\_\_\_\_ (i)}$$

For the network N_2,

$$\begin{bmatrix} V_1' \\ I_1' \end{bmatrix} = \begin{bmatrix} A_2 B_2 \\ C_2 D_2 \end{bmatrix} \begin{bmatrix} V_2' \\ -I_2' \end{bmatrix}$$

Since $V_1' = V_2$ and $I_2' = -I_2$, we can write

$$\begin{bmatrix} V_2 \\ -I_2 \end{bmatrix} = \begin{bmatrix} A_2 B_2 \\ C_2 D_2 \end{bmatrix} \begin{bmatrix} V_2' \\ -I_2' \end{bmatrix} \text{\_\_\_\_\_\_\_\_\_\_ (ii)}$$

Combining equations (i) and (ii),

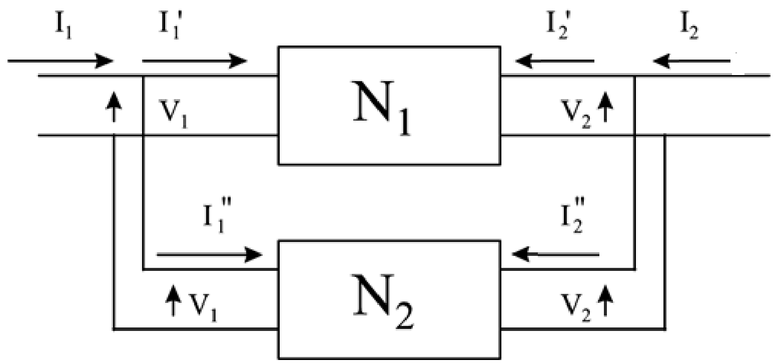
$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A_1 B_1 \\ C_1 D_1 \end{bmatrix} \begin{bmatrix} A_2 B_2 \\ C_2 D_2 \end{bmatrix} \begin{bmatrix} V_2' \\ -I_2' \end{bmatrix} = \begin{bmatrix} AB \\ CD \end{bmatrix} \begin{bmatrix} V_2' \\ -I_2' \end{bmatrix}$$

Hence, $$\begin{bmatrix} AB \\ CD \end{bmatrix} = \begin{bmatrix} A_1 B_1 \\ C_1 D_1 \end{bmatrix} \begin{bmatrix} A_2 B_2 \\ C_2 D_2 \end{bmatrix} \text{\_\_\_\_\_\_\_\_\_\_ (iii)}$$

Equation (iii) shows that the resultant *ABCD* matrix of the cascade connection is the product of the individual *ABCD* matrices.

Parallel Connection:

Figure: 6.8.2 shows two-port networks connected in parallel. In the parallel connection, the two networks have the same input voltages and the same output voltages.



Parallel Connection

Let $Y_{11}^{'}, Y_{12}^{'}, Y_{21}^{'}, Y_{22}^{'}$ be the Y-parameters of the network N_1 and $Y_{11}^{''}, Y_{12}^{''}, Y_{21}^{''}, Y_{22}^{''}$ be the Y-parameters of the network N_2.

For the network N_1, $\begin{bmatrix} I_1^{'} \\ I_2^{'} \end{bmatrix} = \begin{bmatrix} Y_{11}^{'} Y_{12}^{'} \\ Y_{21}^{'} Y_{22}^{'} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$

For the networkN_2, $\begin{bmatrix} I_1^{''} \\ I_2^{''} \end{bmatrix} = \begin{bmatrix} Y_{11}^{''} Y_{12}^{''} \\ Y_{21}^{''} Y_{22}^{''} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$

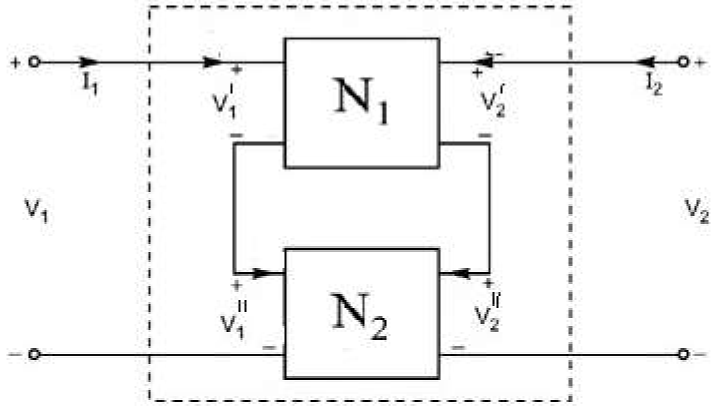
For the combined network, $I_1 = I_1^{'} + I_1^{''}$ and $I_2 = I_2^{'} + I_2^{''}$

$$\text{Hence, } \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} I_1^{'} + I_1^{''} \\ I_2^{'} + I_2^{''} \end{bmatrix} = \begin{bmatrix} Y_{11}^{'} + Y_{11}^{''} Y_{12}^{'} + Y_{12}^{''} \\ Y_{21}^{'} + Y_{21}^{''} Y_{22}^{'} + Y_{22}^{''} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Y_{11} Y_{12} \\ Y_{21} Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

Thus, the resultant Y-parameter matrix for parallel connected networks is the sum of Y matrices of each individual two-port networks.

Series Connection:

Figure: 6.8.3 shows two-port networks connected in series. In a series connection, both the networks carry the same input current. Their output currents are also equal.



Series Connection

Let $Z_{11}^{'}, Z_{12}^{'}, Z_{21}^{'}, Z_{22}^{'}$ be the Z-parameters of the network N_1 and $Z_{11}^{''}, Z_{12}^{''}, Z_{21}^{''}, Z_{22}^{''}$ be the Y-parameters of the network N_2.

For the network N_1,
$$\begin{bmatrix} V_1^{'} \\ V_2^{'} \end{bmatrix} = \begin{bmatrix} Z_{11}^{'} Z_{12}^{'} \\ Z_{21}^{'} Z_{22}^{'} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

For the network N_2,
$$\begin{bmatrix} V_1^{''} \\ V_2^{''} \end{bmatrix} = \begin{bmatrix} Z_{11}^{''} Z_{12}^{''} \\ Z_{21}^{''} Z_{22}^{''} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

For the combined network $V_1 = V_1^{'} + V_1^{''}$ and $V_2 = V_2^{'} + V_2^{''}$

$$\text{Hence, } \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} V_1^{'} + V_1^{''} \\ V_2^{'} + V_2^{''} \end{bmatrix} = \begin{bmatrix} Z_{11}^{'} + Z_{11}^{''} Z_{12}^{'} + Z_{12}^{''} \\ Z_{21}^{'} + Z_{21}^{''} Z_{22}^{'} + Z_{22}^{''} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Z_{11} Z_{12} \\ Z_{21} Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

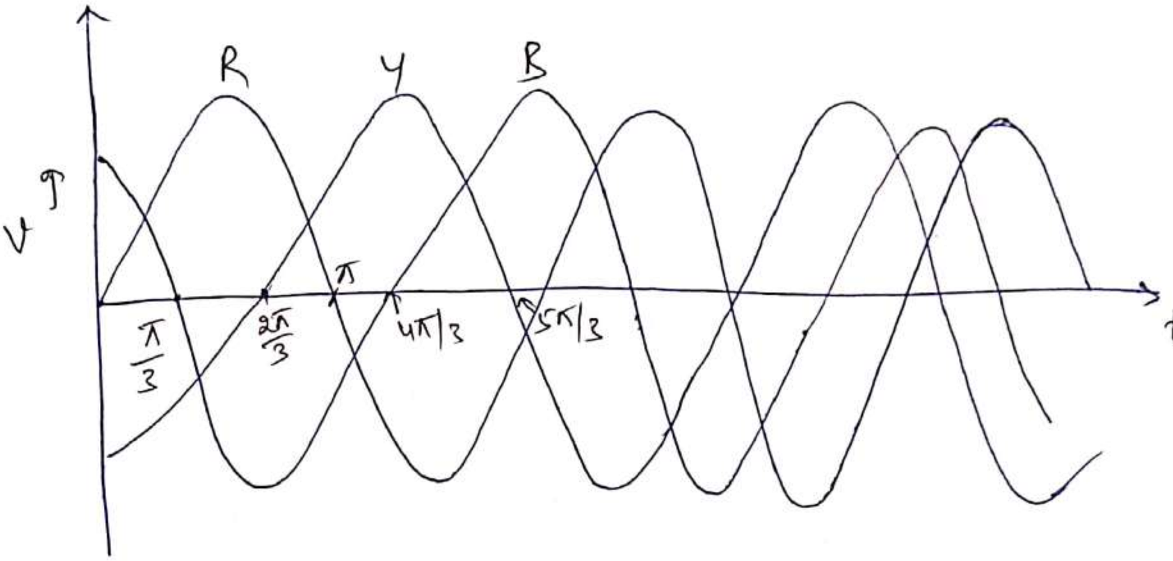
Thus, the ersultant Z-parameters matrix for the series-connected networks is the sum of Z matrices of each individual two-port network.

UNIT-V

ANALYSIS OF THREE PHASE CIRCUITS

In an ac system it is possible to connect two or more number of individual circuits to a common polyphase source. Though it is possible to have any number of phases sources in a polyphase system, the increase in available power is not significant beyond the three phase system. The power generated by the same machine increases 41.4 per cent from single phase to two phase and increase in the power is 50 per cent from single phase to three phase. Beyond three phase, the maximum possible increase is only seven per cent, but the complications are many. So an increase beyond three phase does not justify the extra complications. In view of this, it is only in exceptional cases where more than three phases are used. Circuits supplied by six, twelve and more phases are used in high power radio transmitter stations. Two phase systems are used to supply two phase servo motors in feedback control systems.

In general, a three phase system of voltages (currents) is merely a combination of three single phase systems of voltages (currents) of which the three voltages (currents) differ in phase by 120 electrical degrees from each other in a particular sequence. One such three phase system of sinusoidal voltages is shown in figure below.



Advantages of a Three Phase System :-

1. The power in a single phase circuit is pulsating. When the power factor of the circuit is unity, the power becomes zero 100 times in a 50Hz supply. Therefore, single phase motors have a pulsating torque. Although power supplied by each phase is pulsating, the total power (three phase) supplied to a balanced 3-ϕ circuit is constant at every instant of time. Because of this three phase motors have an absolutely uniform torque.

2. To transmit a given amount of power over a given length a 3-ϕ transmission circuit requires less conductor material than single phase circuit.

3. In a given frame size a three phase motor or 3-ϕ generator produces more output than its single phase counterpart.

4. Three phase motors are more easily started than single phase motors. Single phase motors are not self starting whereas 3-ϕ motors are.

5. operating characteristics of 3-ϕ motors (apparatus) are superior than those of a similar 1-ϕ apparatus.

6. All 3-ϕ machines are superior in performance. Their control equipments are smaller, cheaper, lighter in weight and more efficient

Generation of Three Phase Voltages:-

Three phase voltages can be generated in a stationary armature with a rotating field structure, or in a rotating armature with a stationary field as shown in fig1.3(a) & 1.3(b)

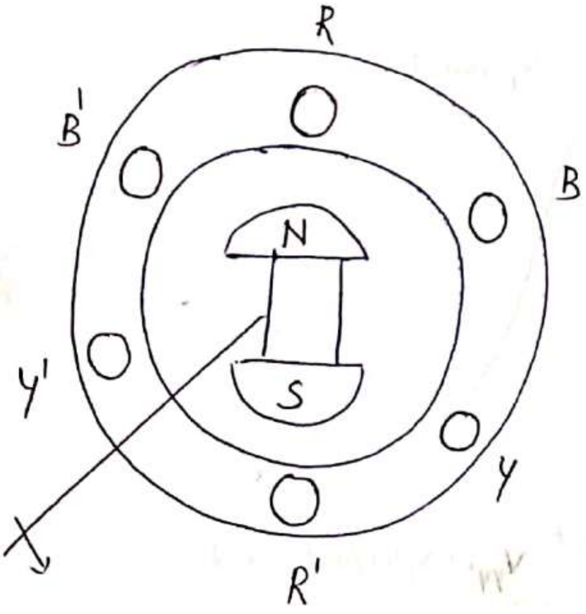


Fig1.3(a)

Stationary Armature

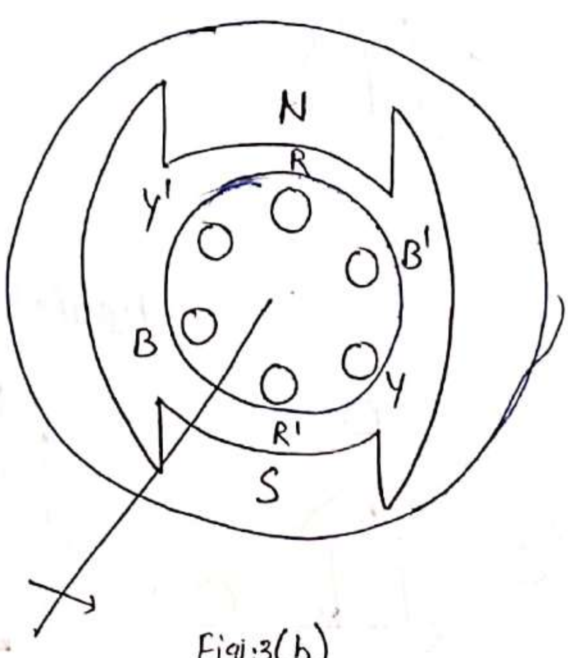


Fig1.3(b)

Stationary field

Single phase voltages and currents are generated by single phase generators as shown in fig 1.1(a). The armature (here a stationary armature) of such a generator has only one winding or one set of coils. In a two phase generator the armature has two distinct windings or two sets of coils that are displaced 90° (electrical degrees) apart so that the generated voltages in the two phases have 90° phase displacement as shown in fig 1.1(b). Similarly three phase voltages are generated in three separate but identical sets of windings or coils that are displaced by 120 electrical degrees in the armature so that the voltages generated in them are 120° apart in time phase. This arrangement is shown in fig 1.1(c). Here RR' constitutes one coil (R-phase); YY' another coil (Y-phase), and BB' constitutes the third phase (B phase). The field magnets are assumed in clockwise rotation

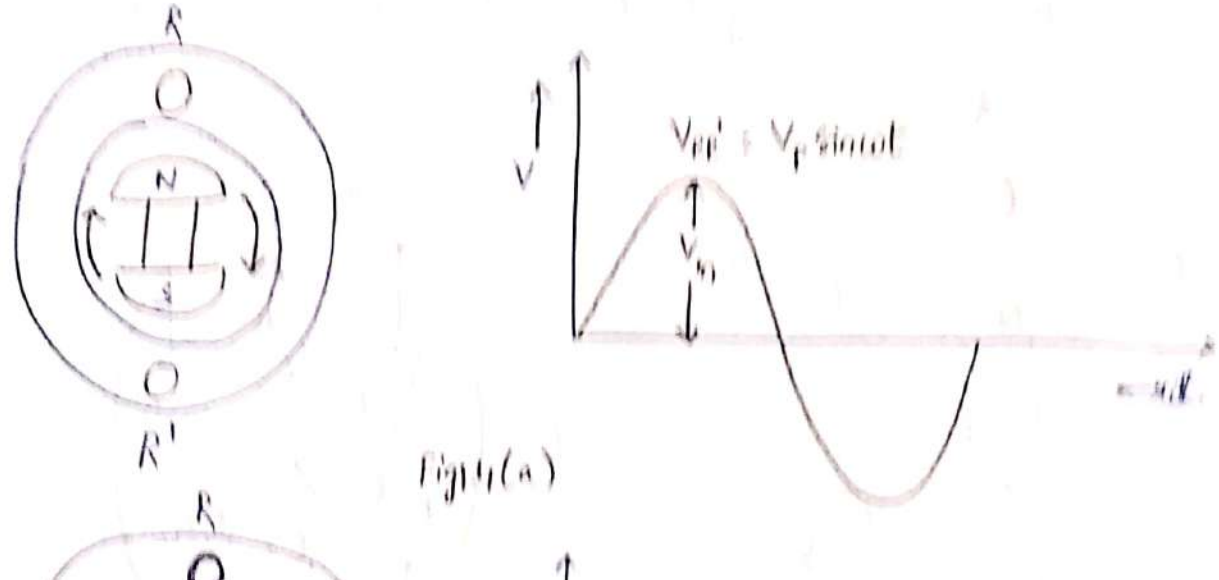


Fig 1.1(a)

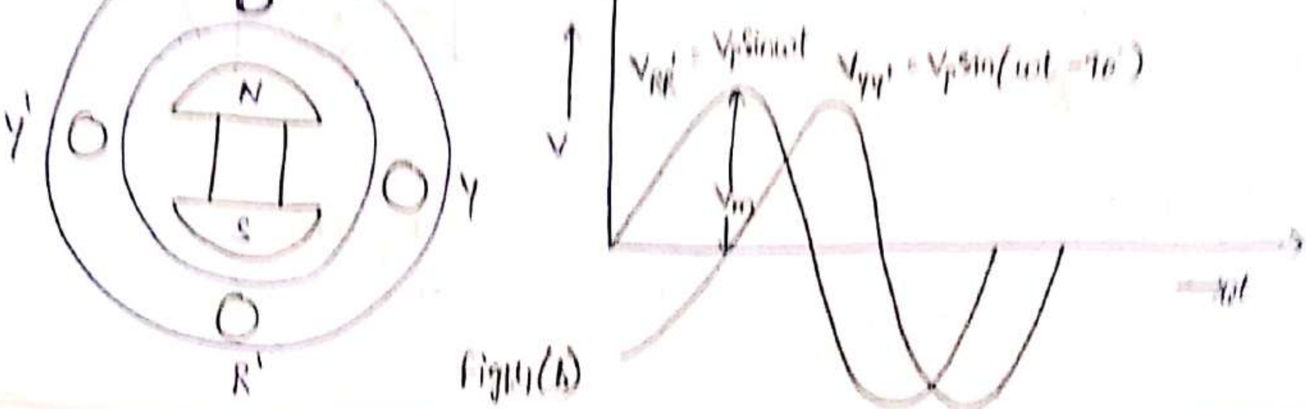


Fig 1.1(b)

Scanned by CamScanner

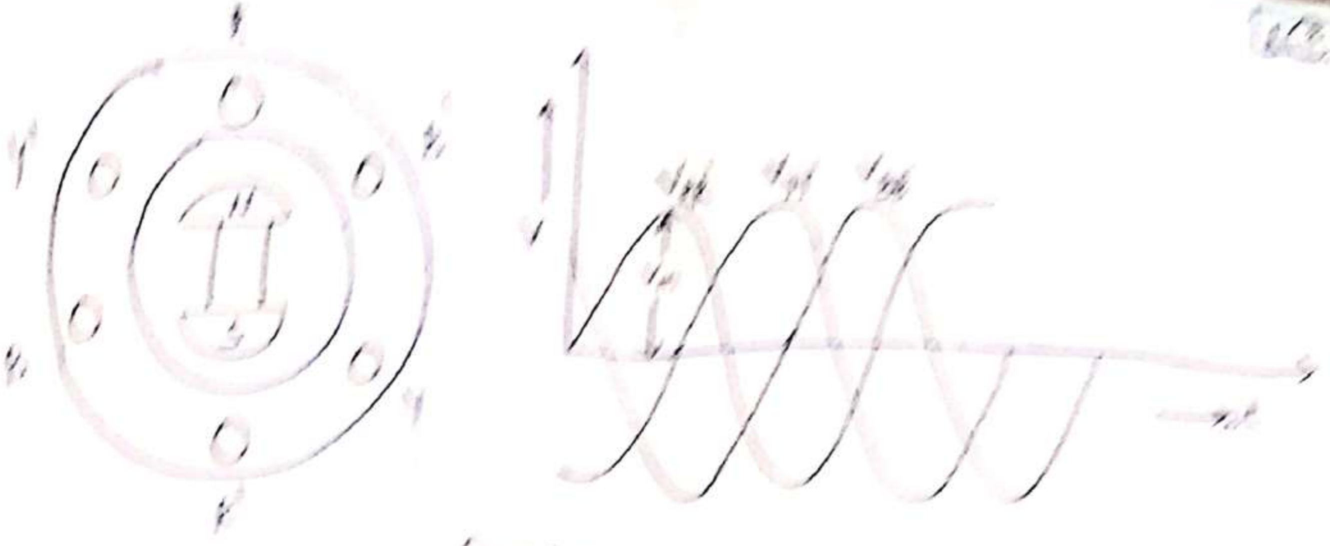


Fig 4(c)

The voltages generated by a three phase alternator is shown in fig 4(c). The three voltages are of same magnitude and frequency, but are displaced from one another by 120°. Assuming the voltage to be sinusoidal, we can write the equations for the instantaneous values of the voltages of the three phases. Counting the time from the instant when the voltage in phase R is zero, the equations are

$$V_{RR'} = V_p \sin \omega t$$

$$V_{YY'} = V_p \sin(\omega t - 120°)$$

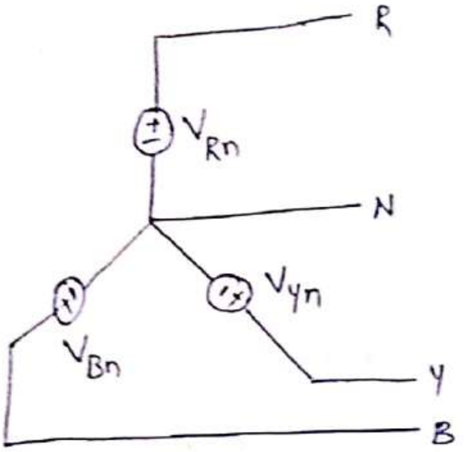
$$V_{BB'} = V_p \sin(\omega t - 240°)$$

Let us consider the Y-connected voltages in Fig 5(a) for now. The voltages V_{Rn}, V_{Yn} and V_{Bn} are respectively between lines R, Y, B and neutral line n. The voltages are called phase voltages. If the voltage sources have the same amplitude and frequency ω and are of out of phase with each other by 120°, the voltages are said to be balanced. This implies

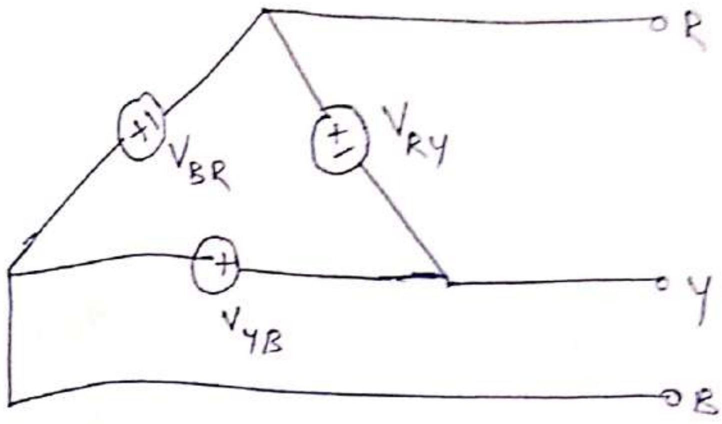
$$V_{Rn} + V_{Bn} + V_{Yn} = 0$$

$$|V_{Rn}| = |V_{Bn}| = |V_{Yn}|$$

*** Balanced phase voltages are equal in magnitude and are out of phase with each other by 120° ***



Three phase voltage sources
Y - Connected
Fig 1.5(a)



Three phase voltage sources
Δ - Connected.
Fig 1.5(b)

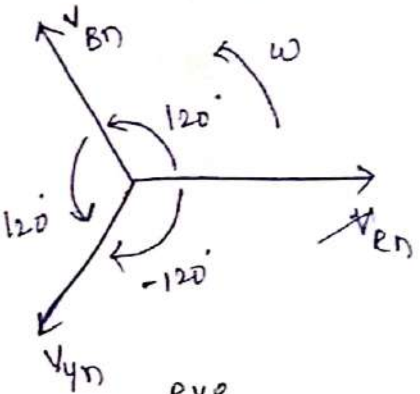
Fig

Since the three phase voltages are 120° out of phase with each other, there are two possible combinations. One possibility is shown in fig 1.6(a) and expressed mathematically as

$$V_{Rn} = V_p \angle 0^\circ$$

$$V_{yn} = V_p \angle -120^\circ$$

$$V_{Bn} = V_p \angle -240^\circ = V_p \angle 120^\circ$$



RYB
Fig 1.6(a) (abc) or +ve sequence

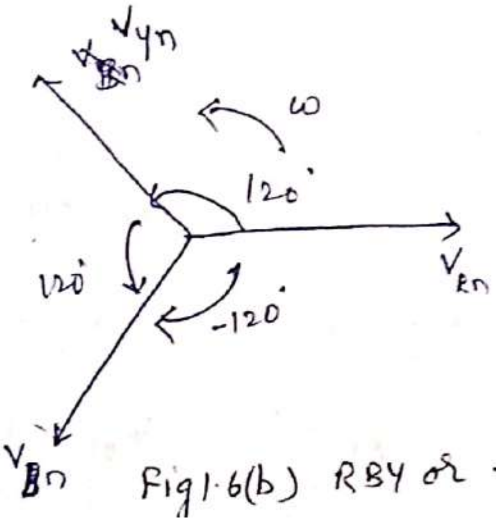


Fig 1.6(b) RBY or -ve sequence

$$V_{Rn} = V_p \angle 0^\circ$$

$$V_{yn} = V_p \angle -120^\circ$$

$$V_{Bn} = V_p \angle -240^\circ = V_p \angle 120^\circ$$

where V_p is the effective or rms value of phase voltages. This is known as RYB sequence or positive sequence. In this phase sequence V_{Rn} leads V_{yn} which inturn leads V_{Bn}. This sequence is produced when the rotor in fig 1.3(b) rotates counterclockwise. The other possibility is shown in fig 1.6(b). and is given by

$$V_{Rn} = V_p \angle 0^\circ$$

$$V_{Bn} = V_p \angle -120^\circ$$

$$V_{yn} = V_p \angle -240^\circ = V_p \angle +120^\circ$$

This is called RBY sequence or negative sequence. For this phase sequence, V_{Rn} leads V_{Bn} which in turn leads V_{yn}. The RBY sequence is produced when the rotor in fig 1.3(b) rotates in the clockwise direction.

*** The phase sequence is the time order in which the voltages pass through their respective maximum values. ***

In fig 1.6(a) as the phasors rotate in counterclockwise rotation with frequency ω, they pass through the horizontal axis in a sequence RYBRYBR...... Thus the sequence is RYB or YBR or BRY. Similarly for the phasors in fig 1.6(b) as they rotate in counterclock wise direction they pass the horizontal axis in a sequence

RBYRBYR..... This describes RBY sequence. It determines the direction of rotation of a motor connected to the power source for example.

Like the generator connections, a three phase load can be either Y-connected or delta connected depending on the end applications. Fig 1.7(a) shows Y-connected load, fig 1.7(b) shows Δ-connected load. The neutral line ing fig 1.7(a) may or maynot be there depending on whether the system is 4-wire or 3-wire.

*** A Y or Δ connected load is said to be unbalanced if the phase impedances are not equal in magnitude or phase ***

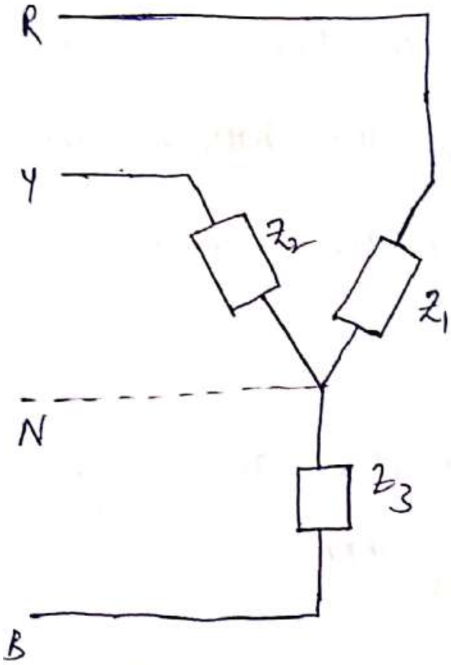


Fig 1.7(a)
Y-connected load

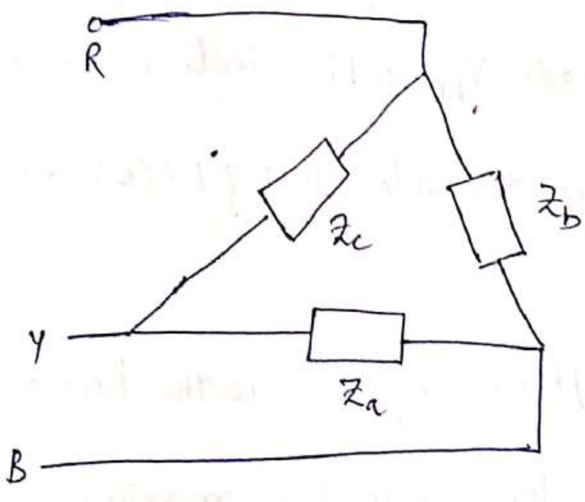


Fig 1.7(b)
Δ-connected load

*** A balanced load is one in which the phase impedances are equal in magnitude and in phase ***

For a balanced Y-Connected load

$$Z_1 = Z_2 = Z_3 = Z_Y$$

Z_Y is the load impedance per phase.

For a balanced Δ-Connected load

$$Z_a = Z_b = Z_c = Z_\Delta$$

where Z_Δ is the load impedance per phase.

Since both three phase source and three phase load can be either Y or Δ Connected, we have four possible connections

* Y-Y (Y-Connected source with Y-Connected load)
* Y-Δ connection
* Δ-Δ Connection
* Δ-Y Connection

Star to delta and delta to Star Transformation :-

while dealing with currents and voltages in loads, it is often necessary to convert a star load to delta load and vice versa. we know that Δ Connection of resistances can be replaced by an equivalent Y connection and vice versa. Similar methods can be applied in case of networks containing general impedances in complex form. So also with AC where some formulae hold good except the resistances are replaced by the impedance. Thus Considering fig 1.8(a) star load can be replaced by an equivalent delta load with impedances as shown

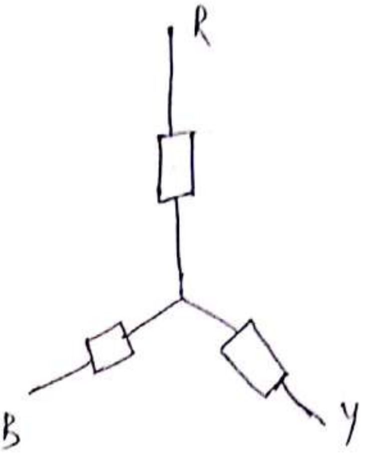


Fig 1.8(a)

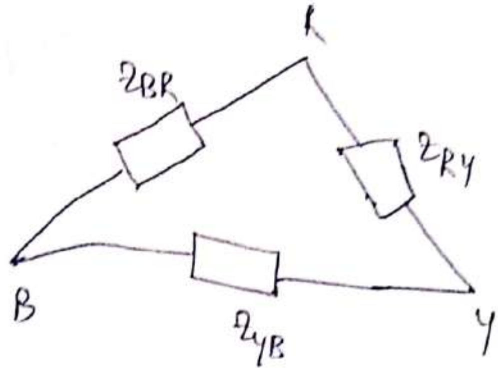


Fig 1.8(b)

Delta impedances in terms of star impedances are

$$Z_{RY} = \frac{Z_R Z_Y + Z_Y Z_B + Z_B Z_R}{Z_B}$$

$$Z_{YB} = \frac{Z_R Z_Y + Z_Y Z_B + Z_B Z_R}{Z_R}$$

$$Z_{BR} = \frac{Z_R Z_Y + Z_Y Z_B + Z_B Z_R}{Z_Y}$$

The converted network is shown in fig 1.8(b). Similarly we can replace the delta load of Fig 1.8(b) by an equivalent star load with branch impedances as

$$Z_R = \frac{Z_{RY} Z_{BR}}{Z_{RY} + Z_{YB} + Z_{BR}}$$

$$Z_Y = \frac{Z_{RY} Z_{YB}}{Z_{RY} + Z_{YB} + Z_{BR}}$$

$$Z_B = \frac{Z_{BR} Z_{YB}}{Z_{RY} + Z_{YB} + Z_{BR}}$$

It should be noted that all impedances are to be expressed in their complex form.

If three phase load is balances then the conversion formulae get simplified. Consider a balanced Y-connected load having an impedance Z_1 in each phase and let the equivalent delta connected load have an impedance of Z_2 in each phase as shown in fig 1.9(b)

Applying conversion formulae

delta impedances in terms of star impedances, we have

$$Z_2 = 3Z_1$$

Similarly we can express star impedance in terms of delta

$$Z_1 = Z_2/3$$

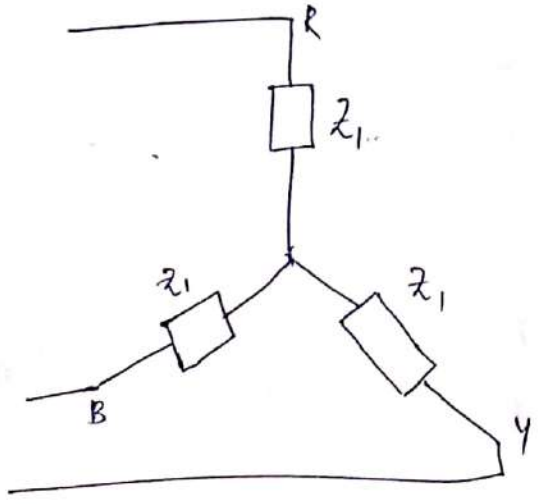


fig 1.9(a)

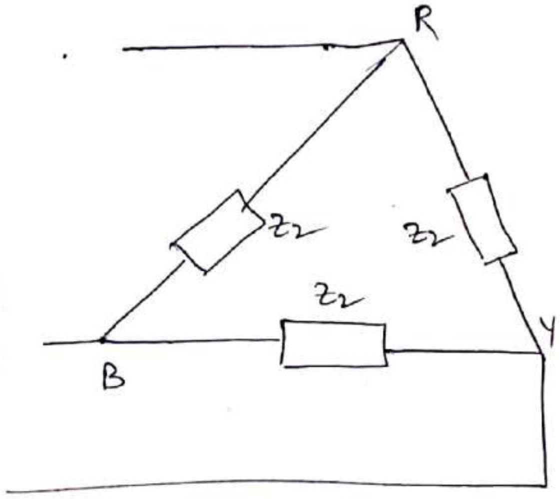


Fig 1.9(b)

Voltage, current and power in a Star Connected System:-

star Connected System :-

Fig 1.10 shows a balanced three phase Y-connected system. The voltage induced in each winding is called the phase voltage (V_{ph}). Likewise V_{RN}, V_{YN} and V_{BN} represent the rms values of the induced voltages in each phase. The voltage available between any pair of terminals is called the line voltage (V_L). Likewise V_{RY}, V_{YB} and V_{BR} are known as line voltages. The double subscript notation is purposefully used to represent voltages and currents in polyphase circuits. Thus V_{RY} indicates the voltage V between points R and Y with R being positive with respect to point Y during its positive half cycle.

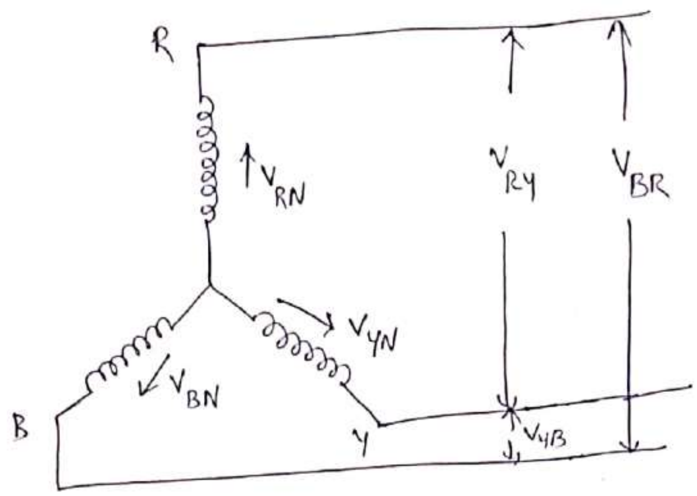


Fig 1.10

Similarly V_{YB} means that Y is positive with respect to point B during its positive half cycle, it also means that

$V_{RY} = -V_{YR}$.

Voltage Relation:-

The phasors corresponding to the phase voltages constituting a three phase system can be represented by a phasor diagram as shown in fig 1.11

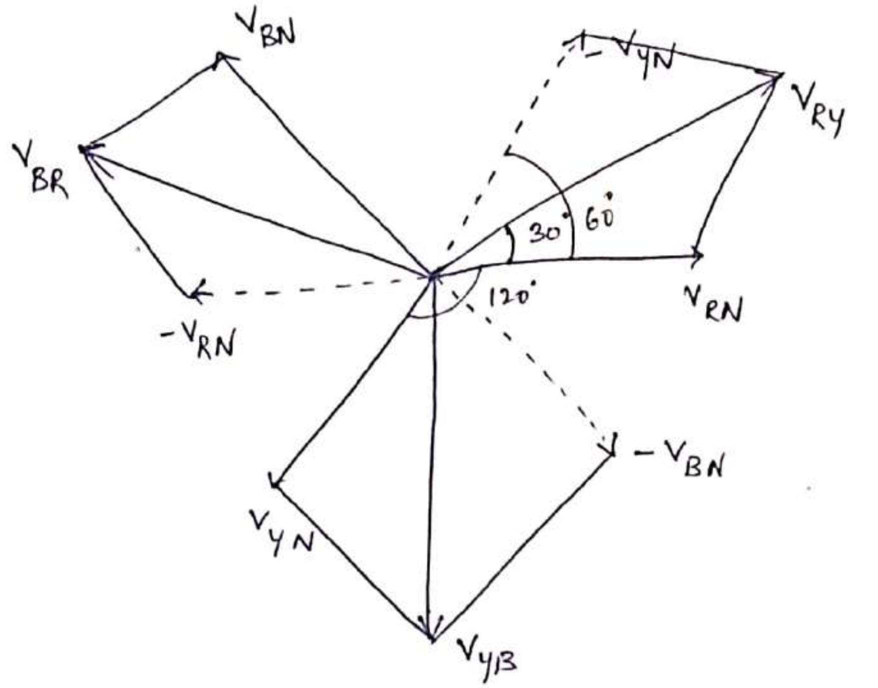


Fig 1.11

From fig 1.11 considering the lines R, Y, B, the line voltages V_{RY} is equal to phasor sum of V_{RN} and V_{NY} which is also Phasor difference of V_{RN} and V_{YN} ($V_{NY} = -V_{YN}$). Hence V_{RY} is found by compounding V_{RN} and V_{YN} reversed. To subtract V_{YN} from V_{RN}, we reverse the phasor V_{YN} and find its phasor sum with V_{RN} as shown in fig. The two phasors V_{RN} and $-V_{YN}$ are equal in length and are 60° apart.

$$|V_{RN}| = -|V_{YN}| = V_{ph}$$

$$V_{RY} = \sqrt{V_{RN}^2 + V_{YN}^2 + 2V_{RN}V_{YN}\cos 60^\circ}$$

$$= \sqrt{V_{ph}^2 + V_{ph}^2 + \cancel{2} \times V_{ph}^2 \times \frac{1}{\cancel{2}}} = \sqrt{3}\,V_{ph}$$

$$V_{RY} = \sqrt{3}\,V_{ph}$$

Similarly $V_{YB} = \sqrt{3}V_{ph}$, $V_{BR} = \sqrt{3}V_{ph}$

Hence in a balanced star connected system

(i) Line voltage = $\sqrt{3}$ times phase voltage

$$V_L = \sqrt{3}V_{ph}$$

(ii) All line voltages are equal in magnitude and are displaced by 120°

(iii) All line voltages are 30° ahead of their respective phase voltages.

Current Relations :-

From fig 1.12(a) shows a balanced three phase, Y-connected system indicating phase currents and line currents. The arrows placed alongside the currents, I_R, I_Y and I_B flowing in three phases indicate the directions of currents when they are assumed to be positive and not the directions at that particular instant. The phasor diagram for phase currents with respect to their phase voltages is shown in fig 1.12(b). All the phase currents are displaced by 120° with respect to each other, 'ϕ' is phase angle between phase voltage and phase current (lagging load is assumed). For a balanced load, all the phase currents are equal in magnitude. It can be observed from fig 1.12(a) that each line conductor is connected in series with its individual phase winding. Therefore the current in a line conductor is same as that in the phase to which the line conductor is connected.

$\therefore \quad I_L = I_{ph} = I_R = I_Y = I_B$

It can be observed that from fig 1.12(b) the angle between the line (phase) current and the corresponding line voltage is $(30+\phi)^\circ$ for lagging load. Consequently if the load is leading then the angle between the line (phase) current and corresponding line voltage will be $(30-\phi)^\circ$

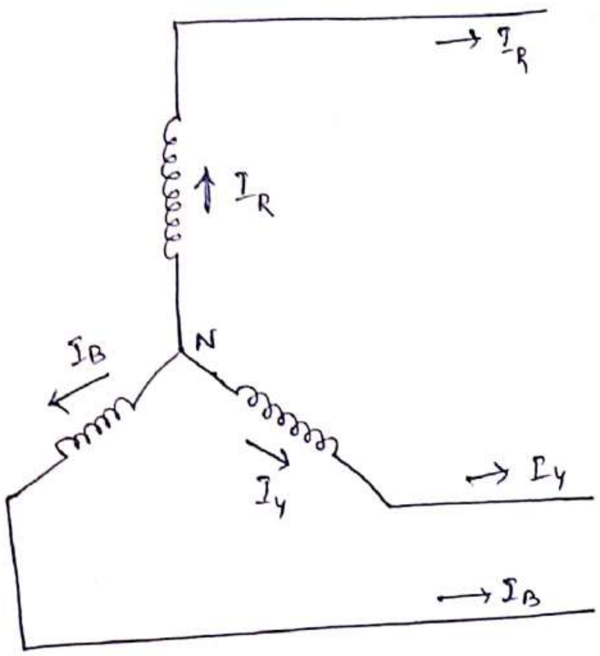


Fig 1.12(a)

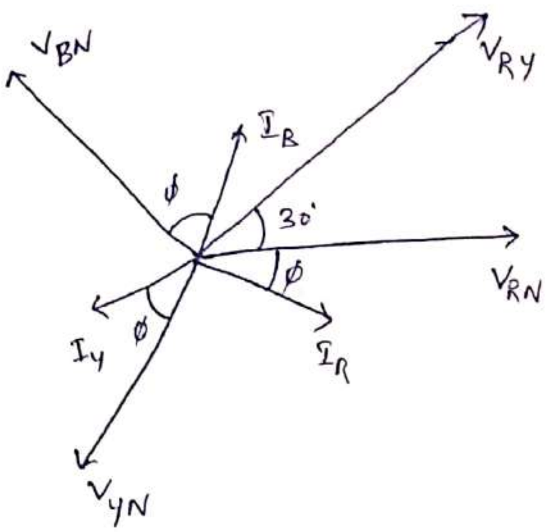


Fig 1.12(b)

Power in the Star Connected Network :-

The total active power or true power in the three phase load is the sum of the powers in the three phases. For a balanced load, power in each phase is same, hence

total power = 3 × power in each phase

$$P = 3 \times V_{ph} \times I_{ph} \cos\phi$$

It is usual practice to express the three phase power in terms of line quantities as follows.

$$V_L = \sqrt{3}\, V_{ph}$$

$$I_L = I_{ph}$$

$$P = \sqrt{3}\, V_L I_L \cos\phi \ \text{W}$$ or active power

Total reactive power is given by

$$Q = \sqrt{3}\, V_L I_L \sin\phi \ \text{VAR}$$

Total apparent power or Volt-amperes

$$= \sqrt{3}\, V_L I_L \ \text{VA}$$

N-Phase Star System :-

It is to be noted that star and mesh are general terms applicable to any no. of phases, but wye and delta are special cases of Y and mesh when the system is a three phase system. Consider an n-phase balanced star system with two adjacent phases as shown in fig 1.13 (a). Its vector diagram is shown in fig 1.13(b)

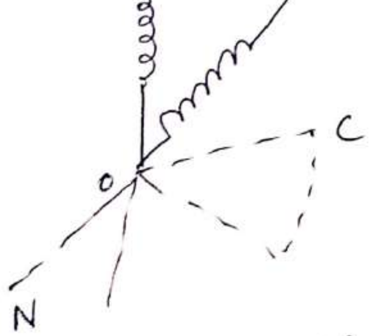


Fig 1.13(a)

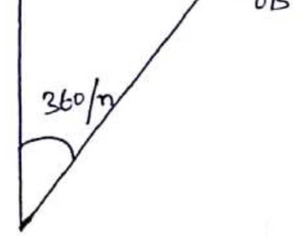


Fig 1.13(b)

The angle of phase difference between adjacent phase voltages is $360°/n$. Let E_{ph} be the voltage of each phase. The line voltage i.e., the voltage between A and B is equal to $E_{AB} = E_L = E_{AO} + E_{OB}$. The vector addition is shown in fig 1.13(c)

It is evident that line current and phase current are same

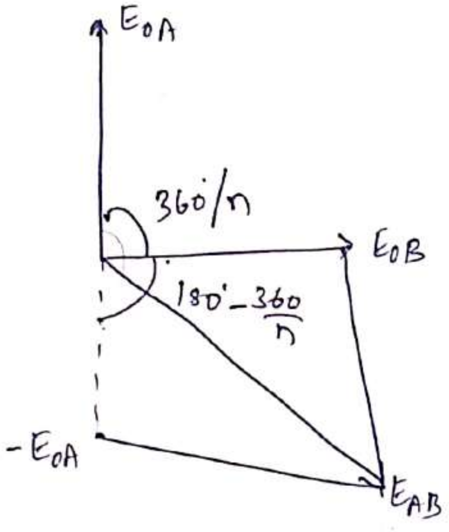


Fig 1.13(c)

$$E_{AB} = E_{AO} + E_{OB}$$

$$E_{AB} = \sqrt{E_{OB}^2 + E_{OA}^2 + 2E_{OA}E_{OB}\cos\left(180° - \frac{360}{n}\right)}$$

$$= \sqrt{E_{ph}^2 + E_{ph}^2 - 2E_{ph}^2\cos(360°/n)} = \sqrt{E_{ph}^2 + E_{ph}^2 - 2E_{ph}^2\cos 2\times\frac{180°}{n}}$$

$$= \sqrt{E_{ph}^2 + E_{ph}^2 - 2E_{ph}^2\cos 2\left(\frac{180°}{n}\right)} = \sqrt{2E_{ph}^2 - 2E_{ph}^2\cos 2\left(\frac{180°}{n}\right)}$$

$$= \sqrt{2}E_{ph}\sqrt{1 - \cos 2\left(\frac{180°}{n}\right)} = \sqrt{2}E_{ph} \times \sqrt{2}\sin\frac{180°}{n}$$

$$= 2E_{ph}\sin\frac{180°}{n}$$

$$E_L = 2E_{ph}\sin\frac{180°}{n}$$

If $n = 3$, $E_L = \sqrt{3}E_{ph}$

Voltage, Current and Power in a Delta Connected System :-

Delta Connected System:-

Fig 1.14 shows a balanced three phase, three-wire delta Connected system. This arrangement is referred to as mesh connection because it forms a closed circuit. It is also known a delta connection because the three branches in the circuit can also be arranged in the form of delta (Δ)

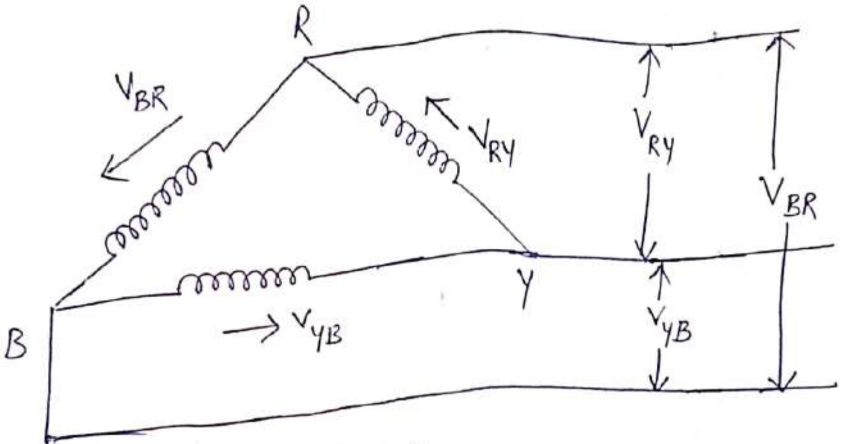


Fig 1.14

From the manner of interconnection of three phases in the circuit, it may appear that the three phases are short circuited among themselves. However this is not the case. Since the system is balanced, the sum of the three voltages round the closed mesh is zero; consequently no current can flow around the mesh when the terminals are open.

The arrows placed along side the voltages, V_{RY}, V_{YB} & V_{BR} of the three phases indicate that the terminals R, Y & B are positive with respect to Y, B and R respectively during their respective +ve half cycles.

Voltage Relation:-

From fig 1·15 we notice that only one phase is connected between any two lines. Hence the voltage between any two lines (V_L) is equal to the phase voltage (V_{ph})

$$V_{RY} = V_L = V_{ph}$$

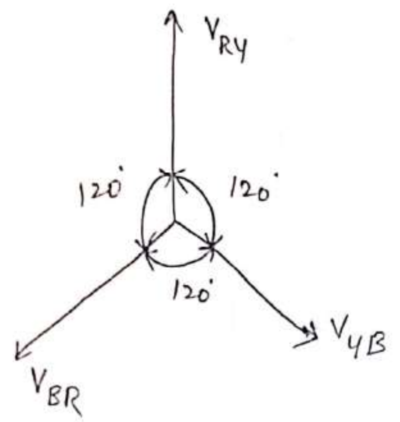


Fig 1·15

Since the system is balanced, all the phase voltages are equal but displaced by 120° from one another as shown in phasor diagram of fig 1·15. The phase sequence RYB is assum

$$|V_{RY}| = |V_{YB}| = |V_{BR}| = V_L = V_{ph}$$

Current Relation:-

In Fig 1·16(a) since the system is balanced, the three phase currents (I_{ph}) i,e., I_{RY}, I_{YB}, I_{BR} are equal in magnitude but are displaced by 120° from one another as shown in fig 1·16(b). I_R, I_Y and I_B are line currents (I_L) i,e., I_R is the line current in line connected to the common point R. Similarly I_Y and I_B are the line currents in lines Y and B connected to common points Y & B respectively. Though here all line currents are directed outwards at no instant will all the line currents flow in the same direction either outward or inwards. Because the three line currents are displaced 120° from one another, when one is positive, the other two might both be negative or one positive and one negative. Also it is to be noted that arrows placed alongside phase currents in fig 1.16(a) indicate the direction of currents when they are assumed to be positive and not their actual direction at a particular instant. We can easily determine the line currents in fig 1.16(a), I_R, I_Y & I_B by applying KCL at the three terminals R, Y and B respectively. thus the current in line R, $I_R = I_{RY} - I_{BR}$ i.e., current in any line is equal to the phasor difference of currents in the two phases attached to that line. Similarly the current in line Y, $I_Y = I_{YB} - I_{RY}$ and the current in line B, $I_B = I_{BR} - I_{YB}$

The phasor addition of these currents is shown in fig 1.16(b).

From the figure

$$I_R = I_{RY} - I_{BR}$$

$$I_R = \sqrt{I_{RY}^2 + I_{BR}^2 + 2 I_{RY} I_{BR} \cos 60^\circ}$$

$$I_R = \sqrt{3}\, I_{ph} \quad \text{since } I_R = I_B = I_{ph}$$

Similarly the remaining two lines currents, I_Y and I_B are also equal to $\sqrt{3}$ times phase currents $I_L = \sqrt{3}\, I_{ph}$

As can be seen from fig 1.16(b) all line currents are equal in magnitude but displaced by 120° from one another and line currents are 30° behind the respective phase currents.

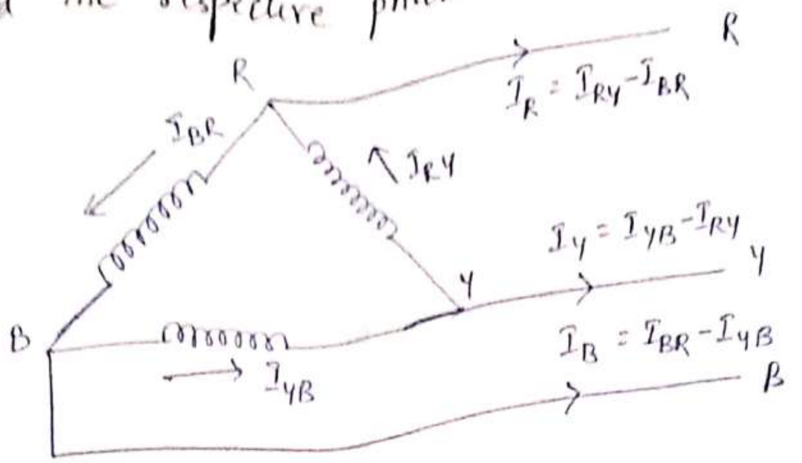


Fig 1.16(a)

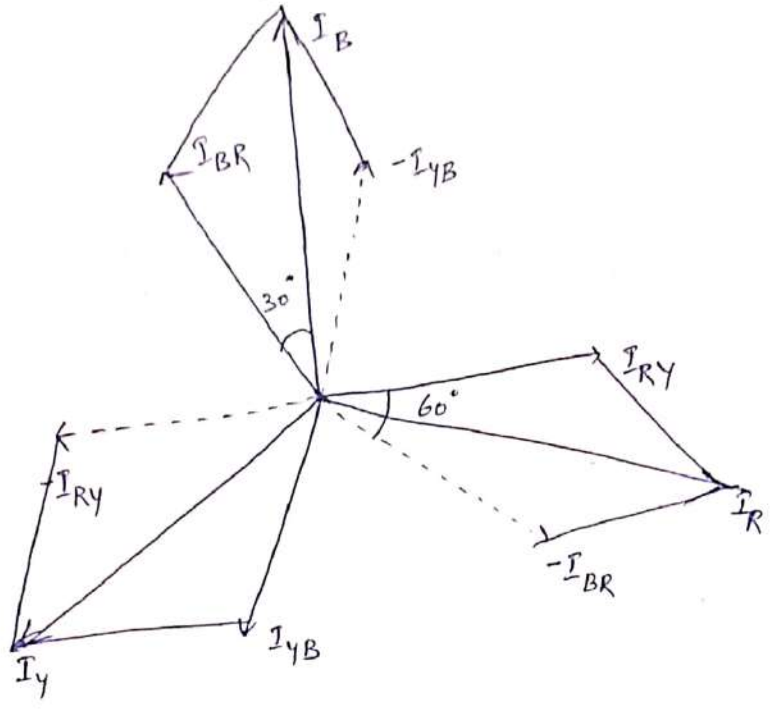


Fig 1.16(b)

Power in the Delta Connected System:-

obviously total power in the delta circuit is the sum of the power in the three phases. Since the load is balanced, the power consumed in each phase is the same. Total power is equal to three times the power in each phase.

$$\text{Power per phase} = V_{ph} I_{ph} \cos\phi$$

where ϕ is the phase angle between phase voltage and phase current.

Total power $P = 3 \times V_{ph} I_{ph} \cos\phi$

In terms of line quantities

$$P = \sqrt{3}\, V_L I_L \cos\phi \quad W$$

and $V_{ph} = V_L$ & $I_{ph} = \frac{I_L}{\sqrt{3}}$

For a balanced system, whether star or delta, the expression for total power is the same.

N Phase Mesh System :-

Fig 1.17(a) shows part of n-phase balanced mesh system. Its vector diagram is shown in fig 1.17(b)

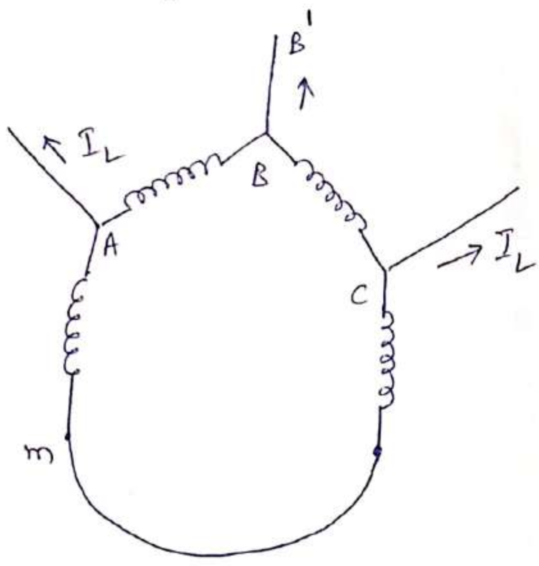


Fig 1.17(a)

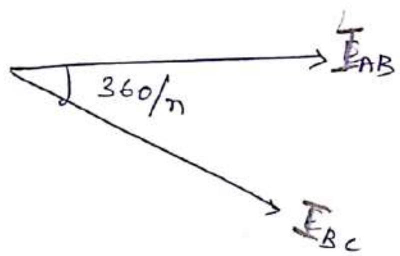


Fig 1.17(b)

Let the current in line BB' be I_L. This is same in all the remaining lines of the n phase system. I_{AB}, I_{BC} are the phase currents in AB and BC phases respectively. The vector addition of the line current is shown in fig 1.17(c). It is evident from fig 1.17(b) that the line & phase voltages are equal.

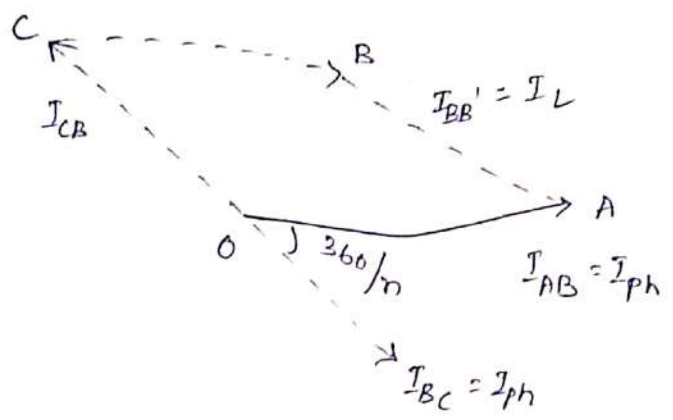


Fig 1.17(c)

$$I_{BB} = I_L = I_{AB} + I_{CB}$$
$$= I_{AB} - I_{BC}$$

Consider the parallelogram OABC

$$OB = \sqrt{OA^2 + OC^2 + 2 \times OA \times OC \cos\left(180 - \frac{360}{n}\right)}$$

$$= \sqrt{I_{ph}^2 + I_{ph}^2 - 2I_{ph}^2 \cos\frac{360°}{n}} = \sqrt{2} I_{ph} \sqrt{1 - \cos 2\frac{180°}{n}}$$

$$= \sqrt{2} I_{ph} \sqrt{2 \sin^2 \frac{180}{n}} = 2 I_{ph} \sin\frac{180}{n}$$

$$I_L = 2 I_{ph} \sin\frac{180}{n}$$

The above equation is a general equation for the line current in a balanced n phase mesh system.

Analysis of Balanced Three Phase Circuits :-

Balanced Three Phase System Delta Load:-

Fig 1.18 (a) shows a three phase, three wire, balanced system supplying power to a balanced three phase delta load. The phase sequence is RYB.

Let us assume the line voltage $V_{RY} = V\angle 0^\circ$ as the reference phasor. Then the three source voltages are given by

$$V_{RY} = V\angle 0^\circ$$
$$V_{YB} = V\angle -120^\circ$$
$$V_{BR} = V\angle -240^\circ$$

These voltages are represented by phasors in fig 1.18(b). Since the load is delta connected, the line voltage of the source is equal to the phase ~~and~~ voltage of the load. The current in phase RY, I_R will lag(lead) behind (ahead of) the phase voltage V_{RY} by an angle ϕ as directed by the nature of the load impedance. The angle of lag of I_Y with respect to V_{YB} as well as the angle of lag of I_B with respect to V_{BR} will be ϕ as the load is balanced. All these quantities are represented in fig 1.18(b)

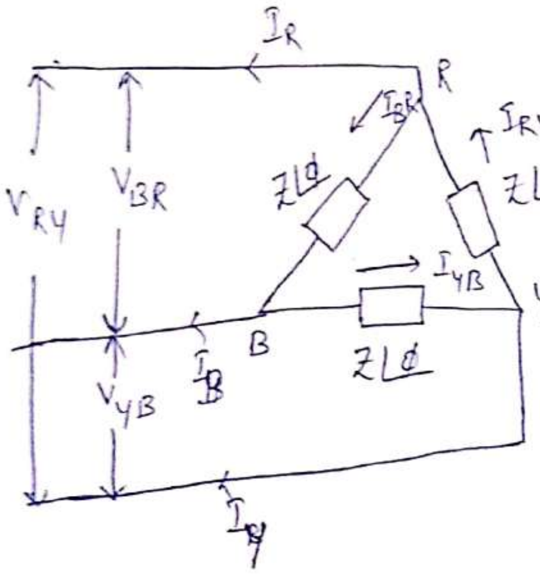


Fig 1.18(a)

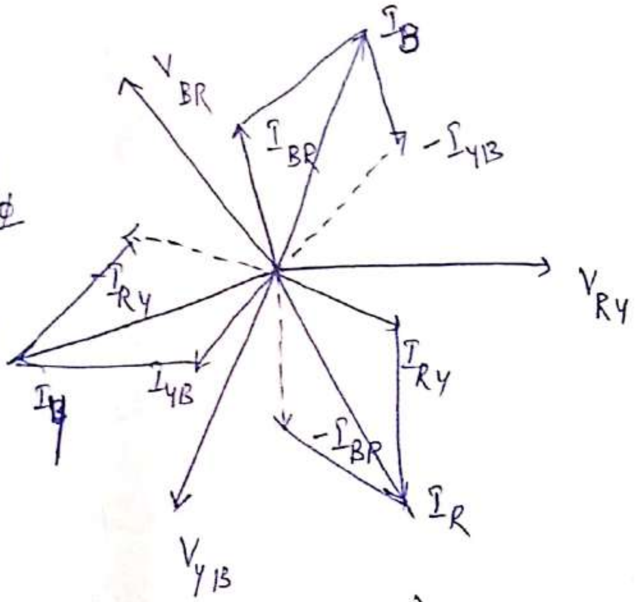


Fig 1.18(b)

If the load impedance is $Z\angle\phi$, the current flowing in the three load impedances are then

$$I_{RY} = \frac{V_{RY}\angle 0^\circ}{Z\angle\phi} = \frac{V}{Z}\angle -\phi$$

$$I_{YB} = \frac{V_{YB}\angle -120^\circ}{Z\angle\phi} = \frac{V}{Z}\angle -120^\circ-\phi$$

$$I_{BR} = \frac{V_{BR}\angle -240^\circ}{Z\angle\phi} = \frac{V}{Z}\angle -240-\phi$$

The line currents are $\sqrt{3}$ times phase currents and are 30° behind their respective phase currents.

$$I_R = \sqrt{3}\left|\frac{V}{Z}\right|\angle -\phi-30^\circ \quad \text{or } I_{RY}-I_{BR} \text{ (Phasor difference)}$$

$$I_Y = \sqrt{3}\left|\frac{V}{Z}\right|\angle -120^\circ-\phi-30^\circ \quad \text{or } I_{YB}-I_{RY}$$

$$= \sqrt{3}\left|\frac{V}{Z}\right|\angle(-150^\circ-\phi)^\circ$$

$$I_B = \sqrt{3}\left|\frac{V}{Z}\right|\angle -240^\circ-\phi-30^\circ \quad \text{or } I_{BR}-I_{YB}$$

$$= \sqrt{3}\left|\frac{V}{Z}\right|\angle(-270^\circ-\phi)^\circ$$

Balanced Three phase System - Star Connected load :-

Fig 1.19(a) shows a three phase, three wire system supplying power to a balanced three phase star connected load. The phase sequence RYB is assumed.

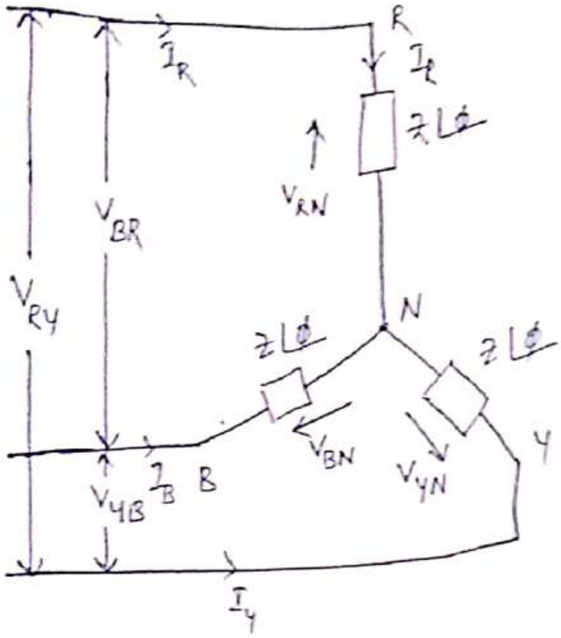


Fig 1.19(a)

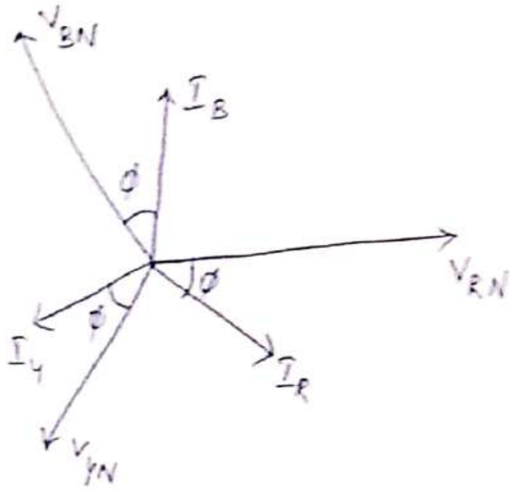


Fig 1.19(b)

In star connection, whatever current is flowing in the phase is also flowing in the line. The three line currents are I_R, I_Y & I_B

V_{RN}, V_{YN} and V_{BN} represent three phase voltages of the network i.e., the voltage between any line and neutral. Let us assume the voltage $V_{RN} = V\angle 0^\circ$ as the reference phasor. Consequently the phase voltage

$$V_{RN} = V\angle 0^\circ$$

$$V_{YN} = V\angle -120^\circ$$

$$V_{BN} = V\angle -240^\circ$$

Hence

$$I_R = \frac{V_{RN}}{Z\angle\phi} = \frac{V\angle 0^\circ}{Z\angle\phi} = \left|\frac{V}{Z}\right|\angle -\phi$$

$$I_Y = \frac{V_{YN}}{Z\angle\phi} = \frac{V\angle -120^\circ}{Z\angle\phi} = \left|\frac{V}{Z}\right|\angle(-120^\circ-\phi)^\circ$$

$$I_B = \frac{V_{BR}}{Z\angle\phi} = \frac{V\angle -240^\circ}{Z\angle\phi} = \left|\frac{V}{Z}\right|\angle(-240^\circ-\phi)^\circ$$

As seen from the above expressions, the currents I_R, I_Y & I_B are equal in magnitude and have a 120° phase difference. The disposition of these vectors is shown in fig 1.19(b). Sometimes a fourthwire called neutral wire is run from the neutral point if the source is also star connected. This gives three phase four wire star connected system. However if the three line currents are balanced, the current in fourth wire is zero; removing this connecting wire between the source neutral and load neutral is therefore not going to make any change in the condition of the system. The availability of the neutral wire makes it possible to use all the three phase voltages, as well as the three line voltages. Usually neutral is grounded for safety and for the design of insulation

Analysis of Unbalanced Three Phase Circuits :-

Types of Unbalanced loads:-

An unbalance exists in a circuit when the impedances in one or more phases differ from the impedances of the other phases. In such a case, line or phase currents are different and are displaced from one another by unequal angles. So far we have considered balanced loads connected to balanced systems. It is enough to solve problems, considering one phase only on balanced loads; the conditions on other two phases being similar. Problems on unbalanced three phase loads are difficult to handle because conditions in three phases are different. However, the source voltages are assumed to be balanced. If the system is three wire system the currents flowing towards the load in the three lines must add to zero at any given instant. If the system is a four wire system, the sum of the three outgoing line currents is equal to the return current in the neutral wire.

Following are the unbalanced loads

(i) Unbalanced delta connected load

(ii) Unbalanced three wire star connected load and

(iii) Unbalanced four wire star connected load

Unbalanced Delta Connected Load:-

Fig 1.20 shows an unbalanced delta load connected to a balanced three phase supply.

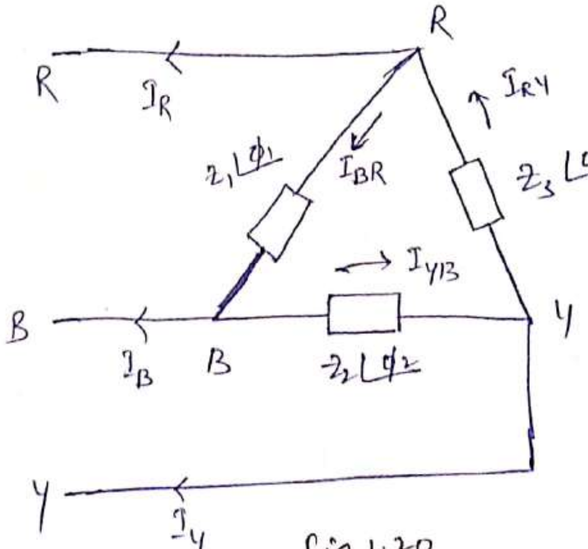


Fig 1.20

The unbalanced delta connected load supplied from a balanced three phase supply does not present any new problems because the voltage across the load phase is fixed. It is independent of the nature of load and is equal to the line voltage of the supply

The current in each load phase is equal to line voltage divided by the impedance of that phase. The line current will be phasor difference of corresponding phase currents taking V_{RY} as reference phasor.

Assuming RYB phase sequence, we have

$$V_{RY} = V\angle 0^\circ, \quad V_{YB} = V\angle -120^\circ, \quad V_{BR} = V\angle -240^\circ$$

Phase currents are

$$I_{RY} = \frac{V_{RY}}{Z_1\angle\phi_1} = \frac{V\angle 0^\circ}{Z_1\angle\phi_1} = \left|\frac{V}{Z_1}\right| \angle -\phi_1^\circ \text{ A}$$

$$I_{YB} = \frac{V_{YB}}{Z_2\angle\phi_2} = \frac{V\angle -120^\circ}{Z_2\angle\phi_2} = \left|\frac{V}{Z_2}\right| \angle(-120^\circ - \phi_2)^\circ \text{ A}$$

$$I_{BR} = \frac{V_{BR}}{Z_3\angle\phi_3} = \frac{V\angle -240^\circ}{Z_3\angle\phi_3} = \left|\frac{V}{Z_3}\right| \angle(-240^\circ - \phi_3)^\circ \text{ A}$$

The three line currents are

$$I_R = I_{RY} - I_{BR}$$

$$I_Y = I_{YB} - I_{RY}$$

$$I_B = I_{BR} - I_{YB}$$

Unbalanced Four Wire Star Connected Load:-

Fig 1.21 shows an unbalanced star load connected to a balanced 3 phase 4 wire supply.

The star point N_L, of the load is connected to the star point N_s of the supply. It is the simplest case of an unbalanced load because of the presence of the neutral wire; the star points of the supply N_s (generator) and the load N_L are at the same potential

It means that the voltage across each load impedance is equal to the phase voltage of the supply (generator) i.e., the voltages across the three load impedances are equalised even though load impedances are unequal. However the current in each phase (or line) will be different. Obviously the vector sum of the currents in three lines is not zero; but is equal to neutral current.
Phase currents can be calculated in similar way as that followed in an unbalanced delta connected load.

Taking the phase voltage $V_{RN} = V\angle 0^\circ$ V as reference and assuming RYB phase sequences, we have the three phase voltages as follows

$$V_{RN} = V\angle 0^\circ, \quad V_{YN} = V\angle -120^\circ, \quad V_{BN} = V\angle -240^\circ$$

The phase currents are

$$I_R = \frac{V_{RN}}{Z_1} = \frac{V\angle 0^\circ}{Z_1\angle \phi_1} = \left|\frac{V}{Z_1}\right| \angle -\phi_1$$

$$I_Y = \frac{V_{YN}}{Z_2} = \frac{V\angle -120^\circ}{Z_2\angle \phi_2} = \left|\frac{V}{Z_2}\right| \angle (-120^\circ - \phi_2)$$

$$I_B = \frac{V_{BN}}{Z_3} = \frac{V\angle -240^\circ}{Z_3\angle \phi_3} = \left|\frac{V}{Z_3}\right| \angle (-240^\circ - \phi_3)$$

Incidentally, I_R, I_Y & I_B are also the line currents, the current in neutral wire is the vector sum of three line currents.

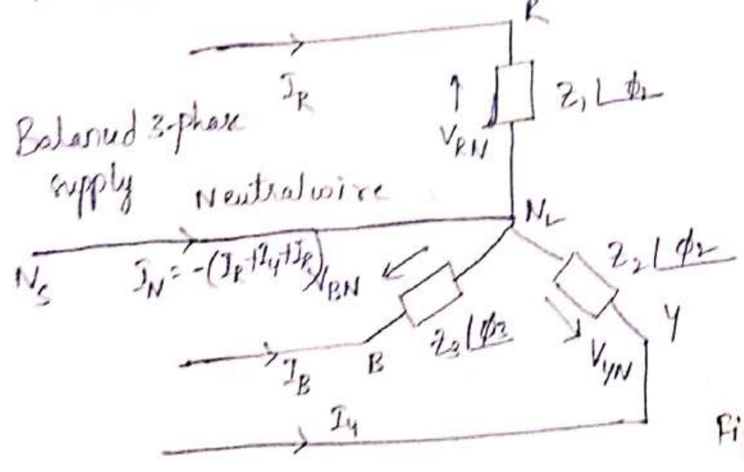


Fig 1.21

Unbalanced Three Wire Star Connected load:

In a three phase four wire system if the connection between supply neutral and load neutral is broken, it would result in an unbalanced three wire star load. This type of load is rarely found in practice, because all the three wire star loads are balanced. Such a system is shown in fig 1.22. Note that the supply star point (N_s) is isolated from the load star point (N_L). The potential of the load star point is different from that of the supply star point. The result is that the load phase voltages is not equal to supply phase voltage and they are not only unequal in magnitude but also subtend angles other than 120° with one another. The magnitude of each phase voltage depends upon the individual phase loads. The potential of the load neutral point changes according to changes in the impedances of the phases, that is why sometimes the load neutral is also called a floating neutral point. All star connected, unbalanced loads supplied from polyphase systems without a neutral wire have floating neutral point. The phasor sum of the three unbalanced line currents is zero. The phase voltage of the load is not $1/\sqrt{3}$ of the line voltage. The unbalanced three wire star load is difficult to deal with. It is because load phase voltages cannot be determined directly

The given supply line voltages. There are many methods to Solve such unbalanced Y-Connected loads. Two frequently used methods are

(i) Star-delta Conversion method and

(ii) Application of Millman's theorem.

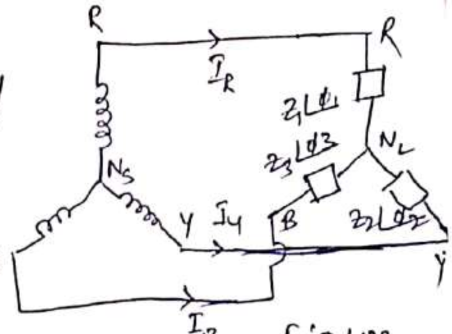


Fig 1.22

Measurement of Active and Reactive Power:-

Power Measurement in a Single Phase Circuit by Wattmeter:-

Wattmeters are generally used to measure power in the circuit. A wattmeter principally consists of two coils, one coil is called the current coil and the other the pressure or Voltage Coil. A diagramatic representation of a wattmeter connected to measure power in a single phase circuit as shown in fig 1.23

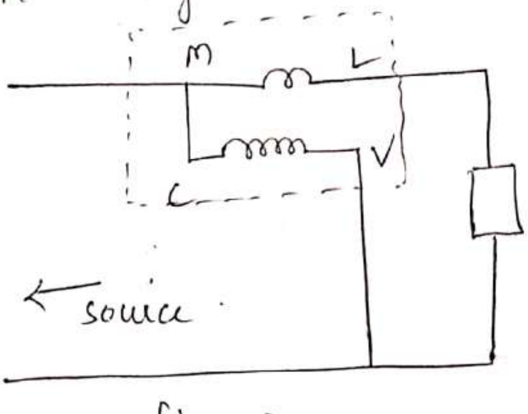


Fig 1.23

The coil represented with less no. of turns between M and L is the current coil, which carries the current in the load and has very low impedance. The coll with more number of turns between the common terminal (comm) and V is the pressure coil which is connected across the load and has high impedance. The load voltage is impressed across the pressure coil. The terminal M denotes the mains side, L denotes load side, Common denotes the common point of current coil and pressure coil and V denotes the second terminal of the pressure coil, usually selected as per the range of the load voltage in the circuit. From the fig 1.23 it is clear that the wattmeter has four terminals, two for current coil and two for potential coil. When the current flow through the two coils, they set up magnetic fields in space. An electromagnetic torque is produced by the interaction of the two magnetic fields. Under the influence of the torque one of the coils moves on a calibrated scale against the action of a spring. The instantaneous torque produced by electromagnetic action is proportional to the product of instantaneous values of currents in the two coils. The small current in the pressure coil is equal to the input voltage divided by the impedance of the pressure coil. The inertia of the moving system ~~denotes~~ does not permit it to follow the instataneou fluctuations in torque. The wattmeter deflection is proportional to the average power ($VI\cos\phi$) delivered to the circuit. Sometimes wattmeter connected to the circuit gives downscale reading or backward deflection. This is due to improper connection of current coil ~~and~~ pressure coil.

To obtain upscale reading, the terminal marked as

Comm' of the pressure coil is connected to one of the terminals of the current coil as shown in fig 1.24. Note that the connection between the current coil terminal and pressure coil terminal is not inherent but has to made externally. Even with proper connections, sometimes the wattmeter will give down scale reading whenever the phase angle between the voltage across the pressure coil and current through the current coil is more than 90°. In such a case connection of either current coil or pressure coil must be reversed.

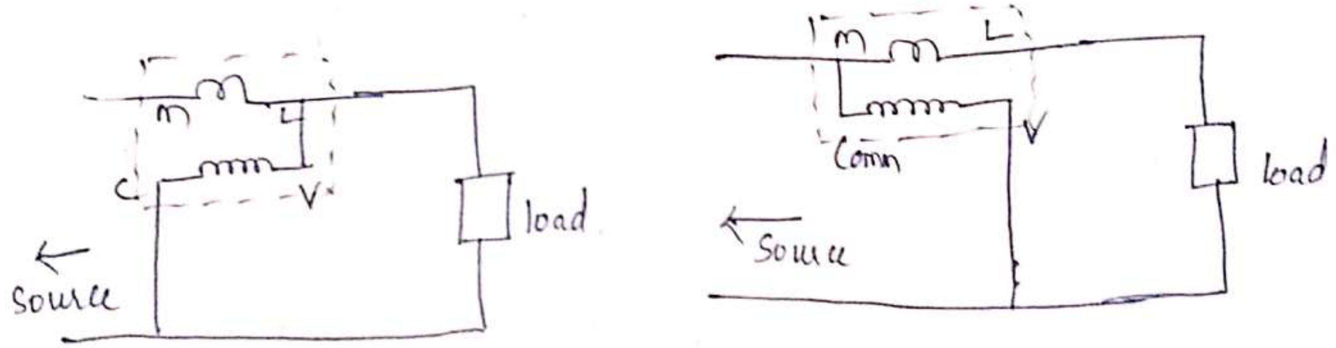


Fig 1.24

Three and Two Wattmeter Method:-

In this method, the three wattmeters are connected in ~~terms~~ three lines as shown in fig 1.25 i,e, the current coils of three wattmeters are introduced in the three lines and one terminal of each potential coil is connected to one terminal of the corresponding current coil, the other three being connected to some common point which forms an effective neutral n.

The load may be either star connected or delta connected.

Let us assume a star connected load, and let the neutral of this load be denoted by N. Now the reading on the wattmeter W_R will correspond to the average value of the product of instantaneous value of the current I_R flowing in line 1 with the voltage drop V_{Rn}, where V_{Rn} is the voltage between points R and n. This can be written as $V_{Rn} = V_{RN} + V_{Nn}$, where V_{RN} is the load phase voltage and V_{Nn} is the voltage between load neutral N, and the common point n. Similarly $V_{Yn} = V_{YN} + V_{Nn}$ and $V_{Bn} = V_{BN} + V_{Nn}$. Therefore average power W_R indicated by the wattmeter is given by

$$W_R = \frac{1}{T}\int_0^T V_{Rn} I_R \, dt$$

where T is the time period of the voltage wave

$$W_R = \frac{1}{T}\int_0^T (V_{RN} + V_{Nn}) I_R \, dt$$

Similarly
$$W_Y = \frac{1}{T}\int_0^T V_{Yn} I_Y \, dt = \frac{1}{T}\int_0^T (V_{YN} + V_{Nn}) I_Y \, dt$$

and
$$W_B = \frac{1}{T}\int_0^T V_{Bn} I_B \, dt = \frac{1}{T}\int_0^T (V_{BN} + V_{Nn}) I_B \, dt$$

Total average power $= W_R + W_Y + W_B$

$$= \frac{1}{T}\int_0^T (V_{RN} I_R + V_{YN} I_Y + V_{BN} I_B) \, dt + \frac{1}{T}\int_0^T V_{Nn} (I_R + I_Y + I_B) \, dt$$

Since the system in the problem is a three wire system, the sum of the three currents I_R, I_Y and I_B at any given instant is zero. Hence the power read by the three wattmeters is given by

$$W_R + W_Y + W_B = \frac{1}{T}\int_0^T (V_{RN} I_R + V_{YN} I_Y + V_{BN} I_B)\,dt$$

If the system has a fourth wire i.e, if the neutral wire is available, then the common point, n is to be connected to the system neutral N. In that case V_{Nn} would be zero, and above equation for power would still be valid. In other words whatever be value of V_{Nn}, the algebraic sum of three currents I_R I_Y & I_B is zero. Hence the term $V_{Nn}(I_R + I_Y + I_B)$ would be zero.

Keeping this advantage in mind, suppose common point 'n' in fig 1.25 is connected to line B. In such case, $V_{Nn} = V_{NB}$; then the voltage across the potential coil of wattmeter W_B will be zero and this wattmeter will read zero. Hence this can be removed from the circuit. The total power is read by the remaining two wattmeters, W_R and W_Y

$\therefore$ Total power $= W_R + W_Y$

Let we verify this fact from fig 1.26

The average power indicated by wattmeter W_R is

$$W_R = \frac{1}{T}\int_0^T V_{RB} I_R \, dt$$

and that by $W_Y = \frac{1}{T}\int_0^T V_{YB} I_Y \, dt$

Also $V_{RB} = V_{RN} + V_{NB}$

$V_{YB} = V_{YN} + V_{NB}$

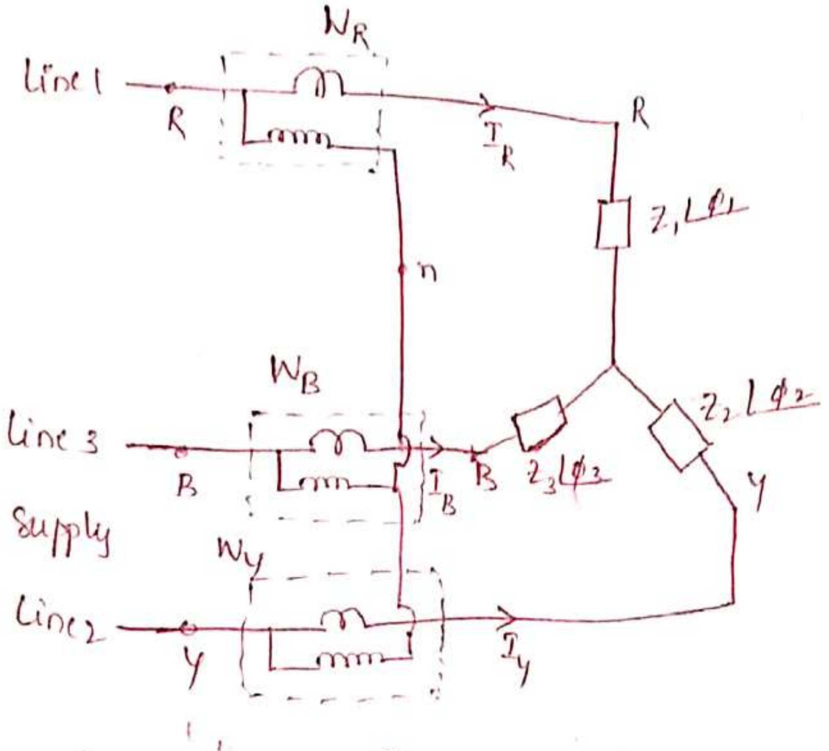


Fig 1.25

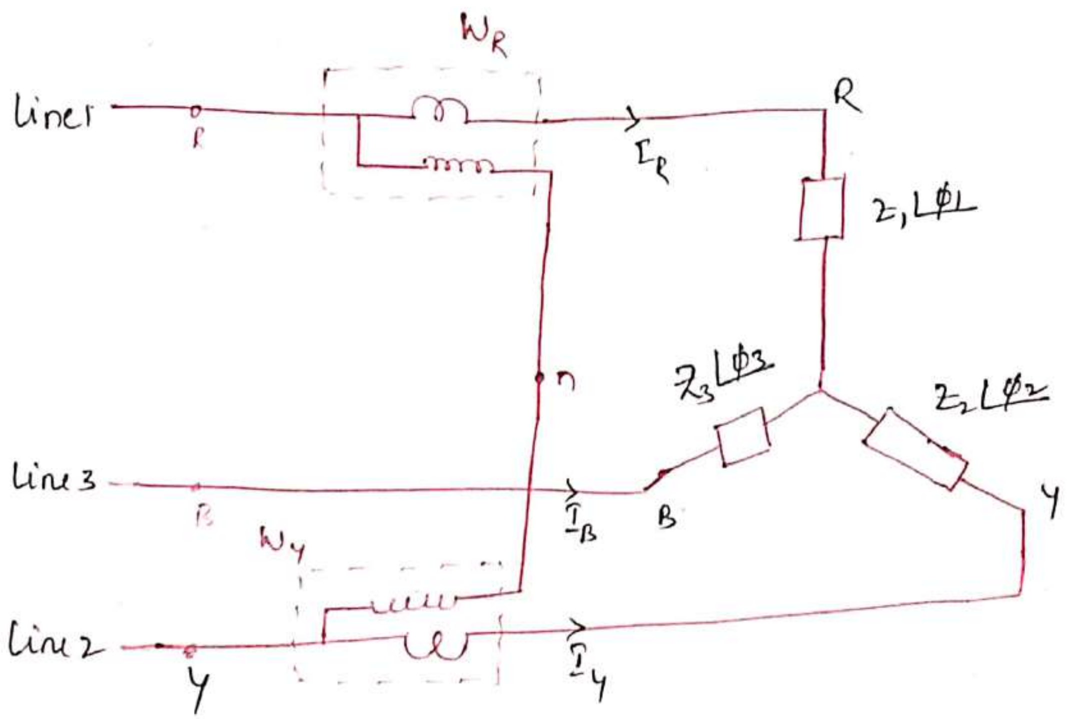


Fig 1.26

$$W_R + W_Y = \frac{1}{T}\int_0^T \left(V_{RB} \times I_R + V_{YB} I_Y\right) dt$$

$$= \frac{1}{T}\int_0^T \left\{(V_{RN} + V_{NB}) I_R + (V_{YN} + V_{NB}) I_Y\right\} dt$$

$$= \frac{1}{T}\int_0^T \left\{(V_{RN} I_R + V_{YN} I_Y) + (I_R + I_Y) V_{NB}\right\} dt$$

We know that $I_R + I_Y + I_B = 0$

$$I_R + I_Y = -I_B$$

Substituting this value in the above equation, we get

$$W_R + W_Y = \frac{1}{T}\int_0^T \{(V_{RN} I_R + V_{YN} I_Y) + (-I_B) V_{NB}\} dt$$

$$V_{NB} = -V_{BN}$$

$$W_R + W_Y = \frac{1}{T}\int_0^T \{V_{RN} I_R + V_{YN} I_Y + V_{BN} I_B\} dt$$

which indicate total power in the load.

The power in a three phase load whether balanced or unbalanced, star connected or delta connected, three wire or four wire can be measured with only two wattmeters. If neutral wire is available in this method it should not carry any current, or the neutral of the load should be isolated from the neutral of the source.

The current flowing through the current coil of each wattmeter is the line current and the voltage across the pressure coil is the line voltage. In case the phase angle between line voltage and current is greater than 90°, the corresponding wattmeter would indicate downscale reading. To obtain upscale reading, the connections of either the current coil or pressure coil has to be interchanged. Reading obtained after reversal of coil connection should be taken as negative. Then the algebraic sum of the two wattmeter readings gives the total power.

Power Factor by Two Wattmeter Method :-

When we talk about the powerfactor in three phase circuits, it implies only to balanced circuits, since the powerfactor in a balanced load is the power factor of any phase. We cannot strictly define the power-factor in three phase unbalanced circuits as every phase has a seperate power factor. The two wattmeter method when applied to measure power in a three phase balanced circuits, provides information that help us to calculate the power factor of the load.

Figure 1.27 shows the vector diagram of the circuit shown in fig 1.26. Since the load is assumed to be balanced, we can take $Z_1\angle\phi_1 = Z_2\angle\phi_2 = Z_3\angle\phi_3 = Z\angle\phi$ for the star connected load. Assuming RYB phase sequence, the three rms load phase voltages are V_{RN}, V_{YN} and V_{BN}. I_R, I_Y and I_B are the rms line (phase) currents. These currents will lag behind their respective phase voltages by an angle ϕ (An inductive load is considered.)

Now consider the readings of the two wattmeters in fig 1.26. W_R measures the product of effective value of the current through its current coil I_R, effective value of the current through its curr coil I_R, effective value of the voltage across its pressure coil V_{RB} and the cosine of the angle between the phasors I_R and V_{RB}. The voltage across the pressure coil of W_R is given as follows

$$V_{RB} = V_{RN} - V_{BN} \quad \text{phasor difference}$$

It is clear from the phasor diagram that the phase angle between V_{RB} & I_R is $(30 - \phi)$

$$\therefore W_R = V_{RB} \times I_R \cos(30 - \phi)$$

Similarly W_y measures the product of effective value of current through its current coil I_y, the effective value of the voltage across its pressure coil, V_{YB} and the cosine of angle between the phasors V_{YB} & I_y

$$V_{YB} = V_{YN} - V_{BN}$$

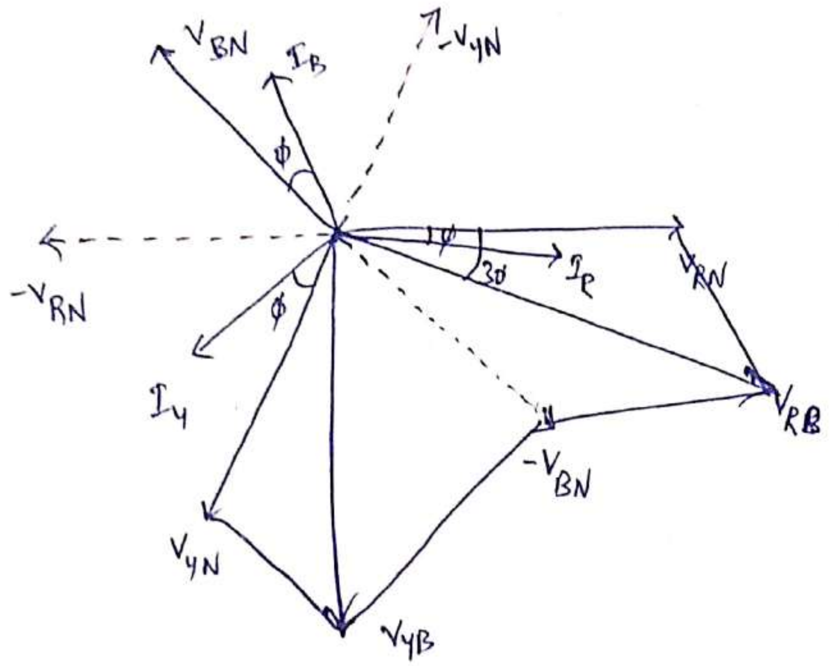


Fig 1.27

From fig 1.27 it is clear that the phase angle between V_{YB} & I_y is $(30° + \phi)$

$$W_y = V_{YB} \times I_y \cos(30° + \phi)$$

since load is balanced, the line voltage $V_{RB} = V_{YB} = V_L$ and line current $I_R = I_Y = I_L$

$$W_R = V_L I_L \cos(30° - \phi)$$

$$W_Y = V_L I_L \cos(30° + \phi)$$

Adding W_R & W_Y gives total power in the circuit, thus

$$W_R + W_Y = \sqrt{3} V_L I_L \cos\phi$$

from the two wattmeter readings, it is clear that for the same load angle ϕ, wattmeter W_R registers more power when the load is inductive. It is also connected in the leading phase as the phase sequence is RYB. Therefore W_R is higher reading wattmeter in the circuit of fig 1.26. In other words if the load is capacitive, the wattmeter connected in leading phase reads less for the same load angle. So, if we know the nature of the load, we can easily identify the phase sequence of the system. The higher reading wattmeter always reads positive. By proper manipulation of two wattmeter readings, we can obtain the power factor of the load.

$$W_R = V_L I_L \cos(30° - \phi) \quad \text{(Higher reading)}$$

$$W_Y = V_L I_L \cos(30° + \phi) \quad \text{(Lower reading)}$$

$$W_R + W_Y = \sqrt{3} V_L I_L \cos\phi$$

$$W_R - W_Y = V_L I_L \sin\phi$$

Taking the ratio of the above two values, we get

$$\frac{W_R - W_Y}{W_R + W_Y} = \frac{\tan\phi}{\sqrt{3}}$$

$$\tan\phi = \sqrt{3}\left[\frac{W_R - W_Y}{W_R + W_Y}\right]$$

$$\phi = \tan^{-1}\sqrt{3}\left[\frac{W_R - W_Y}{W_R + W_Y}\right]$$

Thereafter $\cos\phi$ is found.

Variation in Wattmeter Readings with Load Power Factor:-

It is useful to study the effect of the p.f on the readings of the wattmeter. The readings of the two wattmeter depend on the load power factor angle ϕ, such that

$$W_R = V_L I_L \cos(30-\phi)^\circ$$
$$W_Y = V_L I_L \cos(30+\phi)^\circ$$

We can therefore, make the following deductions:

(i) When ϕ is zero i,e., power factor is unity from the above expressions we can conclude that the two wattmeter indicate equal and positive values.

(ii) When ϕ rises from 0 to 60° i,e., upto power factor 0.5, wattmeter W_R reads positive (since it is connected in the leading phase); where as wattmeter W_Y reads positive, but less than W_R. When $\phi = 60°$, $W_Y = 0$ and the total power is being measured ~~by~~ only by wattmeter W_R.

(iii) If the power factor is further reduced from 0.5 i,e., when ϕ is greater than 60°, W_R indicates positive value whereas

W_Y reads downscale reading in such case. The connections of either the current coil or pressure coil of the corresponding wattmeter have to be interchanged to obtain an upscale reading and the reading thus obtained must be given a negative sign. The total power in the circuit would be $W_R + (-W_Y) = W_R - W_Y$.

Wattmeter W_Y reads downscale for the phase angle between 60° and 90°. When the power factor is zero (i.e., $\phi = 90^\circ$) the two wattmeters will read equal & opposite values.

$$\text{i.e.,}\quad W_R = V_L I_L \cos(30-90)^\circ = 0.5 V_L I_L$$

$$W_Y = V_L I_L \cos(30+90)^\circ = -0.5 V_L I_L$$

Leading Power Factor Load :-

If the load is capacitive, the wattmeter connected in leading phase would read less value. In that case, W_R will be the lower reading wattmeter and W_Y will be the higher reading wattmeter. Fig 1.28 shows the phasor diagram for leading p.f.

As the power factor is leading, the phase currents I_R, I_Y & I_B are leading their respective phase voltage by an angle ϕ. From the fig 1.28, the reading of the wattmeter connected in the leading phase is given by

$$W_R = V_{RB} \times I_R \cos(30+\phi)^\circ$$

$$= V_L I_L \cos(30+\phi)^\circ \text{ (lower reading wattmeter)}$$

Similarly the reading of wattmeter Connected in the lagging phase is given by

$$W_Y = V_{YB} I_Y \cos(30-\phi)^\circ$$

$$= V_L I_L \cos(30-\phi)^\circ \text{ (higher reading wattmeter)}$$

Again the total power is given by

$$W_R + W_Y = \sqrt{3} V_L I_L \cos\phi$$

$$W_Y - W_R = V_L I_L \sin\phi$$

Hence $$\tan\phi = \frac{\sqrt{3}[W_Y - W_R]}{W_Y + W_R}$$

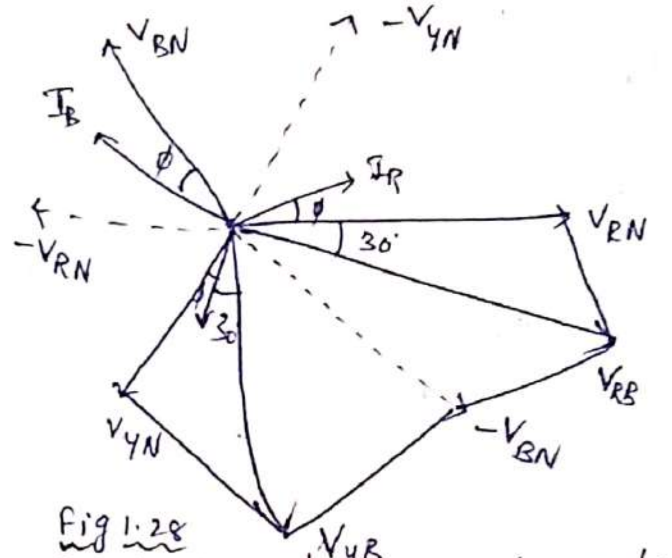


Fig 1.28

A comparision of this expression with that of lagging p.f reveals the fact that the two wattmeter readings are interchanged i.e., for lagging p.f, W_R is higher reading wattmeter & W_Y is the lower reading wattmeter whereas for leading p.f W_R is lower reading wattmeter & W_Y is higher reading wattmeter. While using the expression for p.f, whatever may be the nature of the load, the lower reading is to be subtracted from the higher reading in the numerator. The variation in the wattmeter reading with the capacitive load follows the same sequence as in inductive load, with the change in the roles of wattmeters.

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Reactive Power with Wattmeter :-

We have seen that the difference between higher reading wattmeter and lower reading wattmeter yields $V_L I_L \sin\phi$. So the total reactive power $= \sqrt{3} V_L I_L \sin\phi$. Reactive power in a balanced three phase load can be calculated by using a single wattmeter.

As shown in figure(a) the current coil of wattmeter is connected in any one line (R in this case) and pressure coil across the other two lines (between Y and B in this case). Assuming phase sequence RYB and an inductive load of angle ϕ, the phasor diagram for the circuit is shown in fig (b)

From the fig (a) it is clear that the wattmeter power is proportional to the product of current through its current coil, I_R voltage across its pressure coil, V_{YB} and cosine of angle between V_{YB} and I_R

$$V_{YB} = V_{YN} - V_{BN} = V_L$$

From the vector diagram the angle between V_{YB} and I_R is $(90 - \phi)°$

∴ wattmeter reading $= V_{YB} I_R \cos(90-\phi)$

$= V_L I_L \sin\phi$ VAR

If the above expression is multiplied by $\sqrt{3}$, we get the total reactive power in the load.

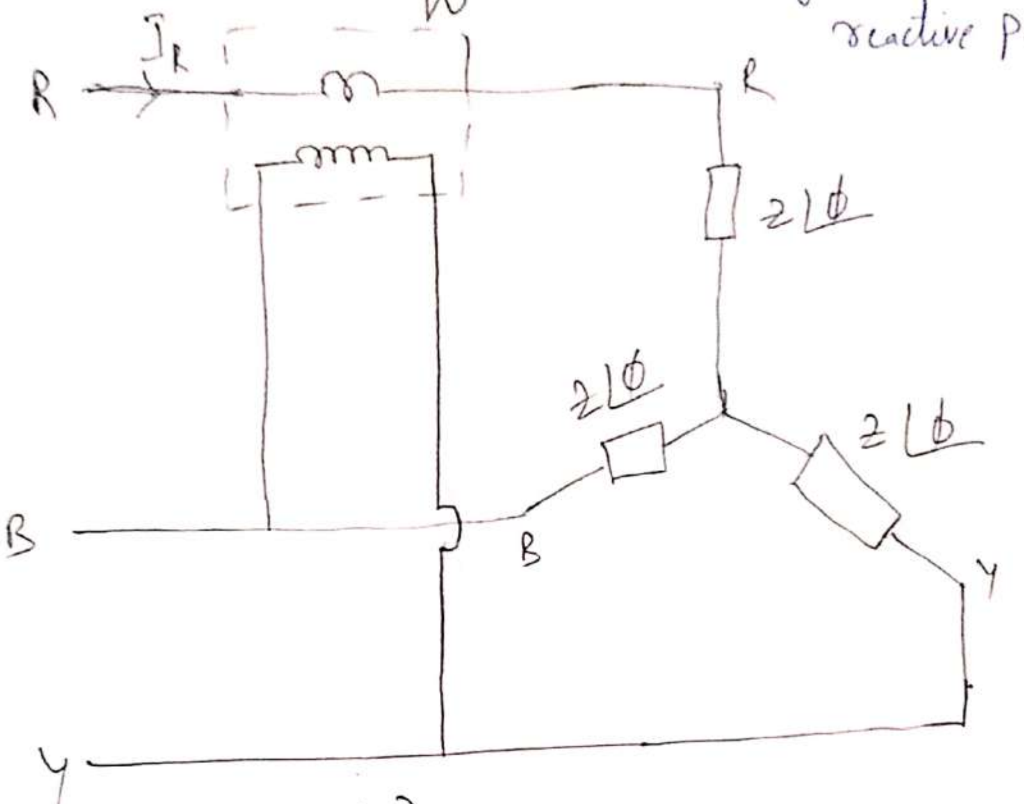


Fig (a)

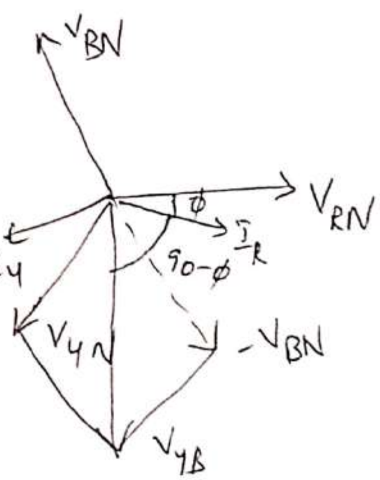


Fig (b)

Problems:-

→ A 3 phase load has a resistance of 10Ω in each phase and is connected in

(i) Star (ii) Delta against a 400V, 3 phase supply

Compare the power consumed in both the cases.

Sol:

Given that

$$R_{ph} = 10\Omega = Z_{ph}$$

$$V_L = 400V$$

(i) Star Connection

$$V_L = 400V, \quad V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{400}{\sqrt{3}} = 231V$$

$$I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{231}{10} = 23.1A$$

Power Consumed $P = 3 V_{ph} I_{ph} \cos\phi = 3 \times 231 \times 23.1 \times 1$

$$= 16KW$$

(ii) Delta Connection

$$V_L = V_{ph} = 400V, \quad I_L = \frac{I_{ph}}{\sqrt{3}}$$

$$I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{400}{10} = 40A$$

$$I_L = \frac{40}{\sqrt{3}}$$

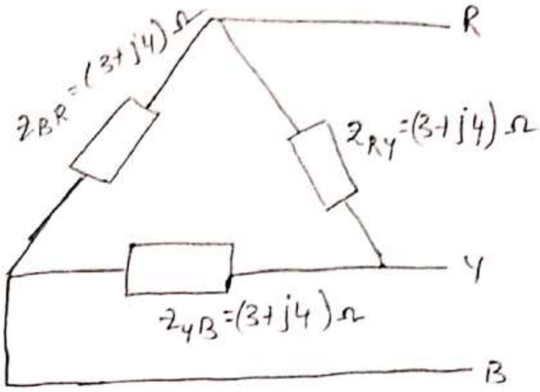
Power consumed $= 3 V_{ph} I_{ph} \cos\phi = 3 \times 400 \times 40 \times 1$

$$= 48KW$$

$$P_{delta} = 3 \times P_{star}$$

→ Three identical impedances of (3+j4) Ω are connected in delta. Find an equivalent star network such that the line current is the same when connected to the same supply

Sol:



$Z = (3+j4)\,\Omega$

$$Z_R = \frac{Z_{RY} \times Z_{BR}}{Z_{RY}+Z_{YB}+Z_{BR}} = \frac{(3+j4)(3+j4)}{9+j12}$$

$= (1+j1.33)\,\Omega = 1.66\angle 53.13^\circ\ \Omega$

$Z_Y = 1.66\angle 53.13\ \Omega$

$Z_B = 1.66\angle 53.13^\circ\ \Omega$

→ A star connected three phase load has a resistance of 8 Ω and a capacitive reactance of 10 Ω in each phase. It is fed from a 400V, 3-ϕ balanced supply.

(i) Find the line current total volt-amperes, active and reactive power

(ii) Draw phasor diagram showing phase voltages, line voltages and currents.

Sol: Given that

$R = 8\,\Omega$

$X_C = 10\,\Omega$

$V_L = 400V$

$Z = R - jX_C$

$= 8 - j10 = 12.8\angle -51.34^\circ$

$I_L = ?$

$S = ?$

$P = ?$

$Q = ?$

$$V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{400}{\sqrt{3}} = 230.94V$$

$V_{RN} = 230.94\angle 0^\circ$, $V_{YN} = 230.94\angle -120^\circ$, $V_{BN} = 230.94\angle -240^\circ$

5.25

$$I_R = \frac{V_{RN}}{Z} = \frac{230.94}{12.8\angle -51.34} = 18.042\angle 51.34 \text{ A}$$

$$I_Y = \frac{V_{YN}}{Z} = \frac{230.94\angle -120^\circ}{12.8\angle -51.34} = 18.042\angle -68.66 \text{ A}$$

$$I_B = \frac{V_{BN}}{Z} = \frac{230.94\angle -240^\circ}{12.8\angle -51.34} = 18.042\angle -188.66 \text{ A}$$

Total volt-amperes $S = 3V_{ph}I_{ph}$

$= 3 \times 230.94 \times 18.042$

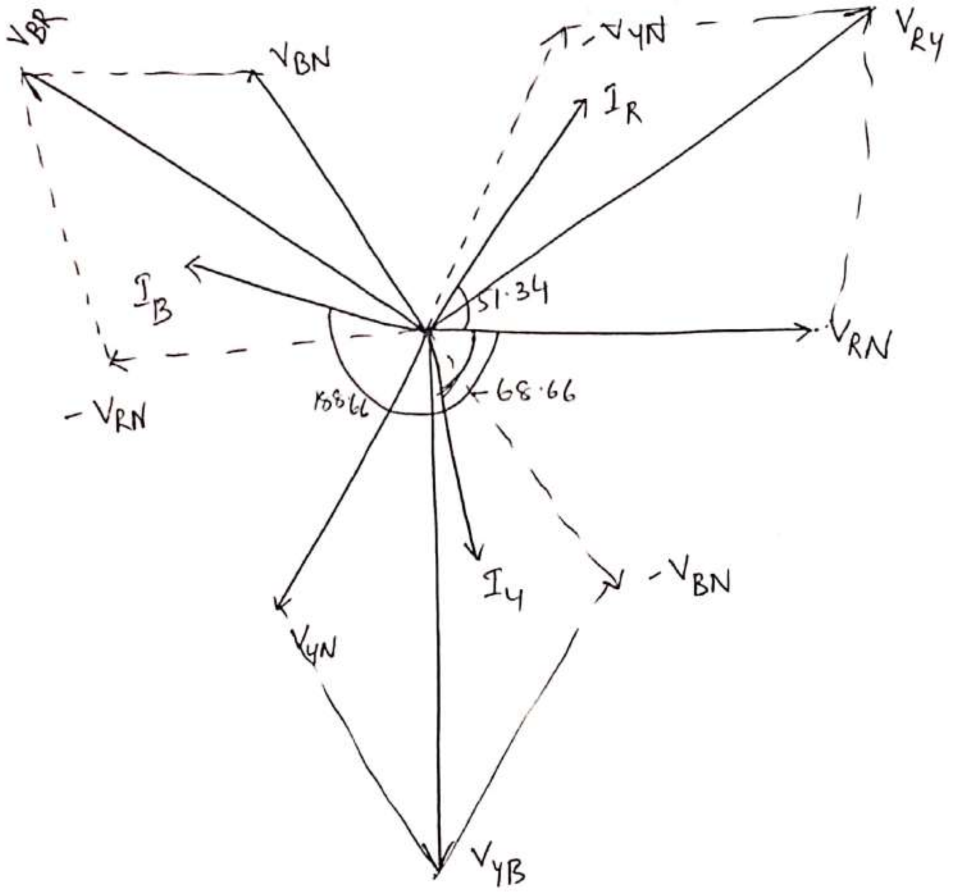
$= 12499.98 \text{ VA}$

$P = 3V_{ph}I_{ph}\cos\phi = 3 \times 230.94 \times 18.042 \cos(51.34)$

$= 7808.63 \text{ W}$

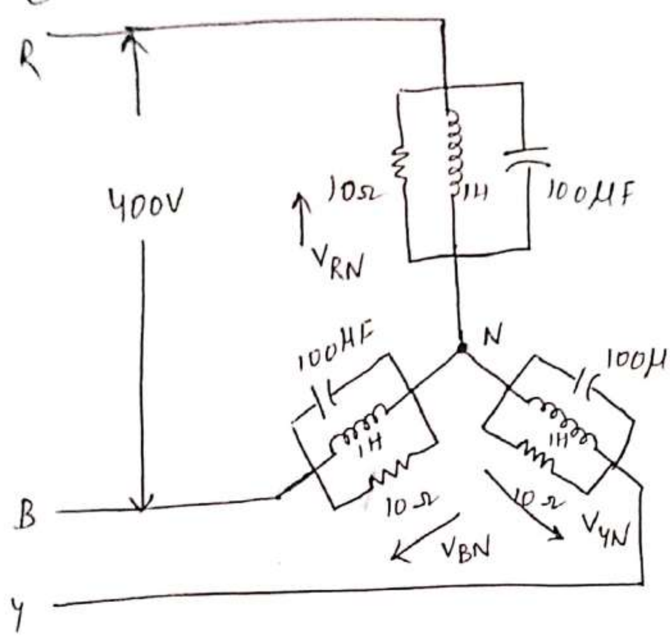
$Q = 3V_{ph}I_{ph}\sin\phi = 3 \times 230.94 \times 18.042 \sin(51.34)$

$= 9760.72 \text{ VAR}$



→ A balanced 3-phase system supplied from 400V, 50Hz supply has R, L & C elements connected parallel in each phase. The values of R, L & C are 10Ω, 1H & 100μF respectively. Calculate the line current, the power & power factor

Sol:



$V_L = 400V$

$f = 50Hz$

$V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{400}{\sqrt{3}} = 230.94V$

$Y_{ph} = \frac{1}{10} + \frac{1}{j\omega L} + j\omega C$

$= 0.1 + \frac{1}{j2\pi f L} + j2\pi f C$

$= 0.1 + \left(-j\frac{1}{2\pi \times 50 \times 1}\right) + j(2\pi \times 50 \times 100 \times 10^{-6})$

$= 0.1 + j0.0282 = 0.1039\angle 15.75\,Ω$

$V_{RN} = 230.94\angle 0^\circ$ V

$V_{YN} = 230.94\angle -120^\circ$ V

$V_{BN} = 230.94\angle -240^\circ$ V

$Z_{ph} = \frac{1}{Y_{ph}} = \frac{1}{0.1039\angle 15.75} = 9.62\angle -15.75\ Ω$

In star $I_L = I_{ph}$

$I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{230.94\angle 0^\circ}{9.62\angle -15.75} = 24\angle 15.75$ A

power $= 3 V_{ph} I_{ph} \cos\phi = 3 \times 230.94 \times 24 \cos(15.75)$

$= 16KW$

Power factor $= \cos\phi = \cos(15.75) = 0.96$ lagging